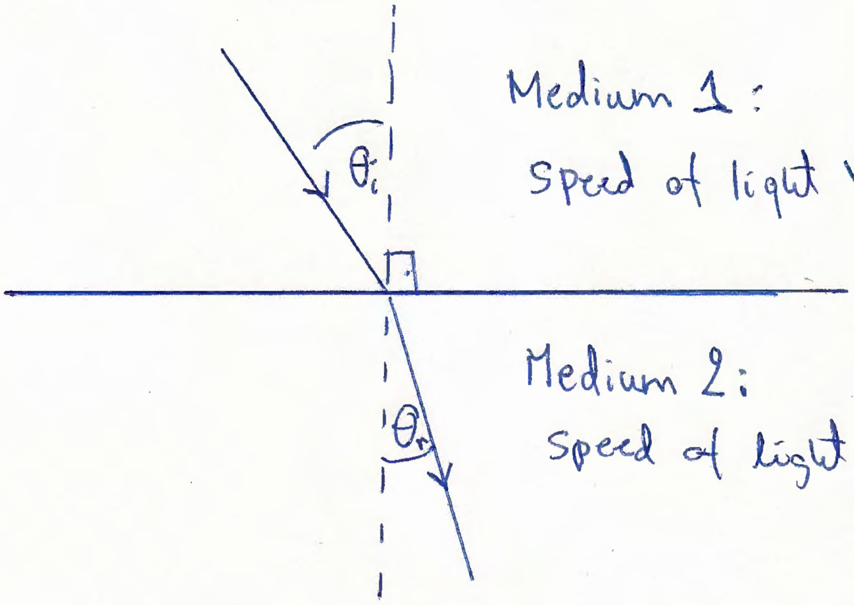
Refraction: review of the fundamentals

1



Medium 1:
Speed of light V_1

Medium 2:
Speed of light V_2

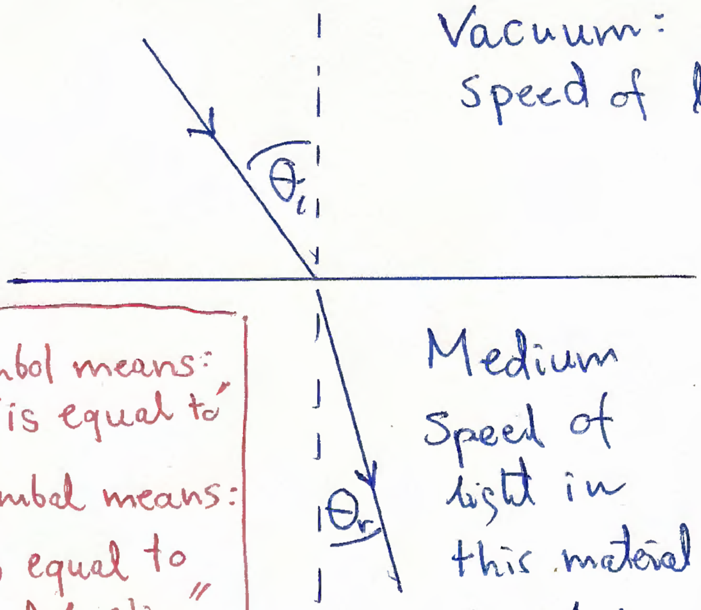
Snell Law: (Equation 1)

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{V_1}{V_2}$$

Why is it so? We will explain when we get to the so-called "Huyghens Principle"

2

If the light is incident from vacuum:



Vacuum:
Speed of light $= c \cong 300{,}000$ km/s

Then Snell Law is:

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{c}{V_{mat}} \quad \text{(Eq. 2)}$$

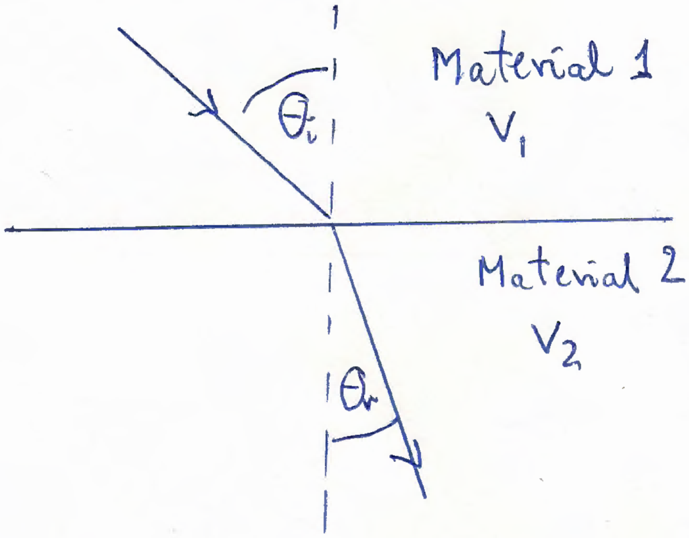
Medium
Speed of light in this material is V_{mat}

$$\boxed{\frac{c}{V_{mat}} \equiv n_{mat}} \quad \text{(Eq. 3)}$$

n_{mat} is called "the index of refraction" of the material

The $=$ symbol means: "is equal to"
The $\equiv$ symbol means: "is equal to by definition"

3 Return to the two-media situation:



$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{V_1}{V_2} \quad \text{Eq. 1, repeated}$$

But from the definition of the index of refraction (a.k.a. "refractive indx") we get:

$$\frac{c}{V_{mat}} \equiv n_{mat} \Rightarrow V_{mat} = \frac{c}{n_{mat}} \quad (\text{Eq. 4})$$

Plugging it into Equation 1, we obtain:

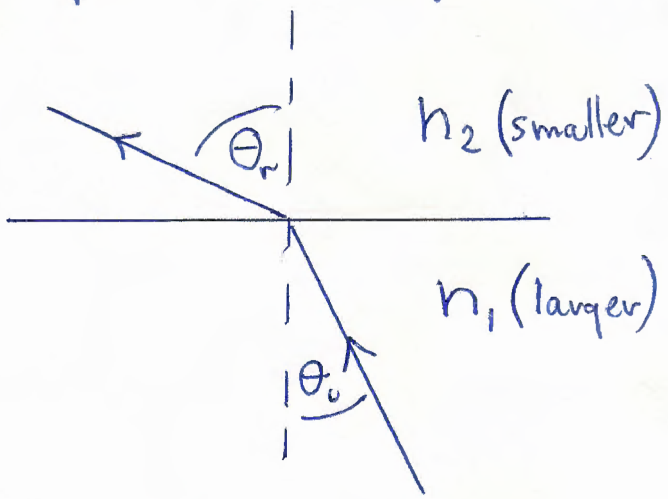
$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1} \quad (\text{Eq. 5})$$

Is this consistent with Eq. 2? Sure! Because the refractive index of vaccum is $n_{vac} = \frac{c}{c}$ (from Eq. 3) so $n_{vac} = 1$.

4. "Optical density": if in material A the speed of light is higher than in material B, we say that "B is optically <u>denser</u> than A".

In other words, it is n, the refraction index, that is the measure of the "optical density": the higher the n value is, the <u>denser</u> the material is

5. Now, let's consider a very important situation: Light passes from a material with higher "optical density" into a material with <u>lower</u> "optical density:



From Eq. 5:

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_2}{n_1}$$

So:

$$\boxed{\sin\theta_r = \frac{n_1}{n_2}\cdot\sin\theta_i} \quad \text{Eq. 6}$$

Note that n_1/n_2 is a number greater than 1!
So, if we keep increasing θ_i, $\sin\theta_r$ grows <u>faster</u> than $\sin\theta_i$!

At some moment, the θ_r angle becomes $90°$, when $\sin\theta_r$ reaches the value of 1.

Note that one is the maximum value of the sine function

The θ_i value, for which $\theta_r = 90°$, is called "the critical angle". Since $\sin 90° = 1$ from Eq. 6 we get $1 = \frac{n_1}{n_2}\sin\theta_{i\,critical}$,

so that:

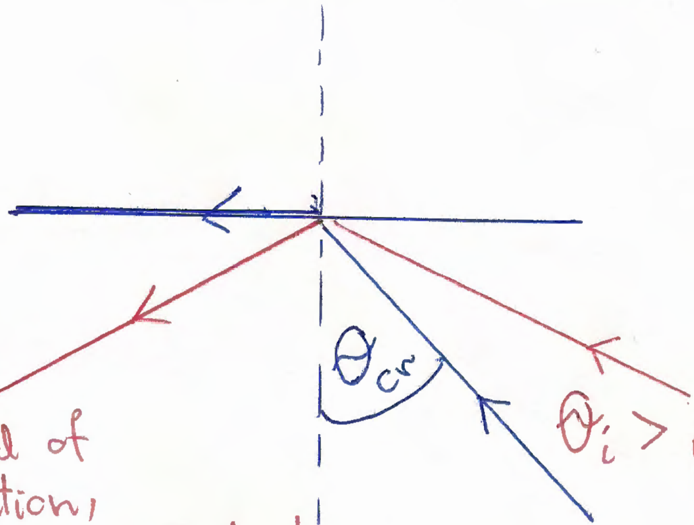
$$\boxed{\sin\theta_{i\,critical} = \frac{n_2}{n_1}} \qquad \text{Eq. 7}$$

Let's rewrite it in a "totally foolproof" form:

$$\boxed{\sin\theta_{i\,critical} = \frac{n_{\text{of the material the light goes into}}}{n_{\text{of the material the light comes from}}}} \qquad \text{Eq. 7a}$$

For θ_i greater than the critical angle value, the Snell Law is no longer valid !!! Why?

Because there is no longer a refracted ray!



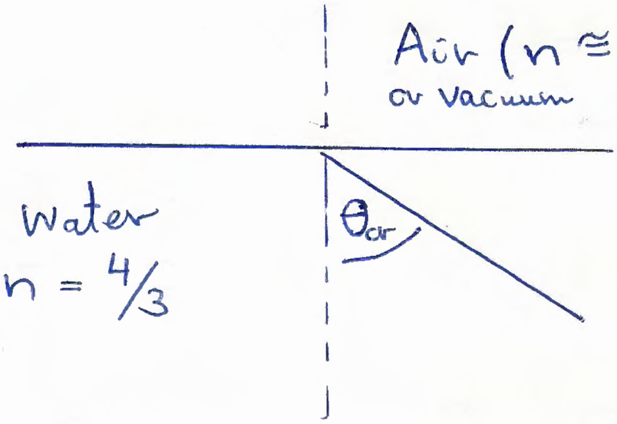
Instead of refraction, symmetric reflection!

This situation is called: "total internal reflection".

Why total? Because exactly 100% of light intensity is reflected

Why internal? Because light does not exit the material it comes from, it stays inside.

In particular: if the "less dense" medium is vacuum (or air) – e.g., light exits water into air



From Eq. 7a:

$$\sin\theta_{cr} = \frac{1}{n_{\text{of the medium}}}$$

Example:

$$(\sin\theta_{cr})_{H_2O} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

From which it follows that $(\theta_{cr})_{H_2O} = 48.6°$