

Spontaneous and Exponential Decay



Projects

The exercises here are intended to provide opportunities for you to work through the scientific and computational thinking associated with the mathematical modeling of spontaneous and exponential decay. Most exercises can be done in either Excel, VenSim or Python.

1. Write your own simulation of the process of radioactive decay as exponential decay. You may also use one of the supplied ones.
 - a. Generate data for the number of nuclei in each generation as a function of time, $N(t)$.
 - b. *Data will look similar to Table 2 in Student Reading.*
 - c. *What do you need to create the simulation? What are the components (the thing that changes) and the parameters (rates of change or process of change) to include?*

2. Using the graphing function of your simulator or using excel, plot the data as the natural logarithm of the number $\ln N(t)$ versus time t (as shown in Figure 4) .
 - a. Describe the characteristics of the plotted data.
 - b. You can either evaluate the logarithm or use a semi logarithmic plot option.
 - c. Hint: An exponential function would be a straight line on this plot, whereas your data should show fluctuations about a straight line.

Verifying your model

3. Vary the initial value $N(0)$ for the number of nuclei, include a very large number, watch how $N(t)$ changes.
 - a. What happens when you use a very large number of nuclei? Describe the characteristics of the plotted data.
 - b. Hint: The $N(t)$ initially falls on a straight line.
 - c. Note: Eventually the probabilistic nature of the simulation becomes evident as $N(t)$ decays to a small value.

4. Run a number of simulations with fixed values of the rate parameter λ , but with different values for the initial number of nuclei $N(0)$. Plot all the different results on the same graph.
 - a. Verify that average slopes of $\ln N(t)$ versus time t are independent of $N(0)$.
 - i. *Hint: Does $N(0)$ affect the slopes? What are the slopes for each plotted set of data?*

5. Vary the rate parameter λ . Run a number of simulations with fixed values of the initial number of nuclei $N(0)$, but with different values for the rate parameter λ .

- a. Plot all the different results on the same graph.
 - b. Are the average slopes of $\ln N(t)$ versus time t proportional to λ ?
 - i. What tells you that they are proportional?
6. (Optional**) The activity (decay rate) $\Delta N/\Delta t$ should also decay exponentially as a function of time. Use your simulation to record the number of decays ΔN that occur in consecutive generations. Verify that a plot of $\ln(\Delta N)$ versus time t has the same average slope as a plot of $\ln N(t)$ versus time (because $\Delta t = 1$ in all our simulations, we do not have to divide ΔN by it) . Note, since ΔN is a small number compared to N , there will be larger statistical fluctuations evident in the activity plot than in the plot of $N(t)$.
7. Explain in your own words how a process that is spontaneous and random at its very heart can lead to exponential decay.
8. Fit an exponential function to the data in Table 1 and thereby deduce value for λ and $\tau_{1/2}$.

Table 1. The form of some typical decay data.					
Generation j	Time Period (seconds)	Δt (seconds)	Number of Parents N_j	Activity ($\Delta N/\Delta t$)	Change ΔN
0	0-0		100		
1	0-1	1	65	35	35
2	1-3	2	43	11	22
3	3-8	5	39	0.8	4