Mirrors and Lenses

CHAPTER 3

3.1 INTRODUCTION

In this chapter we want to apply the principles of reflection and refraction to situations of man's own creation. Here a central problem is to put together optical systems that deliver clear, undistorted *images*. The mirror and lenses used in such systems are most economically and simply manufactured if their surfaces have *spherical* shape, that is, the same shape as a part of an appropriately sized sphere. Therefore, we will mainly discuss such spherical mirrors and lenses.

To find the image in a particular mirror or lens of a point on an object, we must find out what happens to the light rays from this object point that strike the mirror or lens. This procedure is complicated because we have to apply the law of reflection or refraction to each of these rays. Fortunately, there are a few special rays whose reflections or refractions obey simple rules. The process of ***ray tracing*** uses these rays to find the image. Once we have located the image by this process it is an easy matter to find the path of any other ray. We'll illustrate this approach with the simple and familiar case of the flat ***plane mirror***, and then go on to the trickier (and more interesting) case of curved surfaces.

3.2 VIRTUAL IMAGES

When a light ray from an object is reflected by a plane mirror and reaches your eye, your eye and brain make no allowance for this reflection; rather your brain traces rays back in straight, unbroken lines to the point they *seem* to come from. Thus, when you look into a plane mirror, the light seems to be coming from an image *behind* the mirror (Fig. 2.19). (A young kitten, when exposed to a mirror for the first time, will often try to go around behind the mirror in order the get at the "other" kitten.) That is, in the region in front of the mirror, the light behaves exactly as if it were coming from an actual object located at the position of the image (Fig. 3.1). How can we locate the image, using the law of reflection? Let's trace a few rays.

A. Locating the image

Consider an object (the arrow PQ in Fig. 3.2) and an observer's eye, E_1, both in front of a mirror. Light shining on the arrow tip, Q, will be scattered in all directions from it. We can, therefore, consider point Q as a source of light. Consider one ray, QA, leaving the tip. By applying the law of reflection at point A, making the angle of reflection equal to the angle of incidence, we can determine the direction in which the ray travels after it is reflected. The reflected ray happens to go to the eye E_1. (Had we chosen a ray going to a different point on the mirror, say D, the reflected ray would go someplace else.) Where does the eye E_1 see the image of Q? As the eye

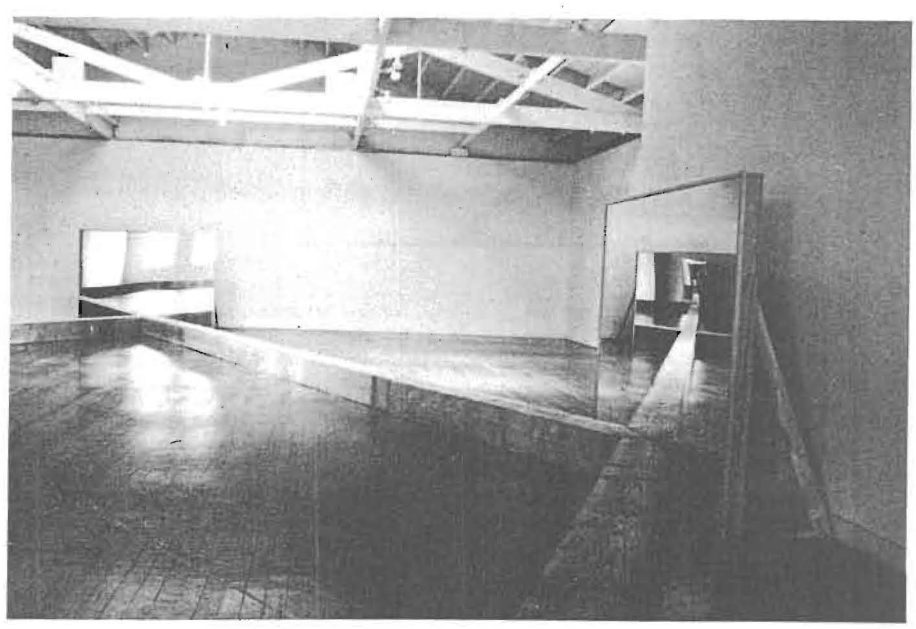
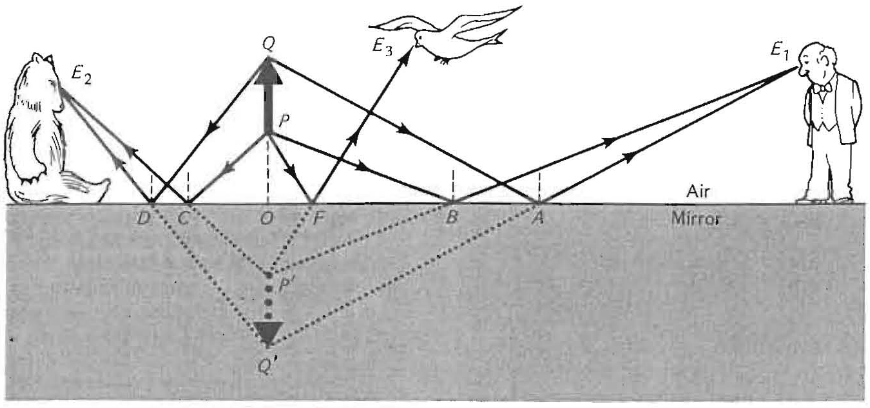


FIGURE 3.1

Where does the object stop and the virtual image begin? Or, ray tracing done with wooden beams. "Untitled," Robert Morris.



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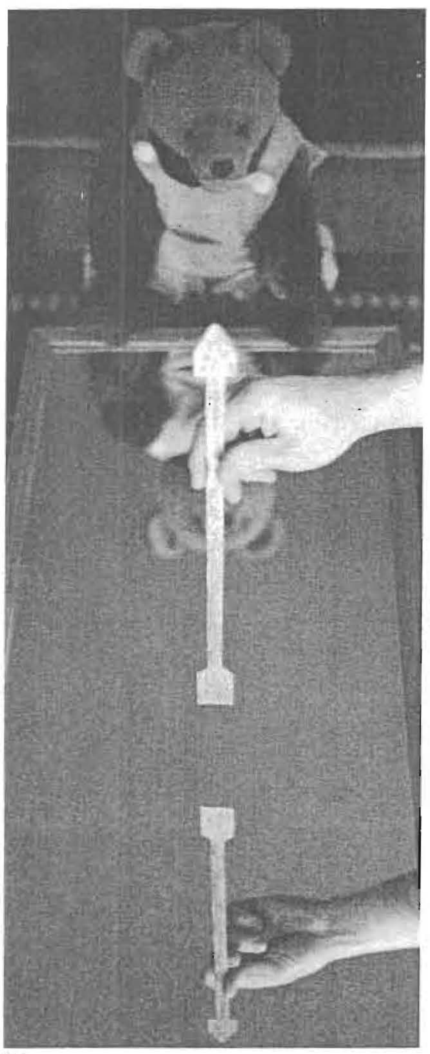


(b)

FIGURE 3.2

(a) Rays from an object that reach three different eyes can be used to locate the image. **(b)** Photograph taken from the position of eye E_3.

assumes that the light reaching it has always traveled in a straight line, it concludes that the source of the ray, AE_1, must be somewhere along the straight line, AE_1 extended. So far we cannot tell where along that line the image lies. If we repeat the construction, however, for a second eye (or the same eye moved), say E_2, we can construct ray QDE_2. Where do the *two* eyes see the image of the arrow's tip? E_1 sees it as lying somewhere along the straight line, AE_1 extended. E_2 sees it as lying somewhere along the straight line, DE_2 extended. The only point that lies on both these straight lines, and on which both eyes can agree, is their intersection Q'. The image of the arrow tip, Q, then must lie at the point Q'. It is, in fact, as far behind the mirror as Q is in front of it. (The distance from Q' to O is the same as from Q to O.)

In the same manner, we see that the image of the tail of the arrow, P, lies at P'. (Note that we didn't *have* to find rays that, after reflection, went to E_1 and E_2. Any pair of rays would do.) We have now located the image of the entire arrow. It lies between P' and Q'. A third eye, E_3, will also see the image of the tail at P' (try the construction). In fact, an eye at *any* position will agree that the image is at $P'Q'$—we have a true image. Knowing this, we need not bother with the law of reflection in order to determine how a ray gets to E_3. We simply draw a straight line from the image point, P', to E_3 (intersecting the mirror at F). Light then actually travels in a straight line from P to F, and again in a straight line from F to E_3. The law of reflection is then automatically obeyed. The light rays always remain in the air, but they *appear* to come from the image $P'Q'$ behind the mirror. Such an image is called a ***virtual image,*** because the light doesn't really come from it, it only appears to do so. A virtual image is found by extending actual light rays back to it. There is no light at the point Q'—it might be embedded in a brick wall on which the mirror hangs. Nevertheless, it looks to all eyes as if Q' were the source of light.

You can convince yourself that your image is behind your bathroom mirror (rather than *on* it) by letting the mirror become steamed up and then tracing the outline of your head on the mirror with your finger. You will notice that it is only half the actual size of your head.

PONDER

Why does this mean that the image is behind the mirror, twice as far as the mirror is from you? (Notice that if you stand farther from the mirror, your image will just fit into the same traced outline, even though it now looks smaller.)

If you remain skeptical, you can measure the distance to your image if you have a camera with a range finder or focusing device. (Another way is described in the TRY IT.)

TRY IT

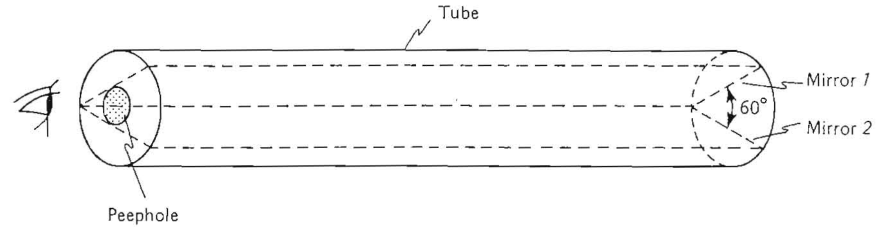
FOR SECTION 3.2A
Locating the virtual image

This way to locate the position of the virtual image produced by a plane mirror requires a small mirror and two identical pencils. Stand the mirror vertically on a table top. Stand one of the pencils upright, a few inches in front of the mirror. The pencil should stick up higher than the mirror, so its image appears cut off. Now position the second pencil behind the mirror so that it appears to be *an extension of the image of the first pencil. Move your eye from side to side to make sure that the second pencil and the image of the first pencil look like one continuous pencil, no matter where your eye is. The second pencil is now at the position of the virtual image of the first pencil, and you can measure its distance from the mirror. Compare that to the distance from the first pencil to the mirror. (If you use a thick glass mirror, remember that the reflecting surface is the back surface of the glass.)*

*B. Kaleidoscopes

As an example of a plethora of virtual images, let's consider the ***kaleidoscope**** (Fig. 3.3a). This is a long tube containing two mirrors running the length of the tube and set at an angle to each other. You look through a peephole between the mirrors at objects at the other end. If the angle between the mir-

*Greek *kalos*, beautiful, plus *eidos*, form, plus *skopeo*, see. Invented by David Brewster.



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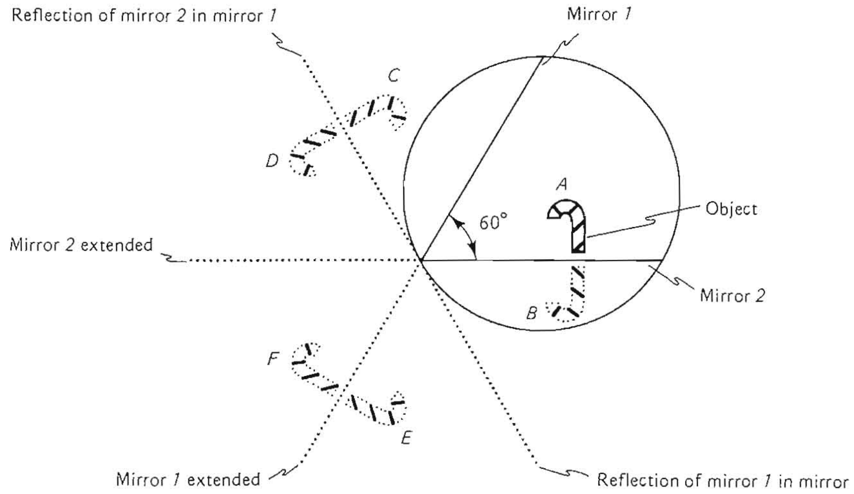

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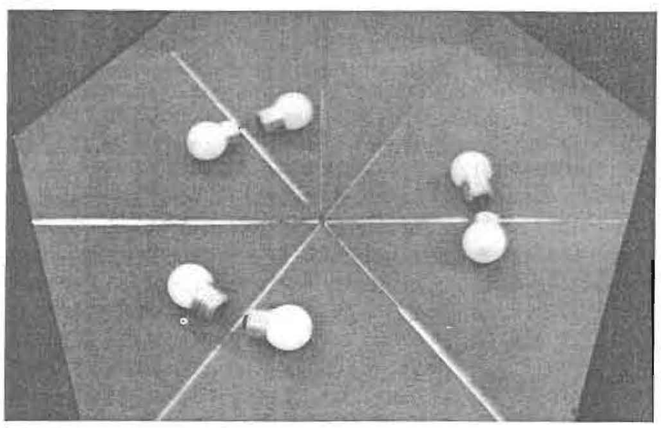

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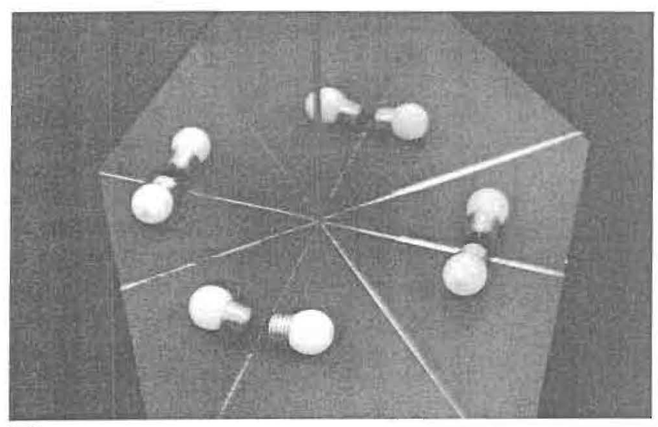

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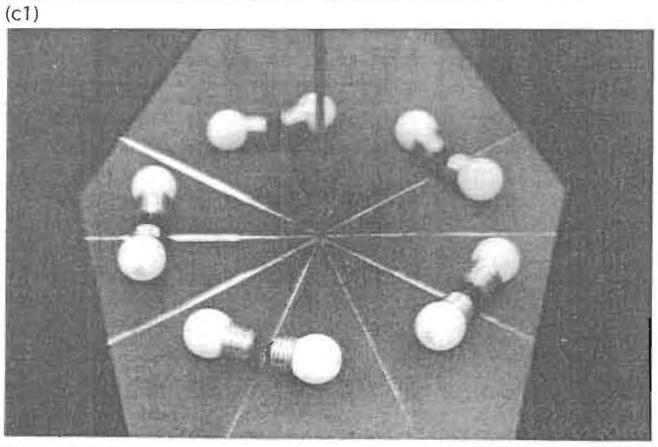

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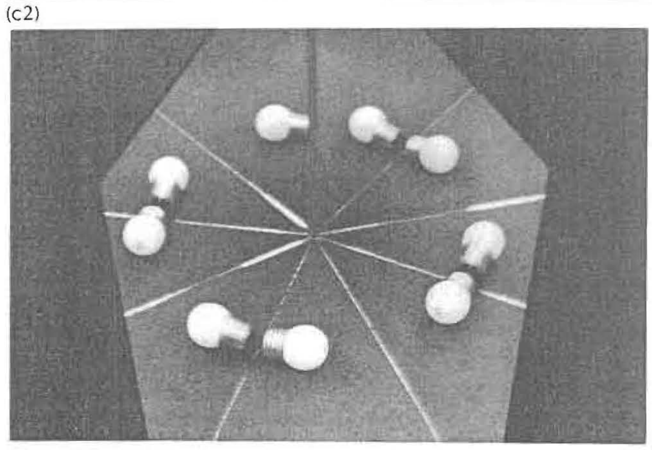


(c4)

FIGURE 3.3

(a) Construction of a kaleidoscope. **(b)** A kaleidoscope view. An object, *A*, is placed between the two mirrors. You see the object directly. The object is also reflected in mirror *2*, so you also see a virtual image at *B*. As this is an image, the light appears to come from *B* just as if there were an object there. Hence both the object, *A*, and the image, *B*, are reflected in mirror *1* and their virtual images are located at C and *D*. (*D* is an image of an image.) Thus, looking in the peephole between the mirrors, you see what appear to be objects at C and *D* (outside the actual tube). Again, these are reflected in mirror *2*, and the new virtual images occur at *E* and *F*. The final pattern observed is then the object, *A*, and the five virtual images, *B* to *F*, arrayed in the threefold symmetric pattern shown. **(c)** Photograph of object reflected in two mirrors: (1) mirrors at 60°, (2) mirrors at 45°, (3) mirrors at 36°, and (4) mirrors at 50°, a nonintegral fraction of 360°.

rors is 60°, 45°, or 30° (or any other angle that goes into 360° an integral number of times) you see a symmetric array of these virtual images. For 60° you see a hexagonal field of view with a pattern of threefold symmetry, consisting of the object and five images (Fig. 3.3b). Figure 3.3c shows how some of these images come about.

If the mirrors are reasonably reflective, the pattern can be quite good, making it difficult to distinguish the original object from the images. When glass mirrors are used, they are usually front-surface mirrors to avoid the extra reflection one normally gets from ordinary glass mirrors, particularly at grazing angles. This is important in a long kaleidoscope, where most of the reflections are at grazing angles.

These toys are supposed to have been used by the weavers of Lancashire to inspire new design patterns for their cloths. They are often used in light shows to project interesting patterns. Walk-in kaleidoscopes (Fig. 3.4) are a source of amusement in fun houses. They are, truly, devices to "multiply variety in a wilderness of mirrors" (T. S. Eliot, "Gerontion").

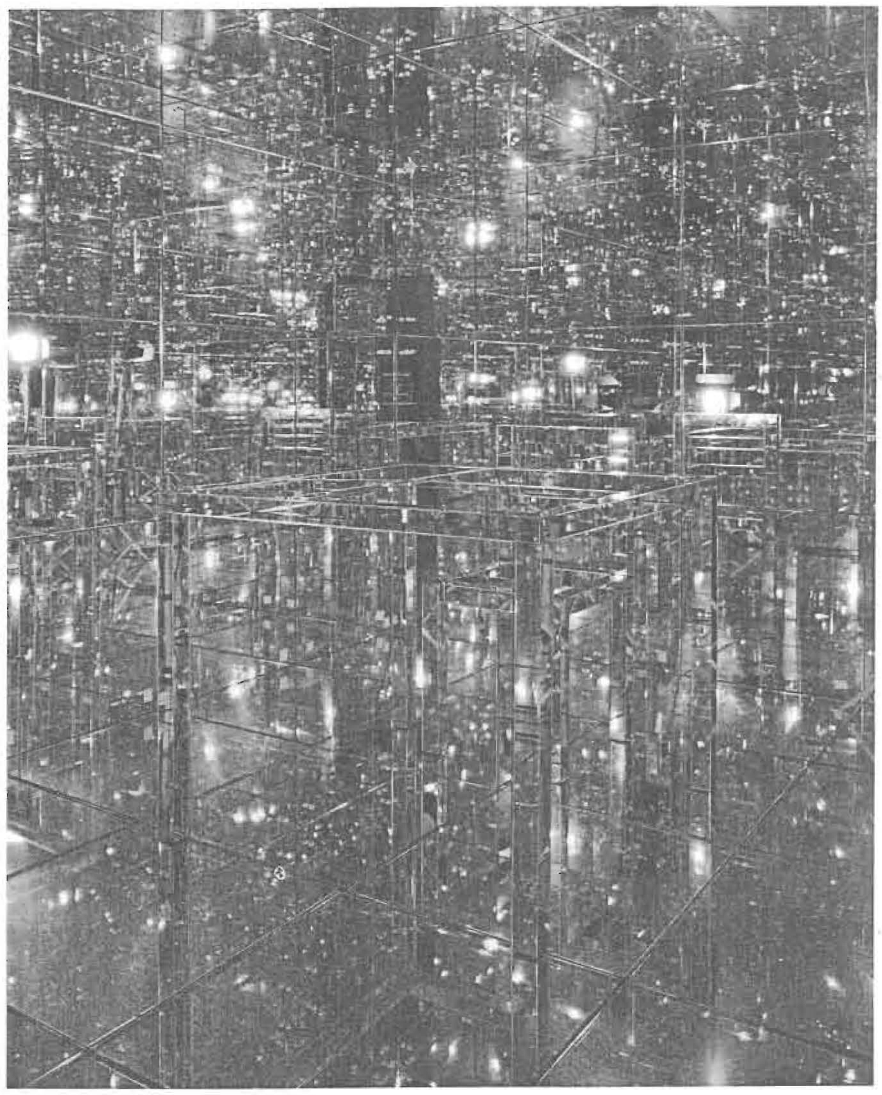


FIGURE 3.4

A walk-in kaleidoscope made of mirrors on a wooden frame: Lucas Samaras, "Mirrored Room."

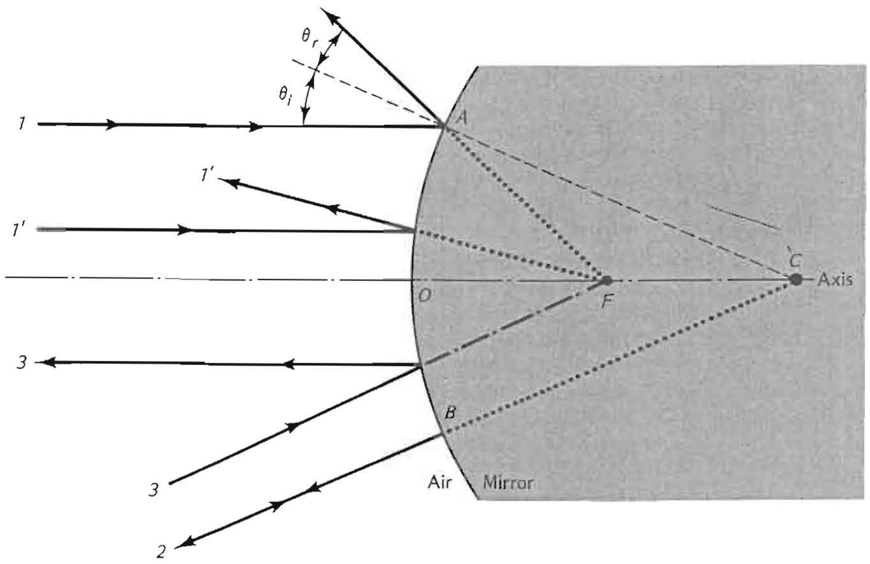
3.3 SPHERICAL MIRRORS

It was easy to locate the image in a plane mirror by following several rays from an object to the mirror. We can also apply this technique to ***spherical mirrors***—pieces of reflecting material in the shape of part of the surface of a sphere. (We draw such surfaces on paper as parts of a circle. You will have to imagine them rotated about their central axes to visualize the spherical surface.) Such mirrors have quite different properties and different uses from plane mirrors, as we'll see.

Let's first understand the properties of spherical mirrors by examining a few special rays for which the law of reflection results in rather simple rules. The rules that we will write down are actually idealizations. But spherical mirrors are *not* ideal optical systems: the images they form are, in general, not perfect but slightly blurred. However, as long as both the position and angles of the rays that reach the mirror are sufficiently close to an ***axis*** (a line passing through the center of the sphere), our approximate rules will be quite good and the images formed quite acceptable—considering only these so-called ***paraxial*** (or near axis) rays gives us a pretty good picture of what actually happens. In Section 3.4B, we'll discuss the blurring that results from the nonparaxial rays.

FIGURE 3.5

(a) Light rays illustrating the three rules, incident on a convex spherical mirror. Note direction of the rays—they are *not* all incident parallel to the axis! In this two-dimensional diagram and throughout the rest of the book, the three-dimensional spherical mirror appears as a part of a circle. **(b)** A shiny copper bowl makes a fine convex spherical mirror.



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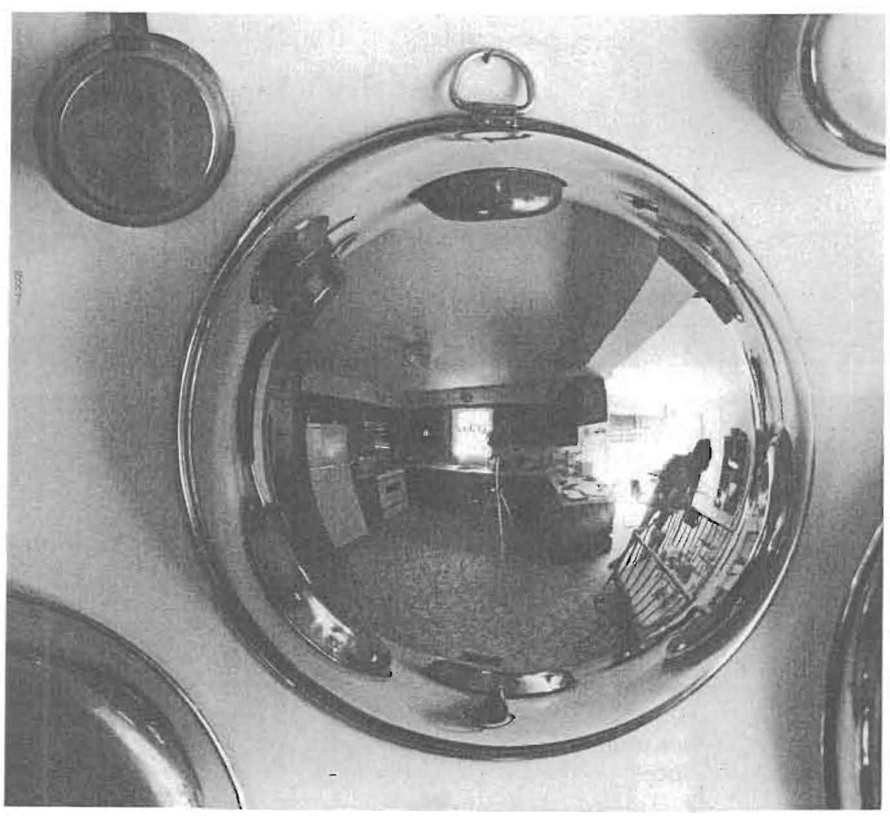


(b)

A. Convex mirrors

Figure 3.5 shows a spherical ***convex*** mirror (a mirror that bulges *toward* the source of light), with the center of the sphere marked *C*, and an axis that intersects the mirror at a point marked *O*. We have also drawn a number of paraxial rays. (Strictly speaking, these rays are not paraxial; they are too far from the axis, and some are at too steep angles. We draw them that way so that you can see them clearly.)

First consider ray *1*, which comes to the mirror parallel to the axis. To apply the law of reflection, we need the normal to the surface at the point *A*, where the ray hits the surface. This is easily found because any radius of a sphere (a straight line from its center to its surface) is perpendicular to its surface. Thus we simply draw a straight line from the center, *C*, through the point *A*, and we have the normal at *A* (shown by a dashed line). The reflected ray is then constructed so that $\theta_r = \theta_i$, according to the law of reflection. When we now extend the reflected ray backward, we see that this extension crosses the axis at some point, labeled *F*. Were we to repeat this construction for another ray coming parallel to the axis, say ray *1'*, we would find that the reflected ray *1'*, when extended backward, intersects the axis at the same point *F* (at least within the paraxial-ray approximation). The point *F* is called the ***focal point*** of the mirror. The distance from *O* to *F* is called the ***focal length***, *f*, of the mirror. For spherical mirrors, the focal length is just one half of the radius of the sphere (see Appendix C for a proof):

$$f = \overline{OF} = \tfrac{1}{2}\overline{OC}$$

so if we know the radius of the sphere, we know the focal length. We now have a general rule:

RAY 1 RULE
All rays incident parallel to the axis are reflected so that they appear to be coming from the focal point, *F*.

Within our approximation, this allows us to draw such a reflected ray quickly. We simply locate F and then draw the reflected ray as if it came from F. We need only a straightedge to do this—we don't need to measure angles once we've located F.

Now consider ray *2*, which is originally aimed at the point C. As such, it arrives perpendicularly to the mirror at B, and therefore is reflected directly back on itself. (That is, $\theta_i = \theta_r = 0$.) This gives us, then, another general rule:

RAY 2 RULE
All rays that (when extended) pass through C are reflected back on themselves.

Finally, consider ray *3*, which is aimed at F. It is just like ray *1* or ray *1'*, except that it is traveling in the opposite direction. The angles θ_i and θ_r, being equal, don't depend on the direction the ray is traveling. So, if you reverse the arrows on a ray of type *1*, you get a ray of type *3*, which must therefore be reflected parallel to the axis. Hence, our third general rule:

RAY 3 RULE
All rays that (when extended) pass through F are reflected back parallel to the axis.

With these three general rules, we are now able to use the techniques of ray tracing to locate the image of any object placed in front of a convex spherical mirror.

B. Locating the image by ray tracing

Using the three convex mirror rules, we can now construct the image of an object, PQ. As before, we treat the object point Q as a source of light. In Figure 3.6a, we've drawn ray *1*, parallel to the axis from Q, reflected as if coming from F. In Figure 3.6b, we've drawn ray *2*, aimed at C from Q, reflected on itself. Both these rays appear to come from the intersection of their extensions, the point Q'. Thus, Q' is the image of the point Q. In Figure 3.6c, we check this result by drawing ray *3*, which leaves Q headed toward F, and is reflected back parallel to the axis. The extension of this reflected ray also passes through Q', confirming our result. Now we can confidently mass produce other reflected rays that originated at Q simply by drawing them as if they came from Q' (Fig. 3.6d). An eye, E, looking at the mirror sees the image of Q at Q', as a result of the ray drawn.

What about the rest of the image? We could repeat this construction for every point on the object between Q and P (except the point P itself), but as the object PQ was per-

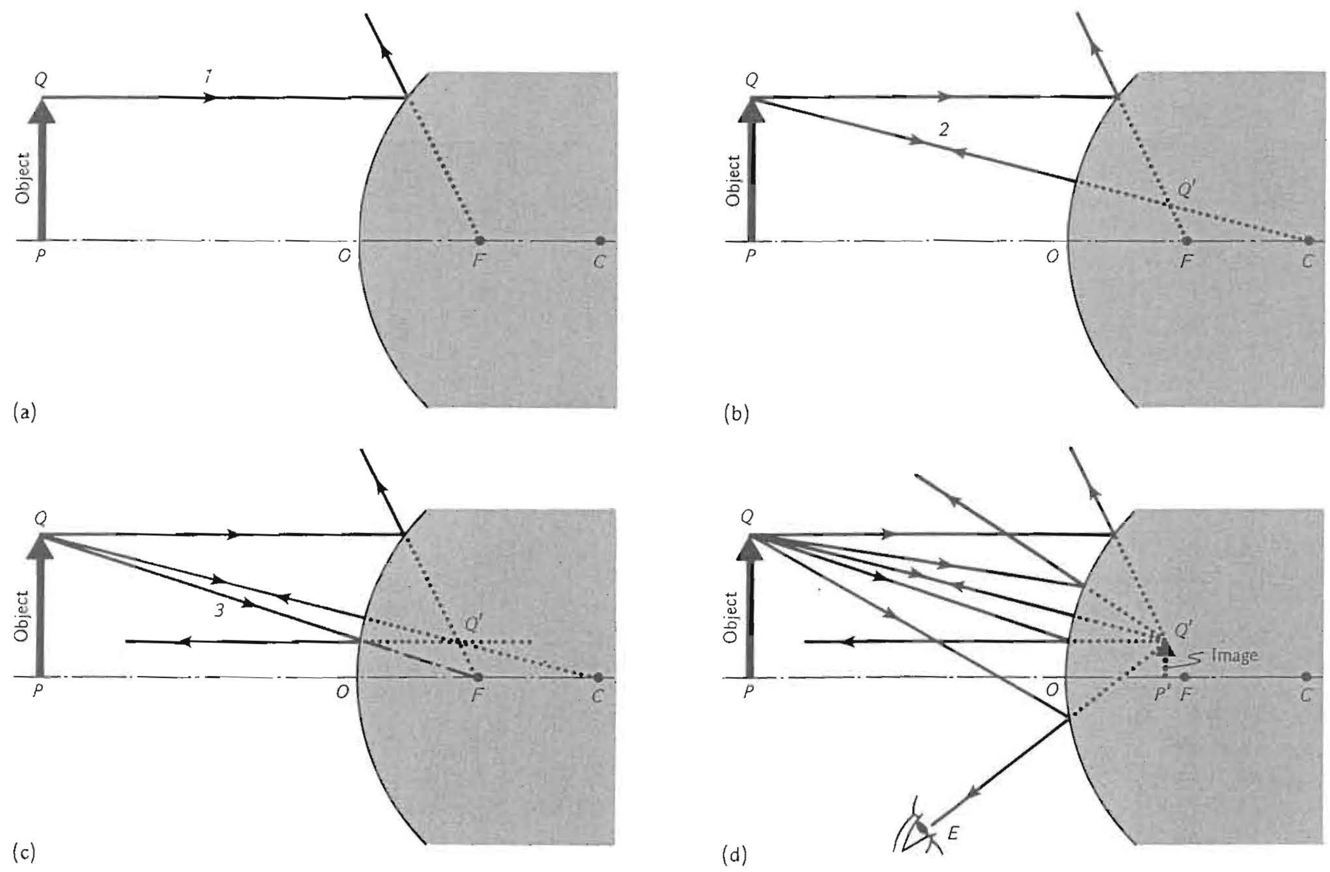


FIGURE 3.6

Construction of the image in a convex mirror: **(a)**, **(b)**, and **(c)** show the use of ray *1*, ray *2*, and ray *3* rules respectively; **(d)** shows how you can mass produce other rays—for example, one going to eye *E*—once you have found the image.

pendicular to the axis it is simpler to drop a perpendicular from Q' to find the image point P'. The image then lies between P' and Q' as shown in Figure 3.6d. (What happens to the three rays if the object point lies on the axis, e.g., point P? As our rules don't help us find the image point in such a case, we must always use an object that is off the axis.)

Consider now the image $P'Q'$. As in the case of the plane mirror, it is a *virtual image:* no light actually comes from it. It is also *erect.* However, unlike the plane mirror case, the image is *closer* to the mirror than the object. Further, the image is *smaller* than the object. The eye sees a smaller image than it would if the convex mirror were replaced by a plane mirror.

Because the image is smaller than the object, we see more of it in the convex mirror than in a plane mirror of the same size. A convex mirror is thus a ***wide-angle*** mirror, giving you a view of a wide angle of the world around the mirror. It is commonly used in stores to protect the merchandise from shoplifters. Some trucks and vans use such wide-angle mirrors as rear view mirrors that give a broader view (Fig. 3.7). The point is succinctly made by the Japanese poet Issa: "Far-off mountains/mirrored in its eye—/dragonfly." He contrasts the tiny convex mirror of the dragonfly's eye with the immense object (the mountains) visible in this "wide-angle mirror."

If the object is very distant (such as a star), the rays from it that hit the mirror must be traveling in essentially the same direction, toward the mirror—they must be parallel to one another. In Figure 3.8 we have drawn a bundle of such parallel rays at an angle to the axis of the mirror. As none of these rays is parallel to the axis, there can be no ray *1*. However, it is possible to select rays *2* and *3* out of the bundle of parallel rays; one aimed at C and one at F. These two rays are sufficient to locate the image at Q'. Note that Q' lies on the plane through F perpendicular to the axis. This plane, called the ***focal plane,*** shows up only as a line in our figure, but actually extends in front of, and behind, the paper on which the figure is drawn. This given us another useful rule:

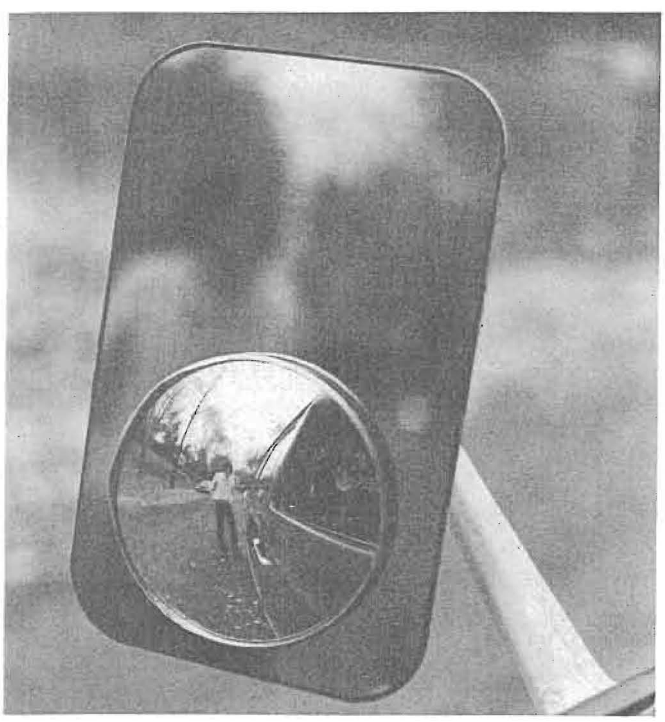
PARALLEL RAYS RULE
Rays parallel to each other are imaged on the focal plane.

The virtual image of the distant star is then at Q'. Any other reflected ray may then easily be constructed by drawing a straight line from Q' to the point on the mirror where the incident ray hits. We have drawn three such rays (other than rays *2* and *3*), omitting the lines between Q' and the mirror so as not to clutter the diagram. Stars in other directions would be imaged at other points on the focal plane.

The method of ray tracing we have illustrated, if done carefully, can give you the correct size, position, and orientation of the image

FIGURE 3.7

Photograph of a convex mirror used to give a wide-angle view. The plane mirror on which it is mounted shows the normal view. **(a)** The camera is focused on the image in the convex mirror, just behind the mirror (see Fig. 3.6). **(b)** The camera is focused on the image in the plane mirror, as far behind the mirror as the object is in front. From a distance, your eye can focus on both images simultaneously.



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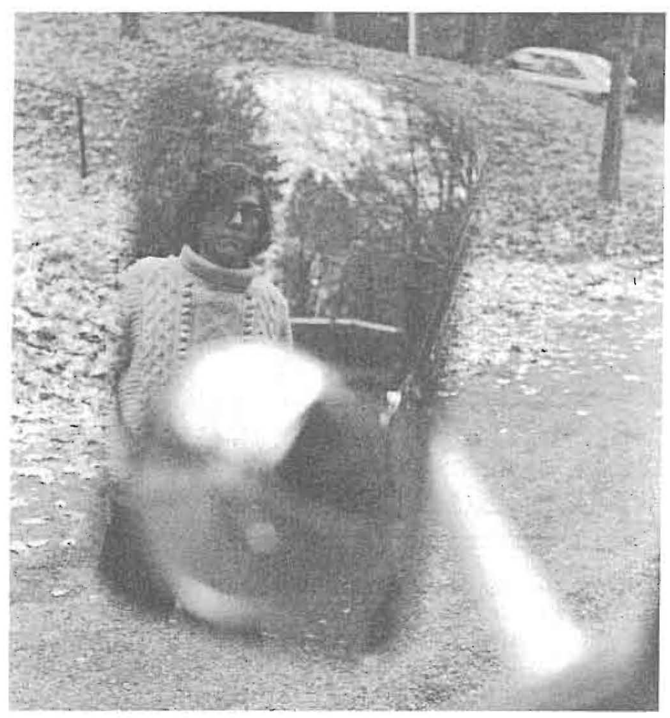

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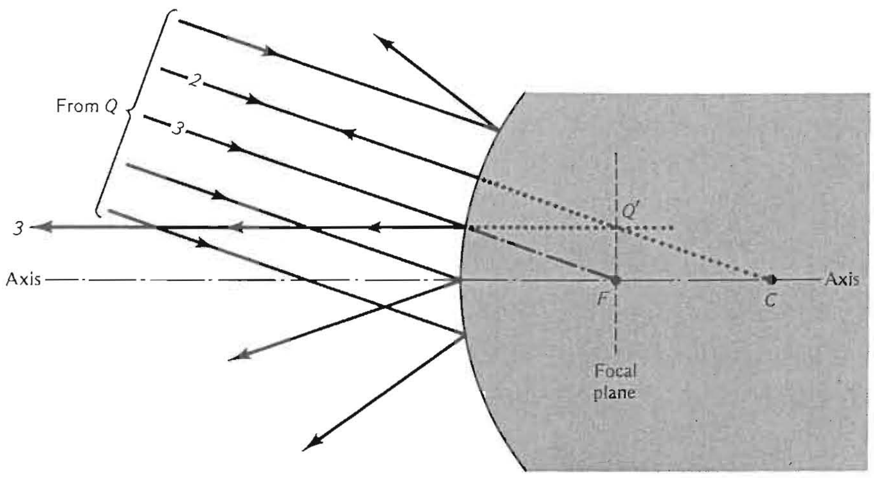


FIGURE 3.8

An object Q, too distant to be shown, sends parallel rays to a convex mirror. Its image, Q', lies in the mirror's focal plane.

of any object. Using a ruler, you can measure the results on your diagram to find these image properties reasonably accurately. You need only know the focal length, f, of the mirror to perform the construction (see also Appendix D).

*C. Deformations in convex mirrors and anamorphic art

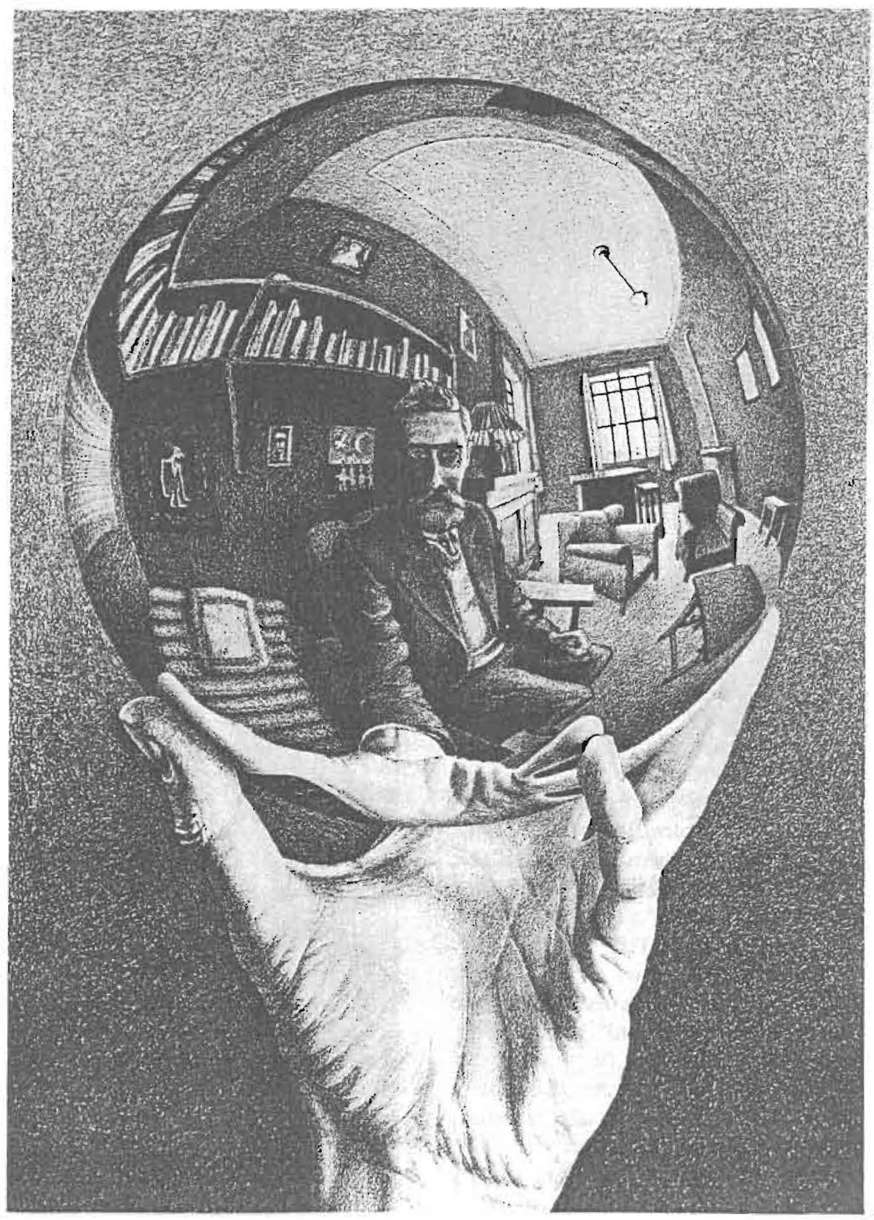
If the object doesn't lie in a plane perpendicular to the axis, or if the mirror is not spherical, the image in a convex mirror will be deformed. The deformations so produced have artistic interest. In Victorian times, a convex mirror was often hung on the wall to give an exotic image of the room around it and the people nearby. Parmigianino, Escher, and other artists have drawn self-portraits in such mirrors (Fig. 3.9).

Later it became fashionable to reverse the process. Instead of using a convex mirror to deform things, one painted a deformed picture that, when viewed with the help of a particular convex mirror, appeared undeformed. Such ***anamorphic art**** developed in the sixteenth century, was popular for several hundred years, and is currently undergoing a revival. As Shakespeare described it

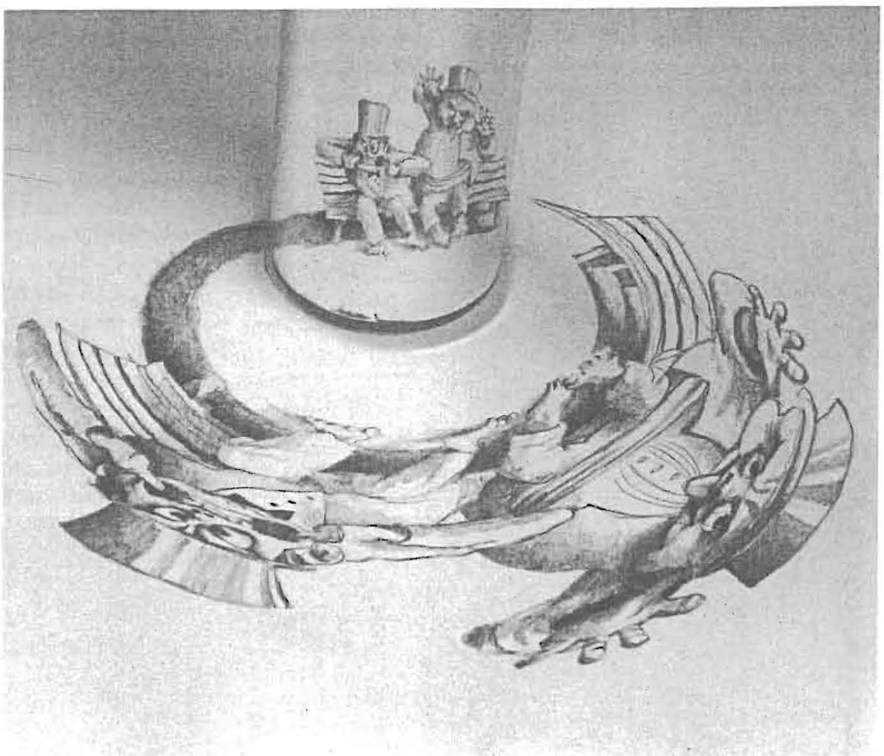
For sorrow's eye, glazed with blinding tears,
Divides one thing entire to many objects;
Like perspectives, which, rightly gazed upon,
Show nothing but confusion, eyed awry,
Distinguish form.

FIGURE 3.9

M. C. Escher, "Hand with Reflecting Globe."



*Greek *anamorphoo*, transform.



picture becomes the inner part of the virtual image (Fig. 3.11). These, too, may be made using an appropriately transformed grid, but they may also be made photographically (see the second TRY IT).

Another type of anamorphic art is made to be viewed without a mirror. Greatly elongated pictures, such as the streak across Holbein's painting "The Ambassadors," when viewed at a glancing angle from the side appear undistorted (Fig. 3.12). This is called a slant anamorphic picture. (This is the type of anamorphic art Shakespeare refers to in the passage cited above.)

FIGURE 3.11

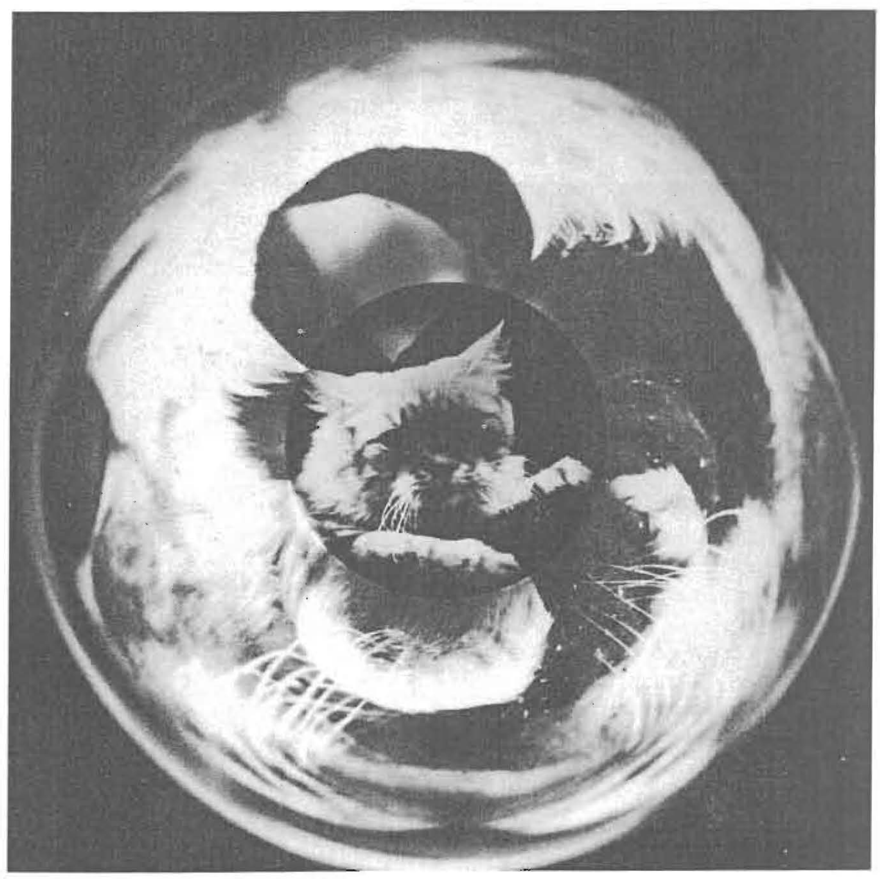
A conical anamorphic photograph, made by the method of the TRY IT. The conical mirror in the center reconstructs the undistorted image from the anamorph surrounding it, so you see the cat's head in the central circle surrounded by the deformed image.

FIGURE 3.10

Cylindrical anamorph, with mirror that reconstructs the image.

Anamorphic art falls into different classes, depending on the way the image is "decoded." One common type requires the use of a cylindrical mirror (a shiny cylinder, such as a tin can). After placing the mirror in the center of the distorted picture, you look into the mirror and see a virtual image of the picture, now undistorted (Fig. 3.10). Concentric circles around the mirror become straight lines in the image, the outer circular boundary of the picture becoming the top of the image. Anamorphic pictures are drawn by means of a special grid, as shown in the first TRY IT.

Conical mirrors can also be used. These are placed, point up, in the center of the conical anamorphic art, and are viewed (with one eye only) from directly above the cone's point. These pictures are particularly perplexing because they are "inside-out"; the outer part of the



These pictures can carry messages that are not readily apparent to a censor, but are easily accessible to their true audiences. They have been used for this purpose ever since their inception. One of the authors' introduction to anamorphic art came in the form of an apparently abstract design in a freshman handbook. When viewed at a slant, however, the design became a very clear (and earthy) student comment on the university administration (an obviously slanted view).

FIGURE 3.12

Hans Holbein's "The Ambassadors." View the streak across the foreground of the picture from the upper right. (Reproduced by courtesy of the Trustees, The National Gallery, London.)

First TRY IT

FOR SECTION 3.3C
Cylindrical anamorphic drawing

To make your own cylindrical anamorphic drawing, you'll need about a square-foot sheet of aluminized mylar (from a graphic arts or plastics store). Figure 3.13 demonstrates the construction technique. An undistorted drawing is made on a rectangular grid. Then each line in the undistorted drawing is translated into a corresponding line on the curved grid.

Make your undistorted drawing on a separate piece of graph paper 12 × 14 squares in size. Use a photocopy or tracing of Figure 3.14 as your curved grid. After completing your transformed drawing, curl and tape your mylar sheet into a cylinder that just fits on the circle inside your curved grid. Stand this cylindrical mirror on the central circle and view the reflected image. Redraw any lines in your drawing that don't appear correct. This is a bit tricky; you must draw while looking at the reflection. When you're satisfied with the anamorphic drawing, erase the grid lines, or trace the drawing onto another sheet. If you color the drawing, be careful to avoid any ridges due to brush strokes.

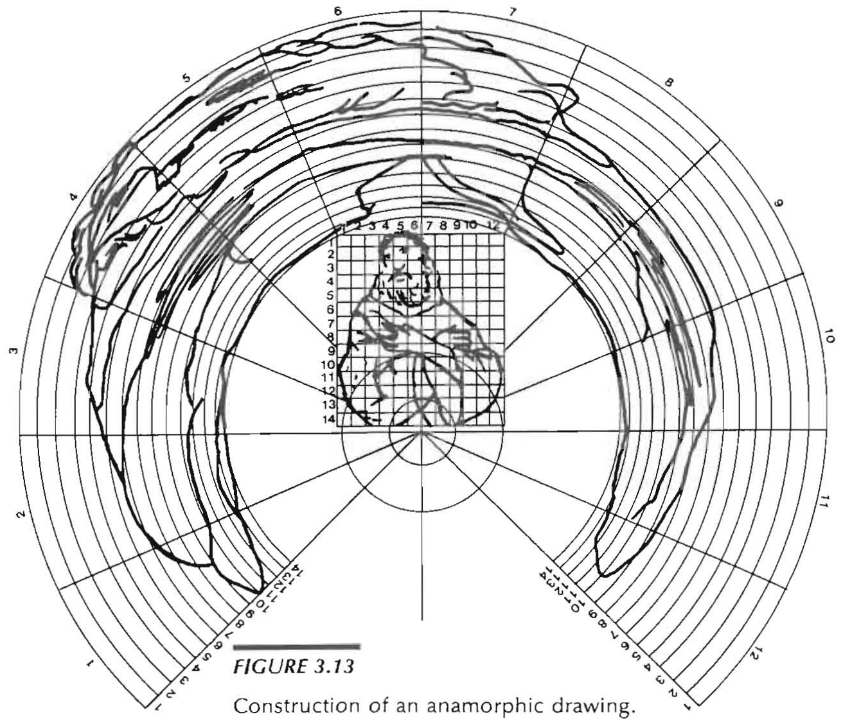


FIGURE 3.13

Construction of an anamorphic drawing.

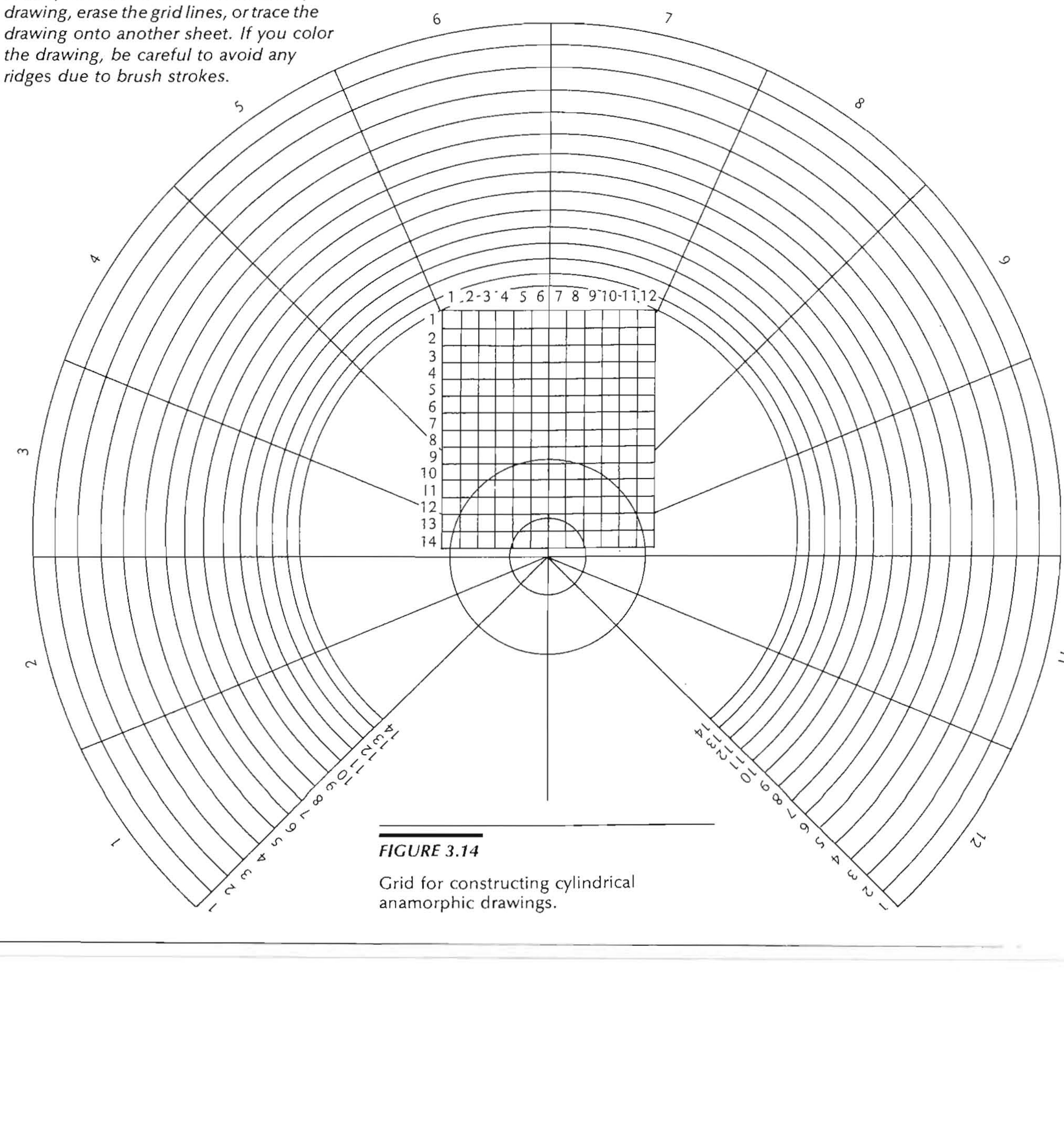


FIGURE 3.14

Grid for constructing cylindrical anamorphic drawings.

Second TRY IT

FOR SECTION 3.3C
Conical anamorphic photograph

If you can make black and white photographic enlargements (8 × 10 inches or larger), you can make a conical anamorphic photograph. Besides a standard enlarger and photographic chemistry, you'll need a ringstand and clamp or other supporting device; a variable aperture or a set of cardboards, each with a different size hole; and a shiny cone of apex angle about 30°. (A cone made from aluminized mylar, taped, is acceptable.)

Choose a negative that has a razor sharp image, in which the subject is fairly small and located at the center of the frame. It is best to have a fairly dark, uniform region at the exact center. (For such a negative, unwanted distortions due to an imperfect cone apex will not be visible on the final print.) Cut a piece of cardboard to fit snugly in your print-paper holder. From the center of the cardboard, remove a circular hole the size of the base of the cone, to form a placement mask. *Put your negative in the enlarger head, turn on the projection lamp, and center the print-paper holder directly below the enlarger lens. Lay the placement mask in the holder, put the cone base down in the hole, then remove the mask. Next, use the ring stand and clamp to hold the variable aperture above the cone. Adjust the aperture opening so that light from the enlarger strikes the cone, but not the print-paper holder directly. Focus the distorted image. In Figure 3.15 the placement mask, hinged to the print-paper holder, is shown tipped away, as for focusing and for exposing the paper.*

For your exposure of the print, use the placement mask to center the cone on the print paper. Expose for the "usual exposure period," the period you would use were the cone not present. Then, slightly reduce the size of the hole in the aperture and expose for the "usual time" again. Repeat this procedure about ten to fifteen times until the aperture hole is as small as possible. If your negative is dark at the very center, you can skip the last few exposure periods. This unusual dodging technique guarantees that each area of the paper receives enough light for a proper exposure.

PONDER

If light from the enlarger were allowed to strike the entire cone for the duration of the exposure, why would areas of the paper near the cone receive much more light than areas farther away (assuming a negative of uniform density)?

To view your print after development, place the cone on the center of the print and look down on the cone, using only one eye (Fig. 3.11).

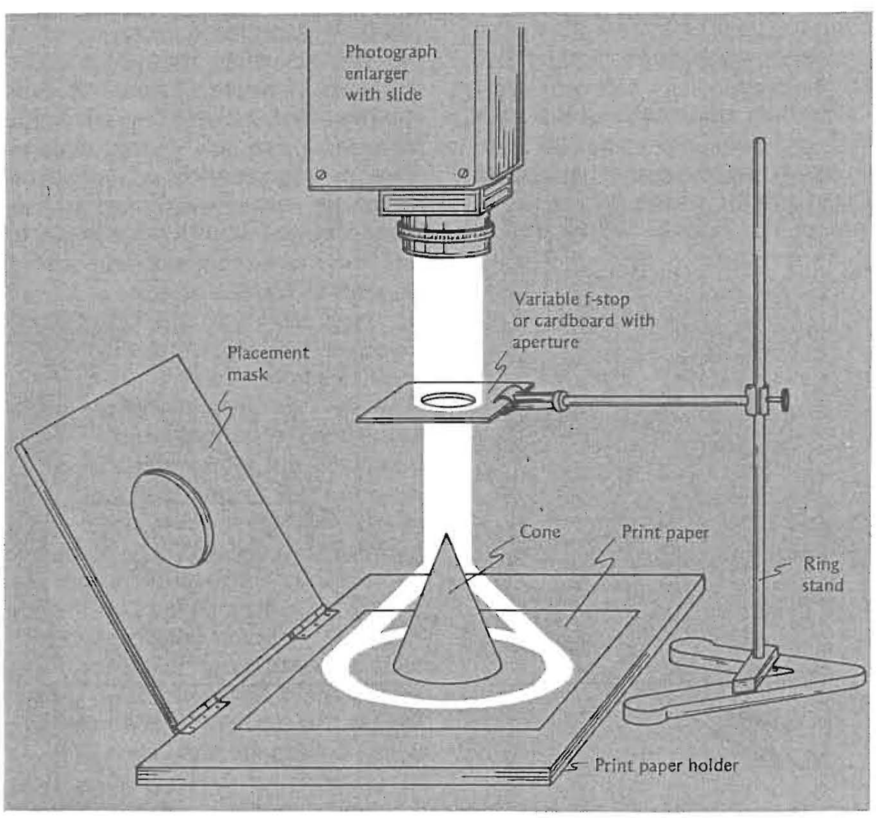


FIGURE 3.15

Set-up for making an anamorphic print. The shiny cone is in place, and the placement mask is tipped back, for exposure.

D. Concave mirrors

The reflecting surface of a **concave** mirror bows *away* from the light source, like a cavity. This means that the center, *C*, of the sphere is in front of the mirror. Following the same steps as we did for the convex mirror, we apply the laws of reflection to ray *1* of Figure 3.16 and discover that the focal point, *F*, also lies in front of the mirror. (Again $\overline{OF} = \frac{1}{2}\overline{OC}$ for a spherical mirror.) In order to distinguish this case from the convex case, we say that the focal length, f, is the *negative* of the distance $\overline{OF}$. Thus f is *negative* for *concave* mirrors, *positive* for *convex* mirrors.

Except for the fact that the points *C* and *F* are in front of the mirror, the general rules for convex mirrors can be taken over directly for the concave mirror. In Figure 3.16, we illustrate each of the three types of rays.

Having *F* and *C* in front of the mirror allows for new possibilities not available with convex mirrors. For example, suppose we put a point source of light at *F*. All rays from this source are then rays of type *3*, as they all come from *F*. Consequently, all such rays will be reflected back parallel to the axis of the mirror. This gives us a way of constructing a parallel beam of light: simply put a small light source at the focal point of a concave mirror. For example, the con-

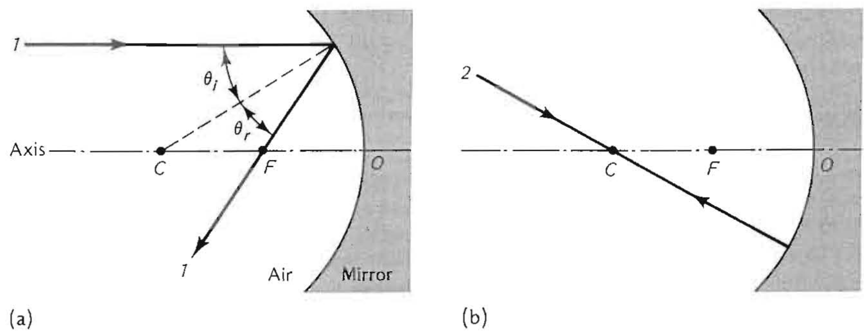


FIGURE 3.16

Three rays obeying their respective rules, for a concave mirror.

cave mirrors in flashlights and headlights make such parallel light beams.

Conversely, as all incident rays parallel to the axis are reflected to pass through F (ray *1*), quite a bit of light energy from a distant source, such as the sun, can be directed to one point. In the third century B.C., Archimedes may have used a concave mirror, constructed of the shiny shields of many soldiers, to burn the attacking Roman fleet. The idea was to focus all the sun's energy striking the large area of the mirror onto the small area of the sun's image on one of the boats. This concentrated energy would set the ship on fire. Whether or not Archimedes did this, such devices are now often used as concentrators in solar heating devices—you can buy small ones to light cigarettes or campfires. (See the FOCUS ON Solar Power.)

Having F and C in front of the mirror has some other rather amusing consequences. We can get rather different types of images depending on the location of the object, but all can be found by the rules of Section B. In our example (Fig. 3.17), we have placed the object between points C and F. Rays *1* and *3* are then easily drawn following the ray tracing rules. Notice now that, to the eye, E, ray *1* not only appears to be *coming from* F after reflection, but it actually *passes through* F. Similarly, ray *3* is not only aimed at F from Q, it actually *goes through* F on its way to the mirror. The rule for ray *2*, however, requires some comment. Starting at Q, no ray can head toward the mirror and pass through C. However, if we draw a ray from Q headed toward the mirror *as if* it had come from C, then this ray is still perpendicular to the mirror and hence is reflected back on itself, as ray *2* normally is. We can thus treat an object lying between C and the mirror. (If the object were closer to the mirror than F, similar comments would apply to the Ray *3* Rule.)

The three reflected rays of Figure 3.17 actually intersect (without having to be extended backward). Therefore, the image lies at Q', in *front* of the mirror. We locate the entire image $P'Q'$, as usual, by dropping a perpendicular to the axis from Q'. It is not a virtual image because the rays from Q really *do* pass through Q'. Such an image, through which the light actually passes, is called a **real image.** To the eye, E, it looks just as if there were an object at Q' blocking the view of the mirror behind. However, if the eye moves to E' and looks toward Q', the "object" disappears because there is then no mirror behind Q' to reflect light to E'. As light rays actually cross at a real image, you can place a screen there that will catch a projected image for you to see. The pattern of bright rippling lines of light one sometimes sees reflected from water is due to this effect (Fig. 3.18). The ripples in the water form little concavities that reflect the sunlight. Because there are many ripples, there are always some of them that image the sun on any nearby surface. Since each concavity tends to

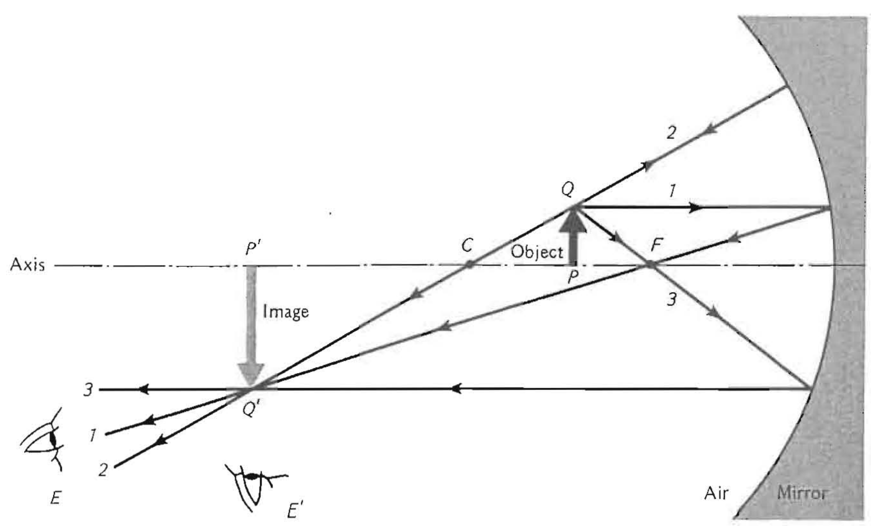


FIGURE 3.17

Construction of the image in a concave mirror by ray tracing.

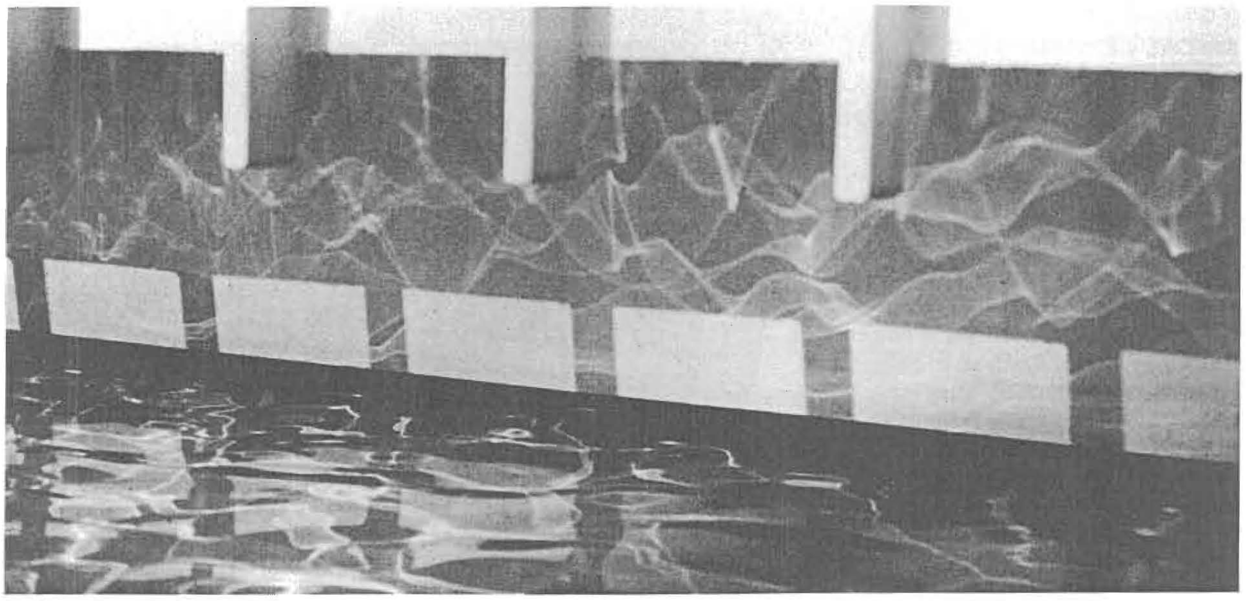


FIGURE 3.18

Rippling lines of light reflected and focused by the uneven surface of the water. Lines can be seen near the surface of the water as well as on the wall, in the shadow of the fence.

be trough-shaped—long, rather than spherical—the image of the sun is elongated into a line. The many ripples produce many such images, which intersect, shimmer, and change as the ripples flutter across the surface of the water.

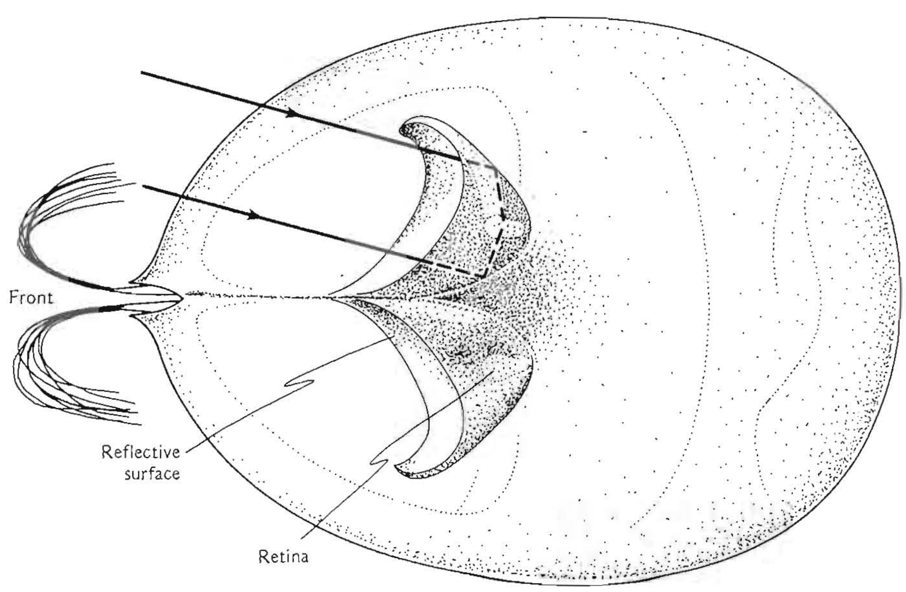
In Figure 3.17 the image formed by the concave mirror is a real image. It is located in front of the mirror—closer to E than the object is! It is inverted, pointing in the opposite direction from the object. It is also ***magnified;*** it is larger than the object. These properties of the image will be changed if the location of the object is changed. For example, if the object were at $P'Q'$, the image would be at PQ. (Why?) In that case the image would still be real and inverted, but smaller than the object and closer to the mirror.

If the object were closer to the mirror than the focal point F, the image would be virtual, behind the mirror, erect, and magnified. Such mirrors are used as shaving or makeup mirrors, allowing you to see a magnified image of your face, provided you are close enough. If you have this type of mirror, examine your image as you move your face closer or farther away from it. Alternatively, you can use the inside of the bowl of a large spoon. In that case it's easier to use your finger tip as the object. Turn the spoon over and look at the back of the bowl as an example of a convex mirror, and compare the two cases.

An unusual example of a concave mirror in nature is the eye of the plankton *Gigantocypris* (Fig. 3.19). This deep sea crustacean has no lens in its eye, but rather a concave nonmetallic mirror (see Sec. 12.3C). While mirror eyes are relatively rare in the animal kingdom, they do occur and we'll see another example in Section 6.4D.

FIGURE 3.19

Drawing of *Gigantocypris*. The dark curves are the reflecting eyes, with the retina in front of the reflector. They look through transparent windows in the carapace.



3.4 SPHERICAL LENSES

We can also create virtual or real images by *refraction*—bending of the light as it enters a new medium. Let's consider two media, air and glass, with a boundary between them shaped like part of a sphere. Suppose light traveling parallel to the axis strikes this boundary from the air side (Figs. 3.20a and c). We have drawn the normals—they are just radial, straight lines from the center of the sphere, *C*. According to Snell's law, the light bends toward the normal so that the transmitted angle, θ_t, is less than the incident angle, θ_i. The two rays in Figure 3.20a, traveling in air parallel to the axis, become converging rays in the glass. This surface is a ***converging*** surface. In Figure 3.20c, the rays become diverging in the glass. This surface is a ***diverging*** surface. Notice also that turning the surface around, so that the light originates from the glass side, does not change the nature of the surface: reversing 3.20a gives 3.20b, still a converging surface. Reversing 3.20c gives 3.20d, still a diverging surface.

A surface like that in Figure 3.20a remains converging when the glass is replaced by water. For this reason, one often sees a pattern of bright rippling lines of light on the bottom of pools, what T. S. Eliot described as "the light that fractures through unquiet water." This pattern is produced by refraction of sunlight at the water surface, much the same as the pattern of lines produced by reflection that was described in Section 3.3D.

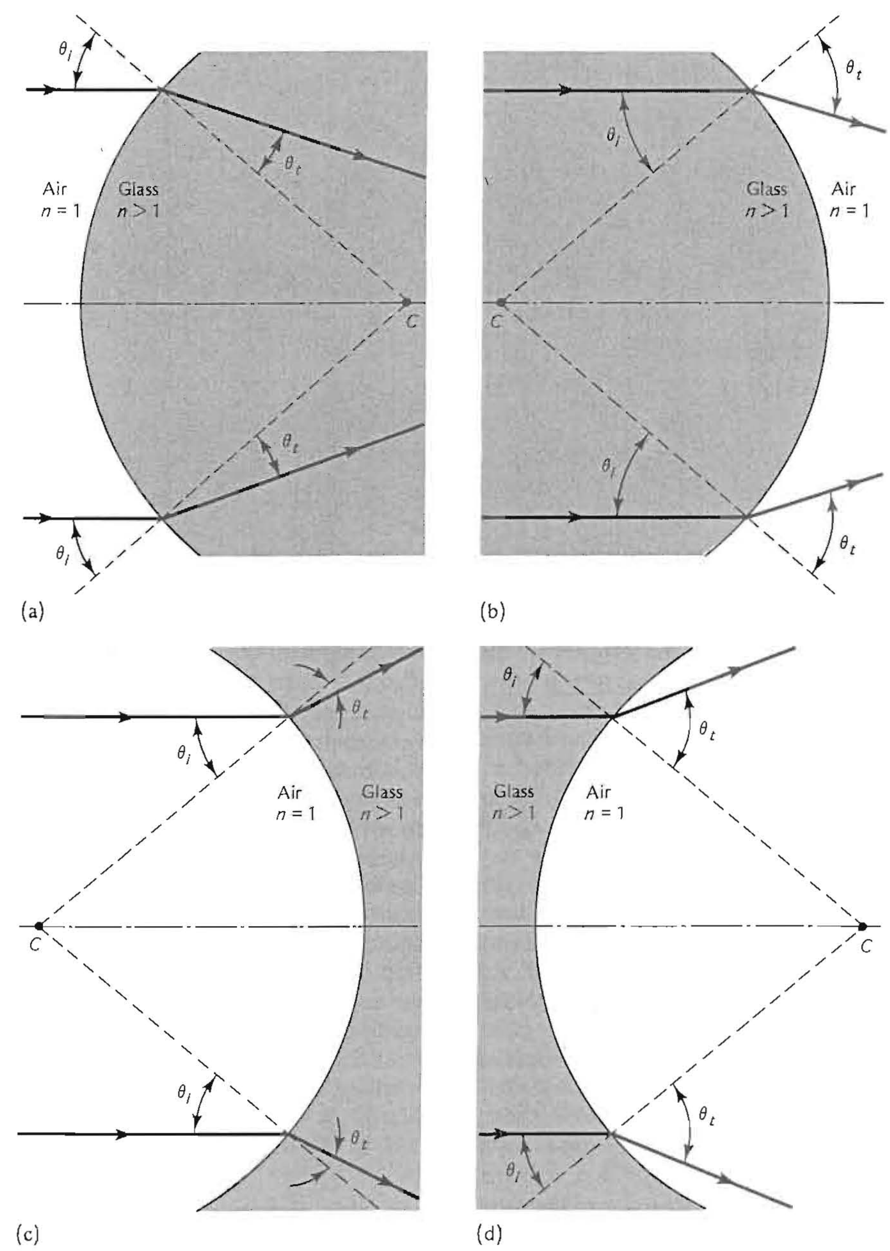


FIGURE 3.20

Effect of a spherical glass surface on light rays incident parallel to the axis:
(**a**) and (**b**) converging surfaces,
(**c**) and (**d**) diverging surfaces.

A. Converging and diverging lenses

Combining the two surfaces of Figures 3.20a and b, we get the ***lens*** of Figure 3.21a. We see, by successive applications of Snell's law, that light originally traveling parallel to the axis is made to converge by refraction at the first surface (points *A* and *B*), and made even more convergent by refraction at the second surface (points *C* and *D*). Emerging from the glass lens, these rays intersect at the point labeled *F'*. In fact, all paraxial incident rays parallel to the axis, including a ray along the axis itself, will intersect at *F'* (see the first TRY IT). The point *F'* is called the ***second focal point*** of this lens. (The location of this point depends on the curvature of both lens surfaces and on the index of refraction of the lens and of the surrounding medium.) Such a lens is called a ***converging*** or ***focusing lens*** (see the second TRY IT). It can be used as a burning glass with which to start fires. The parallel rays of the sun will be brought to a focus at *F'* by this lens, forming a small image of the sun there. All the energy carried by these light rays is

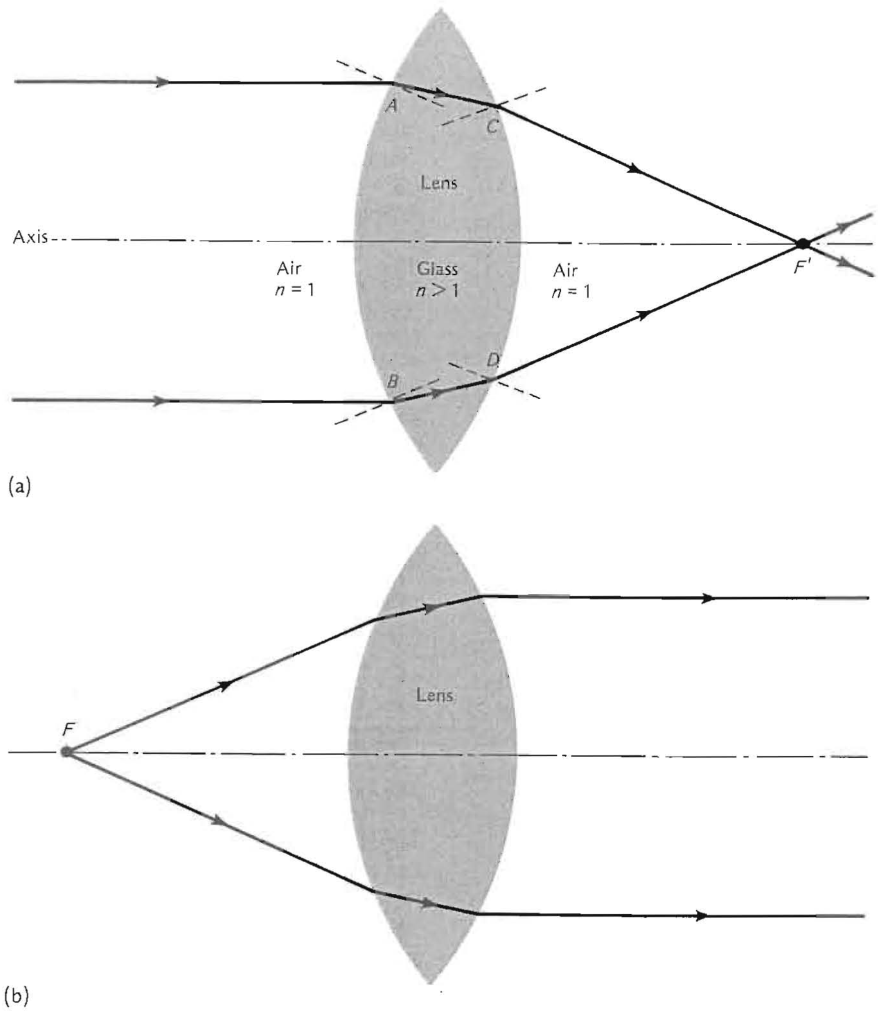


FIGURE 3.21

A converging lens, consisting of two converging surfaces. **(a)** Rays parallel to the axis are focused at F'. **(b)** Rays originating at F are also made to converge and emerge parallel to the axis.

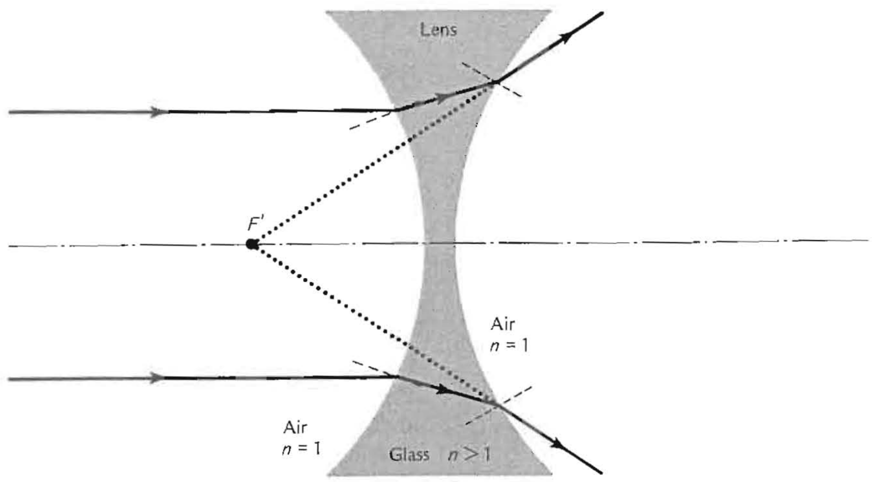
then concentrated in the sun's small image, and will heat that spot (Fig. 3.22). Focusing lenses are quite old. The early ones were glass spheres filled with water. The Greek comic playwrite Aristophanes suggested that burning glasses could be used to annul promissory notes by melting the letters off the wax notes.

A converging lens also has a ***first focal point,*** F, in *front* of the lens (Fig. 3.21b). All rays originating at F and passing through the lens will emerge parallel to the axis. To see that this must be true, notice that Figure 3.21b is simply Figure 3.21a flipped around, with the arrows on the rays reversed.

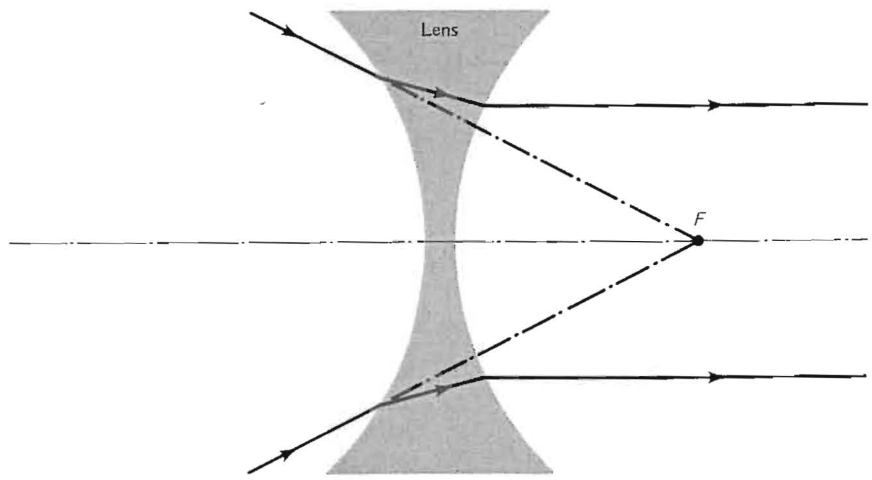
We can make a ***diverging lens*** by combining the two diverging surfaces of Figures 3.20c and d. In Figure 3.23a, we see that the incident rays parallel to the axis are made to diverge. To an eye on the right-hand side of the lens, these divergent rays will *appear* to be coming from their point of intersection (when extended backward). We again label this point F' and call it the second focal point. For the diverging lens, then, F' lies in front of the lens (the reverse of a converging lens, where parallel incident rays actually go *through* F').

Flipping Figure 3.23a over and reversing the arrows (Fig. 3.23b) shows that the first focal point, F, of a diverging lens lies behind the lens. All rays aimed at F that hit the lens emerge parallel to the axis.

FIGURE 3.22



(a)



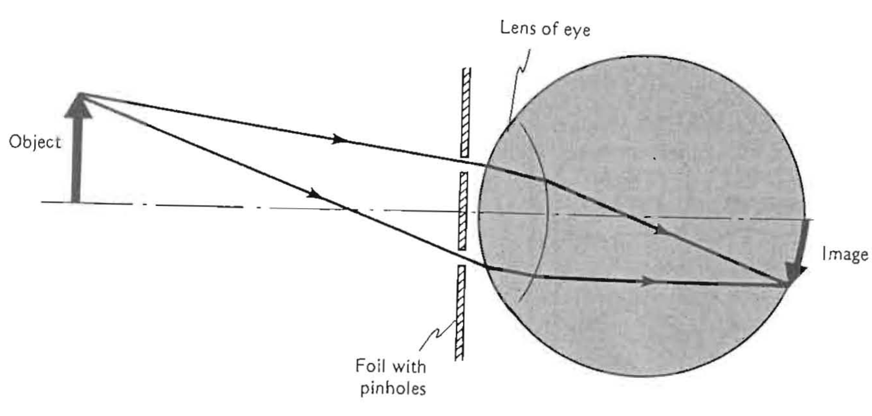
(b)

FIGURE 3.23

A diverging lens, consisting of two diverging surfaces. **(a)** Rays parallel to the axis seem to come from F' after they pass through the lens. **(b)** Rays converging toward F are also made to diverge and emerge parallel to the axis.

FIGURE 3.24

Optics of an eye viewing through multiple pinholes. The lens of the eye creates one image from rays that passed through different pinholes.



First TRY IT

FOR SECTION 3.4A
Focusing of parallel rays

You can verify that a converging lens, such as a magnifying glass, will focus parallel rays. Using the sun as a source, hold a comb perpendicular to a piece of paper so that the shadows of its teeth form a set of long parallel lines on the paper. Now hold the lens so that the light passes through the comb, then through the lens, and hits the paper. Notice that the previously parallel shadows become converging lines that meet at the focal point of the lens. Use a ruler to measure the focal length of the lens.

You can also try this with a cylindrical lens (one shaped liked a cylinder rather than a sphere). A glass of water makes a nice, converging, cylindrical lens.

Second TRY IT

FOR SECTION 3.4A
The focusing ability of a lens

With the aid of the lens in your eye, you can verify that a focusing lens will take rays going in different directions and focus them. In a piece of aluminum foil, make three small pinholes in a triangle, separated from each other by about 2 mm. These will select the three different rays from a point source that are going in three different directions.

First, replace the pinhole in your pinhole camera by these three pinholes (see the TRY IT for Sec. 2.2B). How many images of a distant, small, bright object do you see in this case, where there is no lens between the object and the screen?

Now replace the pinhole camera by your eye, which does have a lens. Hold the three pinholes next to your eyeball and look through them at the distant object. How many images are formed on the "screen" in your eye (the retina)? That is, how many images do you see now (Fig. 3.24)?

If you have a single-lens reflex camera, you can look through the viewfinder to do the same experiment. Look first with the lens removed and the lens opening covered with the aluminum foil containing the three pinholes. Now remove the foil, replace the lens, put the foil right over the lens, and look again.

B. Dew heiligenschein and another type of retroreflector

A drop of water makes a good converging lens. Its second focal point, F', is just a short distance beyond the drop. If a dew drop is held at this distance in front of a blade of grass by the fine hairs on the grass, sunlight is focused on the blade of grass (see Figs. 3.25a and b, and the TRY IT). The blade of grass reflects this light back toward the sun, reversing the path of the incident light—the dew drop on the grass forms a retroreflector.

With the sun behind you, the only retroreflected light you will see will be from the grass straight in front of you, around the shadow of your head. Hence, there will be more light coming from the region around your head's shadow than from any other region of the grass, a phenomenon called ***heiligenschein.**** The sixteenth-century artist Benvenuto Cellini concluded that this was evidence of his genius. Actually, a nearby friend would see the glow only around her own head's shadow, and conclude that she is the genius.

As glass is somewhat more refracting than water, a glass sphere has its focal point just at the edge of the sphere. Therefore, glass spheres with white paint behind them make excellent retroreflectors. They are used in highway signs, license plates, and the paint used for the white lines on highways, because they send so much of the incident light from your headlights back to your eyes (Fig. 3.25c).

*German, light of the holy ones.

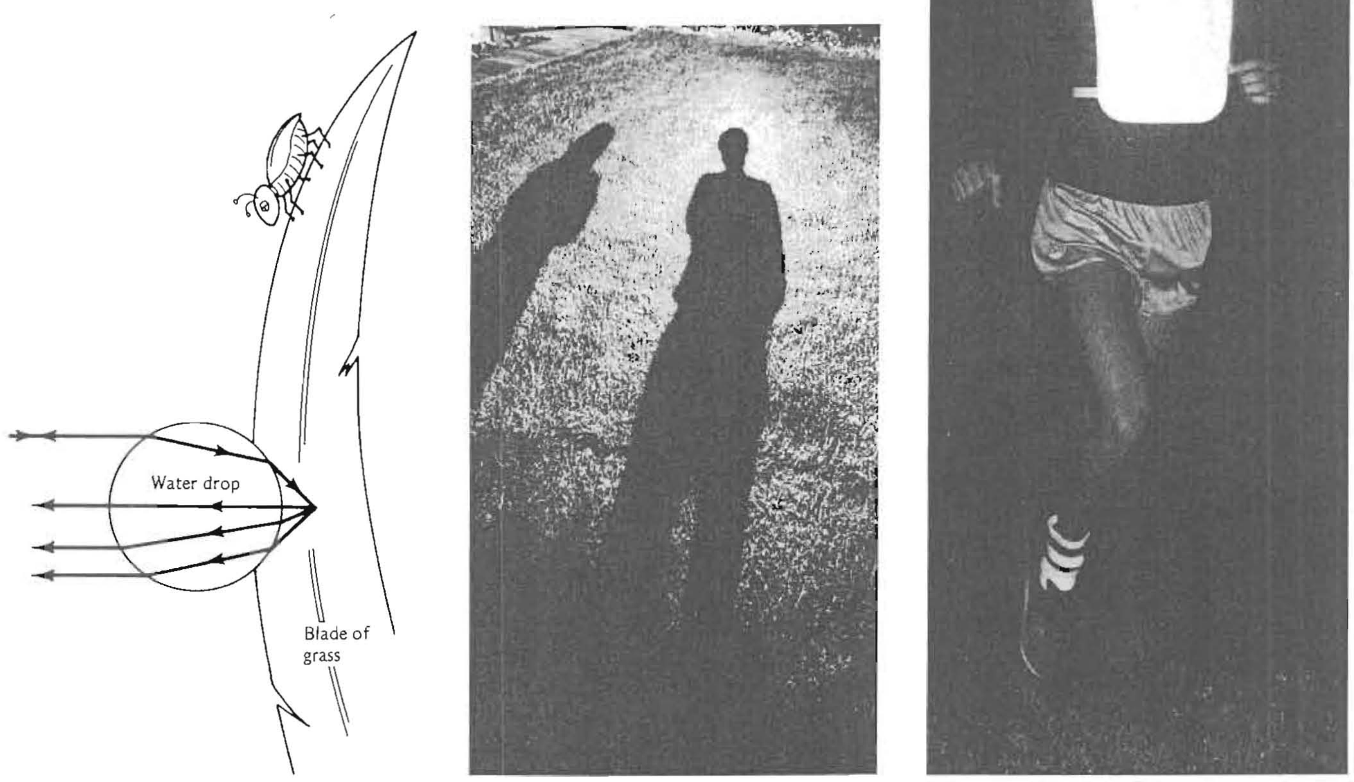
TRY IT

FOR SECTION 3.4B
Focal point of a water drop

You can easily determine the focal point of a water droplet. Make a droplet at the end of an eyedropper or a drinking straw. Bring a piece of paper behind the droplet to catch the sun's rays that have passed through the droplet. Bring the paper closer until you have the sharpest image. (The paper will then almost touch the water, so view the sun's bright image from the side so you don't block the sun's rays with your head.) How far is the focal point from the water droplet?

FIGURE 3.25

(**a**) Water droplet and blade of grass acting as a retroreflector. Only *one* incident ray is shown, and only a few of the many, diffusely reflected rays due to this one incident ray. (**b**) Dew heiligenschein around the shadow of the photographer's head. (**c**) Glass beads used as retroreflectors make this jogger's vest visible in car headlights.



(a) (b) (c)

C. *Ray tracing for thin lenses*

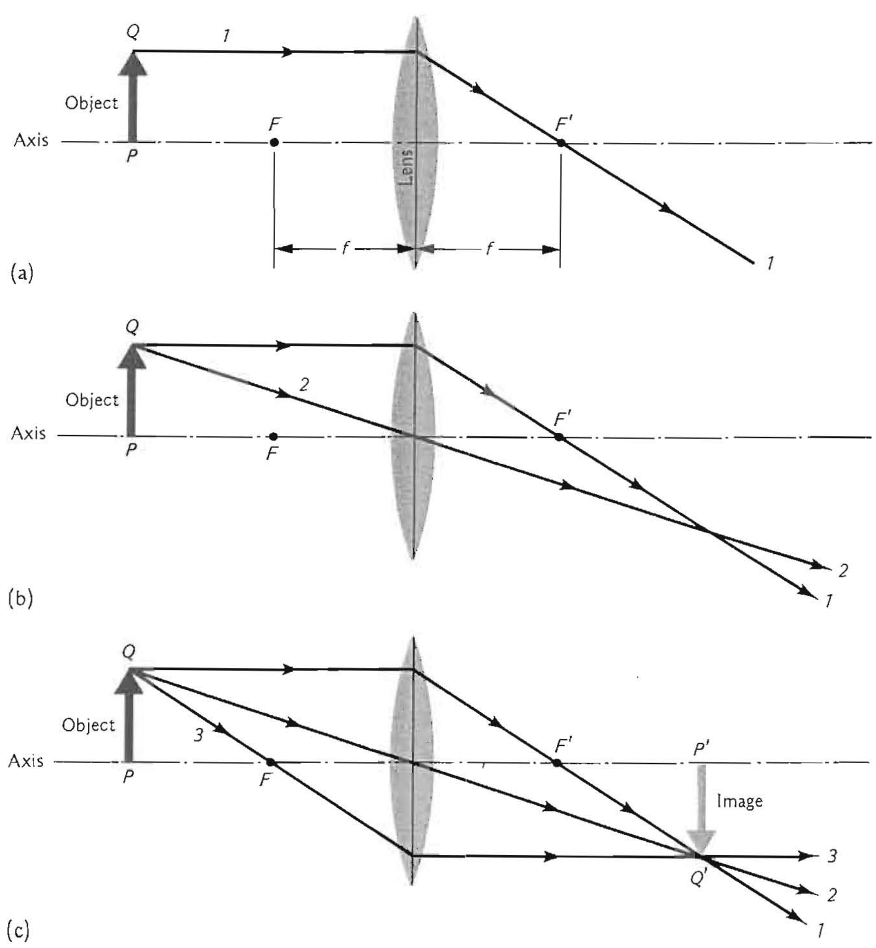
Once we know the location of the focal points F and F', we can construct images by ray tracing, much as we did for mirrors. The situation is simplest if the thickness of the lens is much less than the distance from the lens to either focal point. We will confine our attention to such ***thin lenses*** and, as before, to paraxial rays.

For thin lenses, the distance from the lens to F is equal to that from the lens to F' and is called the ***focal length*** of the lens, f. To distinguish between converging and diverging lenses, f is taken as *positive* for *converging* lenses and *negative* for *diverging* lenses. Another simplification of thin lenses is that any ray passing through the *center* of the lens is undeviated—it continues straight through the lens without being bent.

With this information, we can set up ray tracing rules as we did for mirrors. Again we choose three special rays with particularly simple properties to help us locate the image. We first illustrate the technique for a converging lens, but we'll phrase the rules so as to cover both types of lenses. In Figure 3.26a, we have drawn a lens and the two focal points F and F', each at the distance f from the lens. We've also drawn our standard object PQ. For our first ray from Q, we refer back to Figure 3.21a (and Fig. 3.23a) and write:

FIGURE 3.26

Three stages of construction by ray tracing of the image formed by a converging lens. The lens is shown relatively thick so that you can see it but treated as if it were just the vertical plane through its center.



RAY 1 RULE
A ray parallel to the axis is deflected through F' (or as if it came from F').

We've drawn such a ray from Q, labeled ray *1* in Figure 3.26a.

Ray *2* passes through the center of the lens:

RAY 2 RULE
A ray through the center of the lens continues undeviated.

This rule is illustrated in Figure 3.26b. We'll call ray *2* the ***central ray.***

Finally, we obtain the third rule by examining Figure 3.21b (and Fig. 3.23b):

RAY 3 RULE
A ray to the lens that (when extended, if necessary) passes through F is deflected parallel to the axis.

Ray *3* is illustrated in Figure 3.26c.

Any two of these rays are sufficient to locate the image point Q'. The third ray serves as a check. Again, we find P' by dropping a perpendicular to the axis from Q'. The resultant image in Figure 3.26 is a *real* image (the light actually converges to this image). The image is inverted. Further, in this *particular* case (we took the object distance, from the object to the lens, to be twice the focal length), the image is the same size as the object.

A screen in the plane of $P'Q'$ would show an illuminated inverted image of the object. This is the principle of a slide projector (and of a photographic enlarger). The object, in that case, is the transparency illuminated by the light source in the projector. The lens then projects the inverted image on the screen (thus, the slide must be put in the projector upside down to get an upright image).

Of course, for a slide projector we want the image much larger than the object. This is achieved by mov-

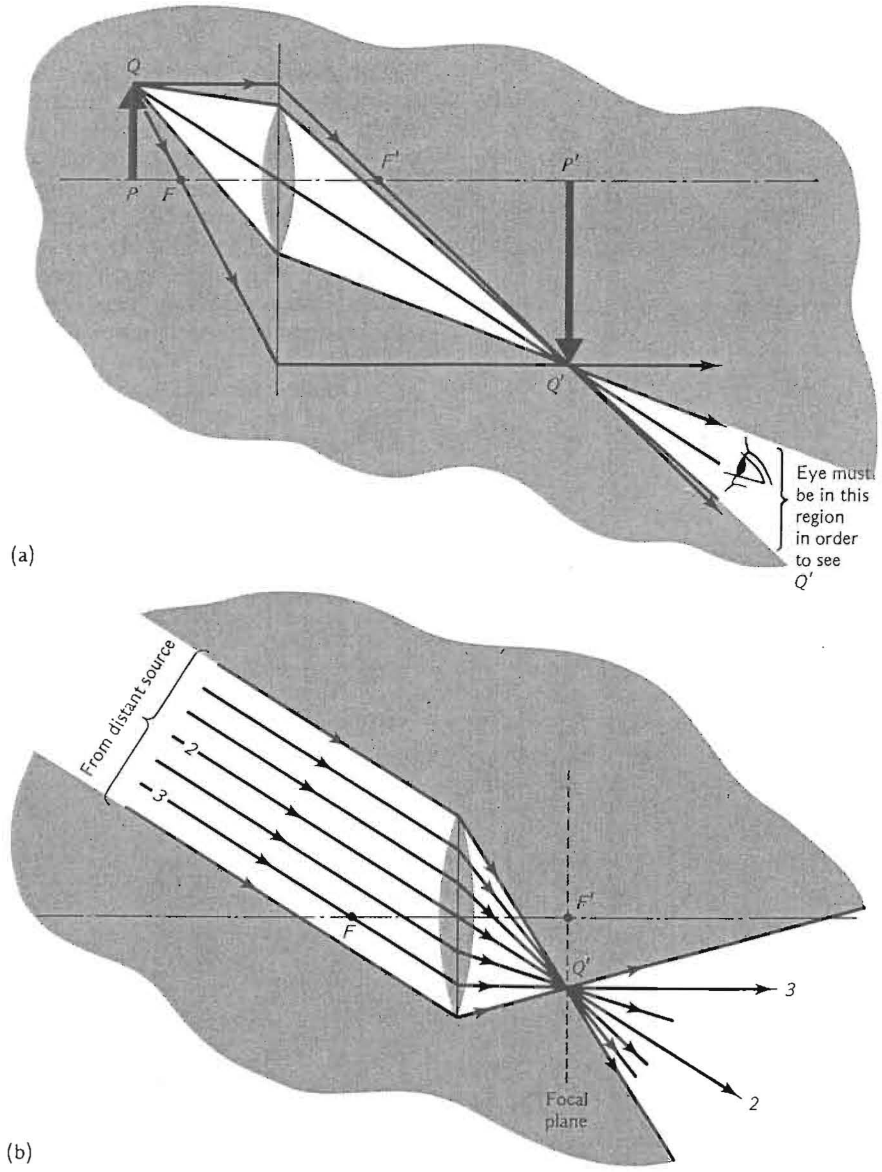


FIGURE 3.27

(a) Another example of ray tracing, with some special twists. Here the lens is too small to transmit all three tracing rays. You draw them anyway, pretending the lens is larger. Once the image is found, it is easy to draw the cone of rays (unshaded) that actually make it through the lens. This cone converges on the image, crosses there, and then diverges from it. In order to see the image, the eye must be within the region of the cone of light on the right of the image. (In a projector, there is a screen at $P'Q'$, so the light does not continue to the right beyond Q' but instead is scattered by the screen so that all eyes to the *left* of $P'Q'$ can see the projected image.)
(b) Parallel rays incident on a converging lens are focused on the focal plane.

ing the object closer to F, as in Figure 3.27a. The closer the object is to F, the larger (and farther) the image will be. This figure also illustrates another point: what to do if the actual lens (or mirror) is too small to catch the three rays used in ray tracing.

Figure 3.27b shows how to trace a bundle of rays parallel to one another. Here, as in Figure 3.8 you do not have any incident rays parallel to the axis, so you cannot construct ray *1*. Rays *2* and *3*, however, are sufficient to locate the image and give us the same rule we had for mirrors:

PARALLEL RAYS RULE
Rays parallel to each other are imaged on the focal plane.

Because the focal plane passes through F', if you want to take a picture of a distant object, you should put your film at the focal plane.

In Figure 3.28 we apply the ray tracing rules to a diverging lens (f negative, F' in front of the lens, F behind it). Compare rays *1* and *3* to the rays in Figures 3.23a and b, respectively. The image in this case is virtual. Any eye to the right of the lens will see rays apparently coming from Q', although most rays do not actually pass through that point (only ray 2 does). The image here is erect and smaller than the object.

Thus, knowing only the focal length, f, of a lens, we can construct the image of any object formed by that lens. The focal length of a lens is given in one of two ways. The lens on your camera may have printed on it, "f = 50 mm." As f is positive, this is a converging lens. However, the prescription for your eyeglasses may read, "−2 D." The D stands for ***diopters,*** a different measure of focal length. The number of diopters (called the ***power of the lens***) is related to the focal length as follows:

$$\text{Power (in diopters)} = \frac{1}{\text{focal length (in meters)}}$$

For example, a −2 D lens is one with a focal length of $-\frac{1}{2}$ m or −50 cm (the negative sign means that it is a diverging lens). The more the lens bends the rays, the higher its power. So power measures the everyday notion of the "strength" of a lens. Once we know the focal length (in either form) we can easily locate F and F' and then trace rays to find the image of any given object (see Appendix E).

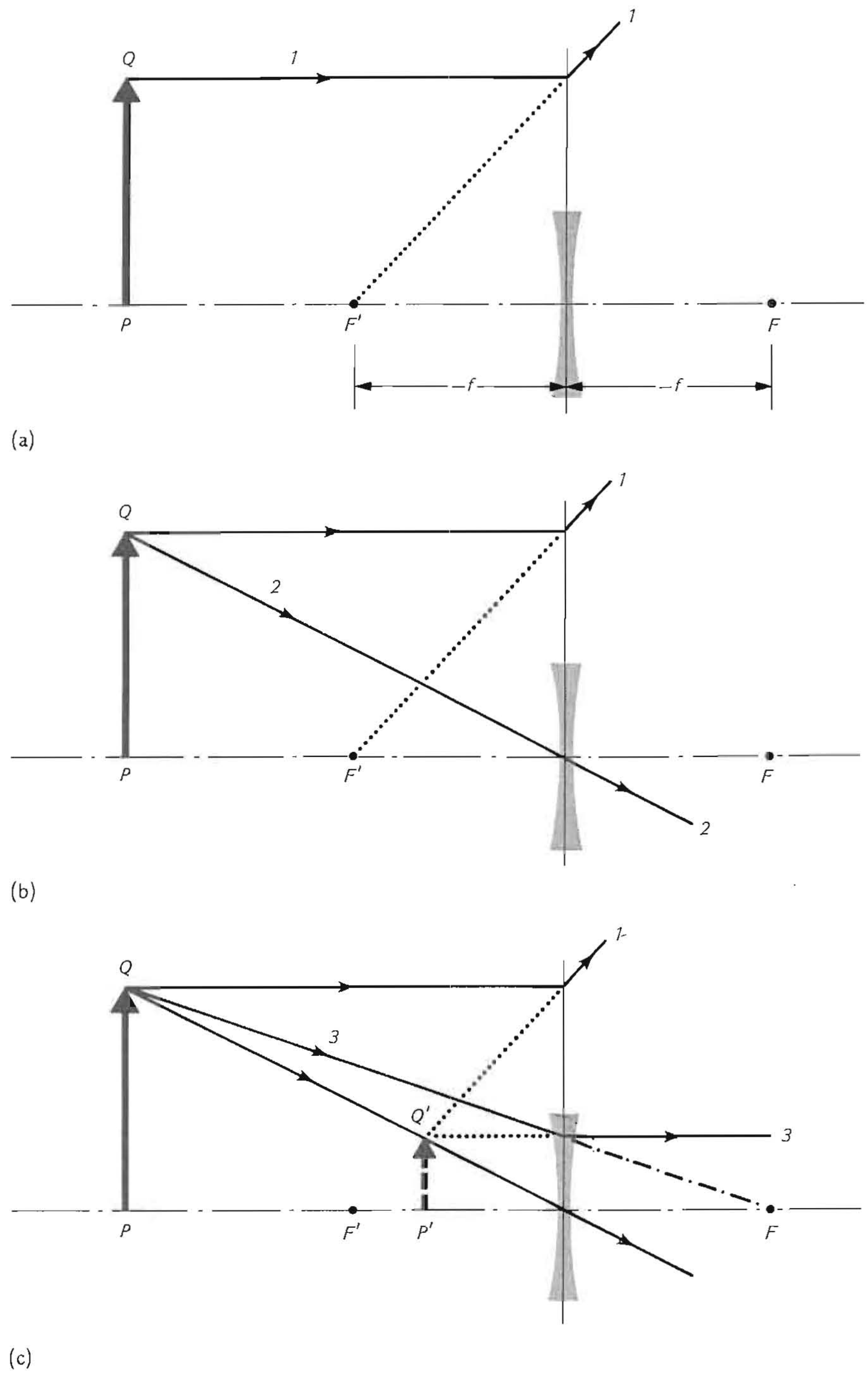


FIGURE 3.28

Three stages of construction, by ray tracing, of the image formed by a diverging lens.

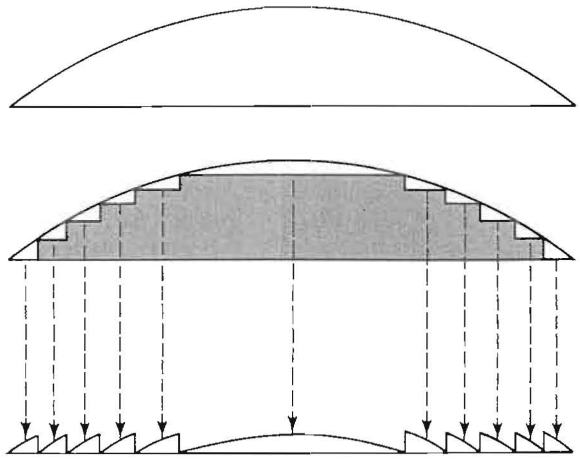
*D. *Fresnel lenses*

In lighthouses, for stage lighting, and whenever a single lens must intercept much of a source's light, one wants a lens with both a large diameter and a short focal length (that is, very curved). Together, these requirements imply a very thick lens. But large thick lenses create problems. They are heavy, take up space, absorb more of the light sent through them, develop large stresses, and often crack.

To avoid these difficulties, Augustin Fresnel conceived of the idea of removing most of the interior glass from a thick lens (Fig. 3.29a). After all, the refraction occurs only at the surfaces of the lens. Therefore, he imagined cutting the lens in sections. By removing most of the glass in each section, he would have a lens with a spherical front surface and a set of circular steps on its back (Fig. 3.29b). Fresnel then collapsed the rings to a common flat back surface (Fig. 3.29c), producing a thin lens with the power and diameter of a large, thick lens.

It is true that this **Fresnel lens** has some spurious refraction at the risers. However, the applications for which it is used do not require precision lenses, so these drawbacks are not important. In fact, such lenses are usually made cheaply by pouring glass into a mold or by stamping them out of plastic.

Glass Fresnel lenses are commonly found in lighthouses and in spotlights for theater lighting (Fig. 3.30). (Spotlight lenses are designated by their diameter and focal length. Thus, a 6 × 9-inch lens has a six-inch diameter and a nine-inch focal length.) Stamped plastic Fresnel lenses are cheap and fit where ordinary lenses of short focal length would not. Novelty stores carry them as curiosities for your window, but they also find use in overhead projectors, camera viewfinders, traffic lights, and numerous other applications. You can recognize them by noticing the pattern of finely spaced rings. Using the same principle, stamped plastic ***Fresnel mirrors*** are flat but behave like concave or convex mirrors.



(a)

(b)

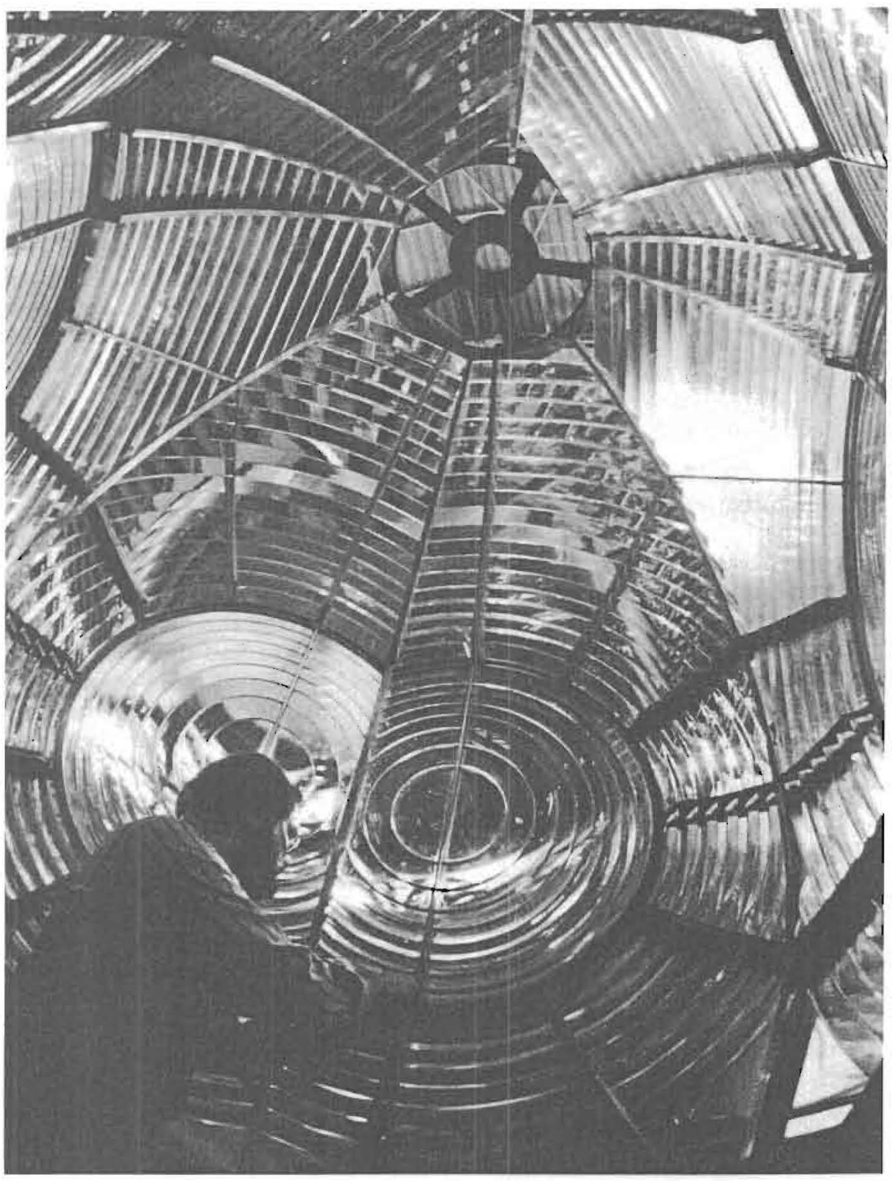
(c)

FIGURE 3.29

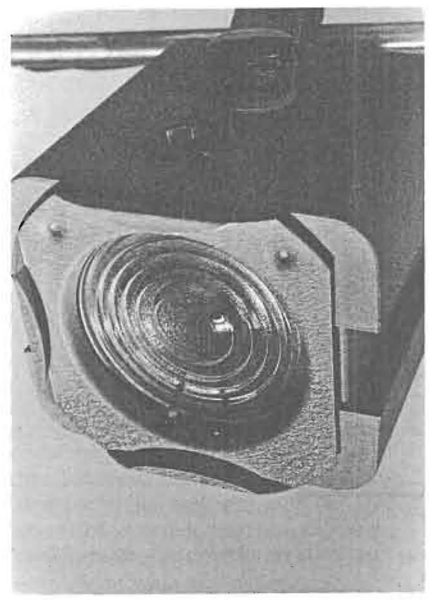
(a) A thick, converging lens (with one flat side). **(b)** Parts of the glass of this lens that have no essential effect on the bending of light (shown shaded). Remember that these sections are really rings oriented perpendicular to the paper. **(c)** After removing the nonessential glass and rearranging the lens, you get a Fresnel lens.

FIGURE 3.30

(a) Photograph of a Fresnel lens designed for use where Fresnel first intended: in a lighthouse. **(b)** Photograph of a Fresnel spotlight.



(a)



(b)

The traffic-light application is of interest in its own right (Fig. 3.31). A lamp, backed by a concave reflector, illuminates a ground glass screen. A Fresnel lens is located about a focal length in front of this screen. The entire apparatus is surrounded by a housing, the back of which (including the lamp and reflector) can be removed. The task is to adjust this light so that it can be seen in some lanes of traffic (say the left-turn lane), but not in other, adjacent lanes. You set the traffic light in place, remove the back, and look in from the back. You see an inverted image of the road on the screen. (Why?) With opaque mask-

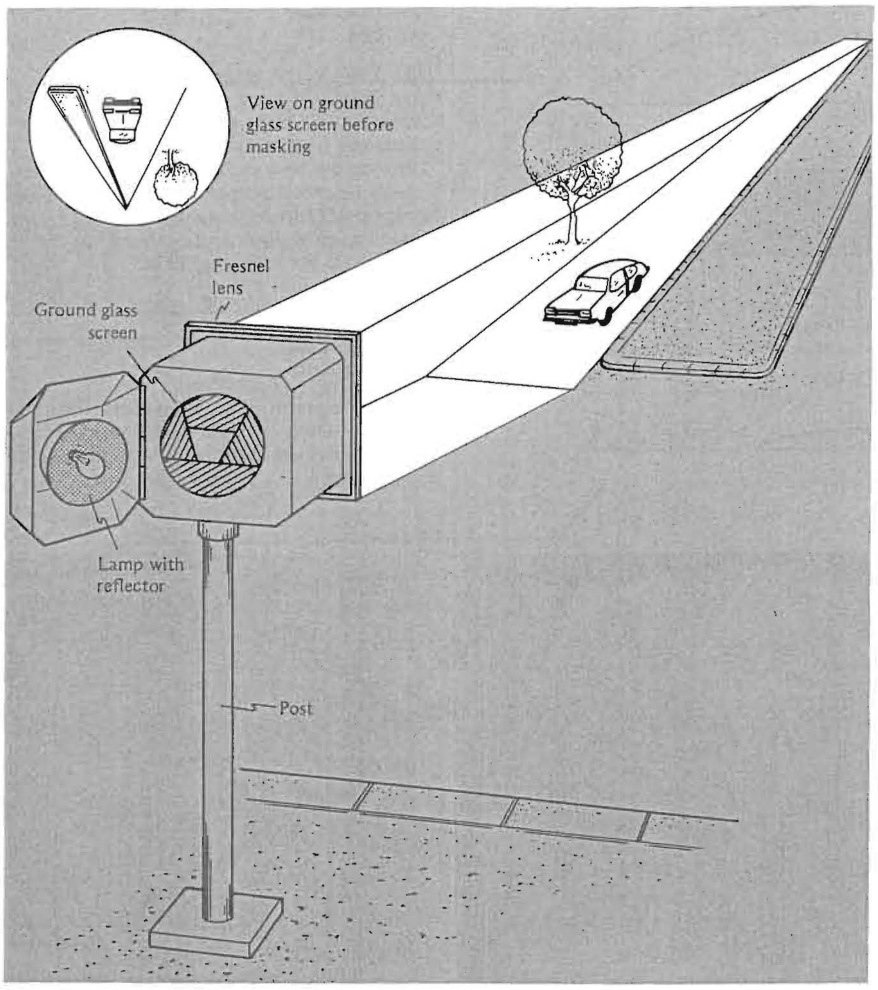


FIGURE 3.31

An "optically programmed" traffic light.

ing tape, you mask off those parts of the image that correspond to the lanes in which you don't want the traffic signal to be seen. When the lamp is replaced, the direction of light travel is reversed; the masked ground glass becomes the object (illuminated by the lamp), and it is projected on the traffic lanes, like a slide in a slide projector. When you stand in a lane where the traffic signal is not meant to be seen, the masked part of the ground glass is projected on you, and the traffic light looks dark. In the proper lane, however, the light shines to your eyes through an unmasked part of the ground glass, and you see the signal.

E. Compound lenses

We often have occasion to use combinations of several lenses. Such ***compound lenses*** are common in photography, almost all optical instruments, and the human eye. To apply the techniques of ray tracing to multiple lens systems, we simply take one lens at a time and apply the rules over and over for as many lenses as we have. Let's illustrate this with the two-lens system of Figure 3.32.

We begin by ignoring the second lens and finding the image produced by the first lens alone. This ***intermediate image,*** $P'Q'$, is located in Figure 3.32a by drawing the three rays labeled *1*, *2*, and *3*. (We've drawn all the rays to the right of the second lens with dashed lines to indicate that they actually will be modified by the second lens.)

Now that we have located the point Q', we can draw other rays that begin at the object point Q, as they must all go to Q'. In particular, we can draw those rays that will be useful for ray tracing through the second lens (Fig. 3.32b). Ray $1'$, aimed at Q', arrives at the second lens traveling parallel to the axis. (It coincides with ray *3*, which left the first lens traveling parallel to the axis.) Ray $2'$ is aimed at Q' through the center of the second lens. Finally, ray $3'$, also aimed at Q', travels between the lenses as if it came from F_2, the first focal point of the second lens. (We have traced rays $2'$ and $3'$ back through the first lens to the object point, Q, so you can see their actual paths. This is not necessary, however, to construct the final image.)

Having located the three rays incident on the second lens, we now apply the ray tracing rules to each of them in Figure 3.32c. Thus, ray $1'$ is deflected through F_2', ray $2'$ is undeflected, and ray $3'$ is deflected parallel to the axis. They intersect at Q'', allowing us to locate the ***final image,*** $P''Q''$.

In sum, we first located the intermediate image, $P'Q'$, using only the first lens. We used this image to construct the three rays needed for ray tracing through the second lens. Having found the three rays, we then ignored the first lens and applied the ray tracing rules at the second lens to find the final image. If there were still more lenses, we would treat $P''Q''$ as an intermediate image and use it to locate the three rays for the next lens. (In general, the next lens may or may not deflect the light before it reaches this intermediate image. The techniques used here still apply.)

The situation becomes simple, however, in one special case: if *two thin lenses* are so close as to *touch* each other, they form a combination that behaves just like another thin lens; one whose *power* is the *sum of the powers* of the individual lenses. (See Appendix F for a proof.) Thus, suppose we have a 1-diopter

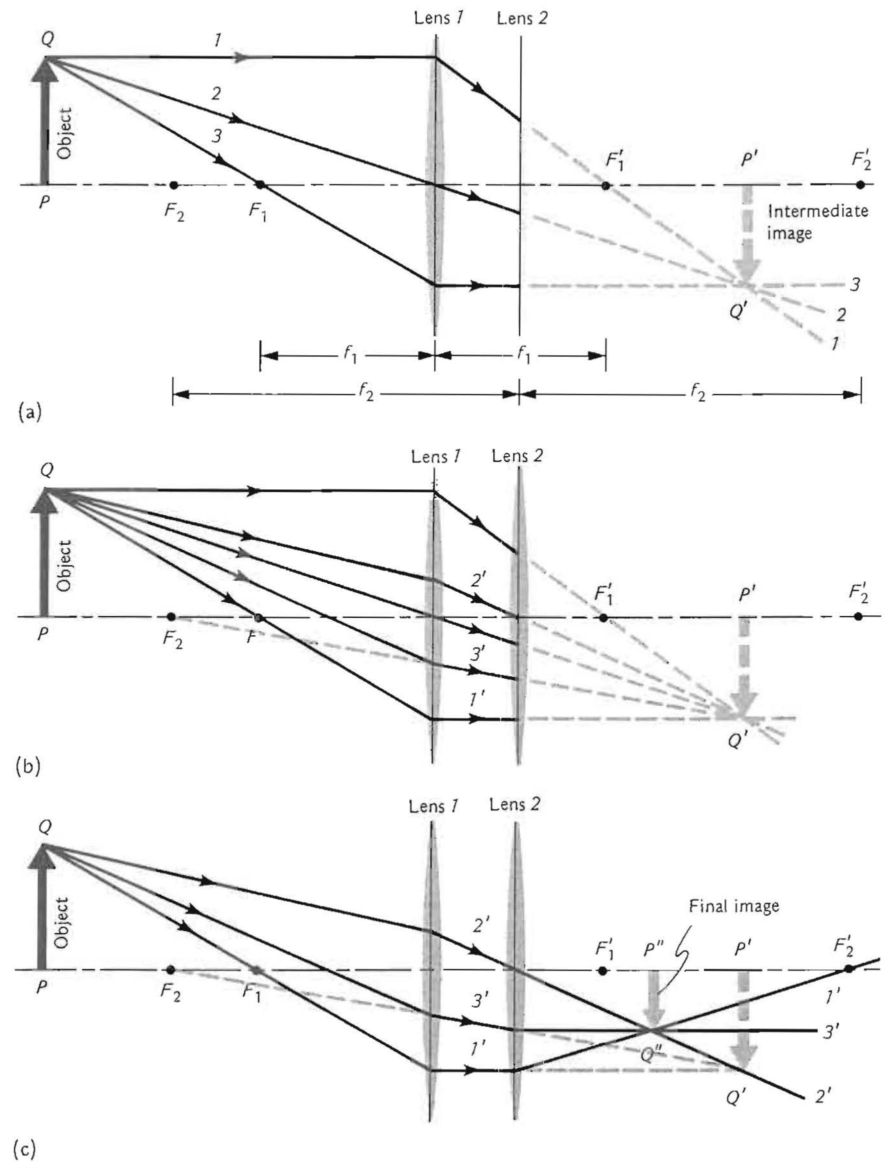


FIGURE 3.32

Construction by ray tracing of the final image of a compound lens consisting of two converging lenses. We have indicated all the focal points and the focal length of each lens. **(a)** Construct the intermediate image, due to the first lens alone. **(b)** Find the three rays incident on lens 2 that are needed for constructing the image *it* forms. **(c)** Use ray tracing rules on lens 2 to find the final image.

lens (focal length is 1 m) and a 4-diopter lens (focal length is $\frac{1}{4}$ m = 25 cm). Placing them together produces an equivalent single lens of 5 diopters (focal length is $\frac{1}{5}$ m = 20 cm). This rule works for negative as well as positive powers, and is the reason that power is a useful way of expressing focal length. In the TRY IT we show how you can measure an eyeglass prescription with the aid of this rule.

TRY IT

FOR SECTION 3.4E

Measuring your eyeglass prescription

Your eyeglass prescription is simply the power (in diopters) of your lenses. If you do not suffer from astigmatism (see Sec. 6.2B, not Sec. 3.5D), your eyeglass lenses are spherical lenses and you need only measure the focal length of each lens to determine its power. We suggest one method, but if you can think of others, try them to see if they all give the same result.

PONDER

How can you tell by sight whether your lens is converging or diverging? How can you tell by touch?

Converging Lenses: *Since objects are imaged on the focal plane, use your lens to form the sharpest image of the sun on a piece of paper, as you would with a burning glass. Measure the distance from the lens to the paper to obtain the focal length. Suppose the distance is 62.5 cm = 0.625 m. The power of that lens, and, therefore, its prescription, is then $\frac{1}{0.625}$ = 1.6 diopters.*

Diverging Lenses: *Find a magnifying glass and measure its focal length by the technique described above. Place it next to your eyeglass lens and measure the focal length, and hence the power, of the combination. (If it isn't converging, find a stronger magnifying glass.) The power of your lens is the power of the combination minus that of the magnifying glass alone (Sec. 3.4E). Suppose the combination has a power of 1.0 diopters (f = 1 m) and the magnifying glass has a power of 4.0 diopters (f = $\frac{1}{4}$ m). Your lens power is then −3.0 diopters.*

3.5 ABERRATIONS

Up to now we have assumed that all rays hitting our mirrors or lenses are paraxial, and we have ignored the fact that the glass in the lenses is dispersive. With these assumptions, spherical mirrors and lenses form sharp, perfect images. In actuality, however, the images formed are slightly blurred and distorted, and may have variously colored edges. That is, spherical lenses and mirrors, even if ground and polished perfectly, do not produce perfect images. The images' deviations from perfection are called ***aberrations.**** By combining various lenses, lens designers attempt to minimize those aberrations that are most important for the particular application in mind, often at the expense of others. For instance, the lens design for a telescope, used primarily for looking at objects very close to its axis, is quite different from that for a wide-angle camera lens, which accepts light from objects very far off-axis.

Clearly, with a bigger diameter lens (gathering more light), more of the accepted rays deviate from paraxial, and, therefore, the aberrations become more important. Cheap cameras have small diameter lenses for which most aberrations are insignificant. Expensive cameras have larger diameter lenses, which may require more than a dozen lens elements to correct for the aberrations. You pay, in large measure, to get rid of the aberrations.

A. Chromatic aberrations

Glass is a dispersive medium, bending blue light more than red light (Table 2.6). Figure 3.33a shows parallel white light being dispersed by a glass lens. The blue and red light have different focal points, F_B' and F_R'. The other colors are focused between. If we place a screen at F_B', we find a sharply focused blue dot surrounded by a halo of the other colors. The largest halo is red (Plate 3.1). Moving the screen to F_R' produces a sharply focused red dot surrounded by a halo of the other colors, with blue spread out the most. This colored blurring is called ***chromatic aberration.***

Such a lens in your camera would produce colored outlines on your

*Latin, *ab-errare* to stray away. Some of the light rays stray from the ideal focus.

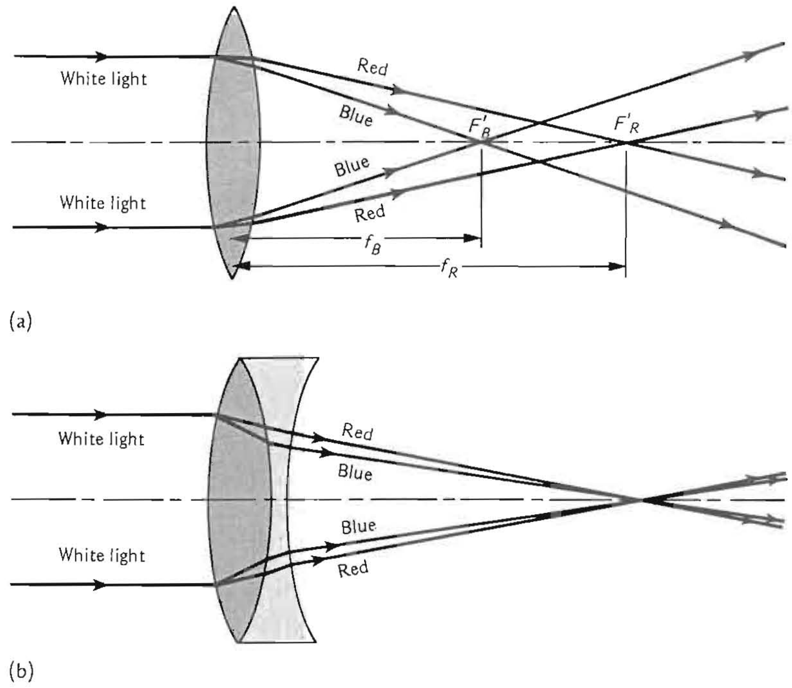


FIGURE 3.33

(a) Chromatic aberration of a converging lens. **(b)** Elimination of this aberration by an achromatic doublet.

photographs. Even if you used black and white film, the incident white light would be spread out into a blurred image.

To correct for this aberration, one uses an ***achromatic doublet:*** two lenses, cemented together, and made of different kinds of glass that have different dispersions (Fig. 3.33b). The converging lens, usually made of crown glass, converges the blue more than the red. The diverging lens, usually of flint glass, diverges the blue more than the red. If the converging lens has the greater power, the combination will be converging and, because of the differences in the glasses used, the red and blue can be focused at the same point. The other colors may not be focused exactly at that point, but will be closer together than in the uncorrected lens of Figure 3.33a. Even the cheapest cameras usually use an achromatic doublet to eliminate this aberration.

Mirrors do not exhibit chromatic aberration because the law of reflection is the same for light of any color. Telescope designers often avoid chromatic aberration by using mirrors instead of lenses.

B. Spherical aberrations of lenses

When we relax the paraxial condition and accept rays far from the axis of a spherical lens, we find that they do not all pass through the same focus. Figure 3.34 shows an exaggerated example of this. No matter where you put your film or screen, the image is not a sharp point, but is spread out forming a circular blur, called the ***circle of confusion.*** The smallest circle of confusion is at the plane marked *A*. This is where you would place the screen to obtain the smallest blur. This blurring is called ***spherical aberration.*** The TRY IT (after Sec. 3.5C) tells you how to demonstrate this phenomenon.

For a given *object distance*, you can correct for spherical aberration by using a lens with surfaces that are not exactly spherical. To minimize this aberration for a *range* of object distances you need a compound lens.

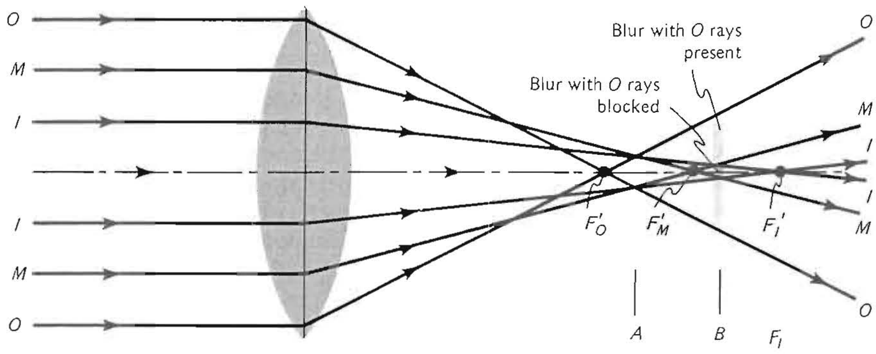


FIGURE 3.34

Spherical aberration in a converging lens. The inner rays, I, closest to the axis, have the farthest focal point, F_I'. The outer rays, O, are bent most, so they have the nearest focal point, F_O'. The rays in the middle, M, are focused between these extremes, at F_M'.

A simpler and cheaper way of diminishing this problem is to use a single, smaller diameter lens. If you block the outer rays, O in Figure 3.34, you can move the screen to the plane marked B and have a smaller blur than you did at A. (With the O rays present, the blur at B would be huge.) Thus, cheap cameras avoid the expense of multielement lenses to diminish spherical aberration by using a small diameter lens.

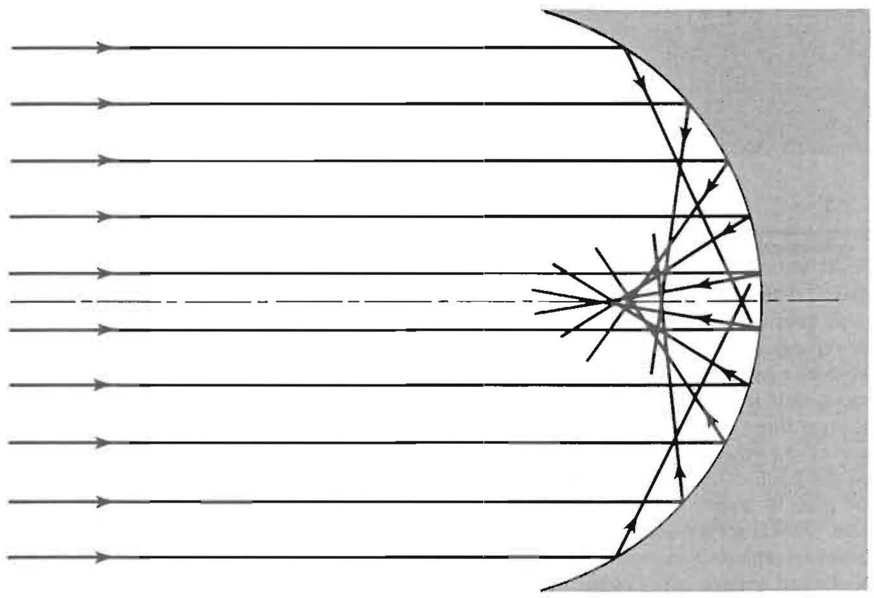


FIGURE 3.35

Spherical aberration in a concave mirror.

C. Spherical aberrations of mirrors

Spherical aberration also occurs for spherical mirrors. Nonparaxial parallel rays incident on such a mirror are not reflected to a sharp focus, but rather spread out in a pattern like the one illustrated in Figure 3.35. (This pattern can easily be seen, as described in the TRY IT.) Note that the more central rays are almost brought to a good focus. When the peripheral rays are needed for brightness, mirrors that are supposed to focus incident parallel rays, as in a telescope, are made flatter at larger distances from the axis, in the shape of a paraboloid (Fig. 3.36). Such ***parabolic reflectors*** are quite common. They are used in headlights and searchlights to take light from a point source and make a parallel beam. Microwave and radar antennas, which may be sending or receiving, are also parabolic. Parabolic microphones, used to pick up sound waves from distant sources, have a small microphone located at

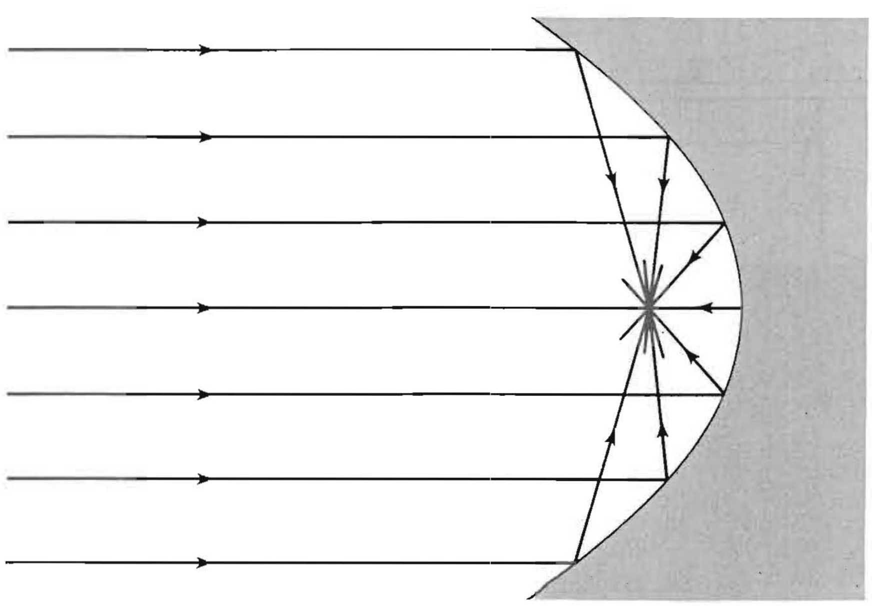


FIGURE 3.36

A parabolic reflector has no aberrations if the object is on the axis and very far away.

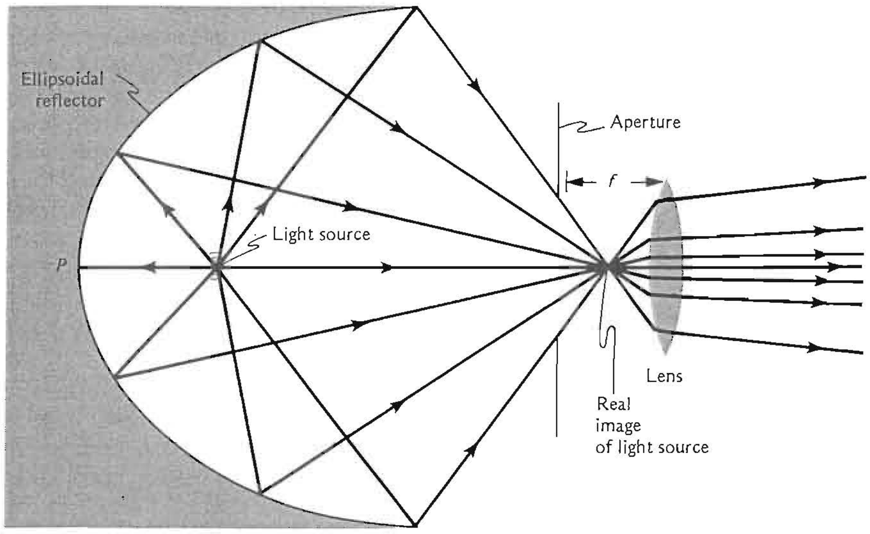


FIGURE 3.37

An ellipsoidal spotlight.

the focus of the (sound) reflecting paraboloid.

To bring most of the light from a nearby source to a focus without aberrations, you need an ***ellipsoidal reflector.*** It has two points called foci, and all light from one focus is reflected to form a real image at the other focus, without aberration. If you replace a part of the ellipsoid by a lens placed in front of that real image, you can make a beam for use as a theater spotlight (Fig. 3.37). There are two advantages of this scheme. First, you can use a short focal length lens without worrying about the lens hitting the light bulb. Also, you can put an aperture behind the real image, and the lens will focus that aperture on the stage. You then have a spotlight beam that, when it is incident on the stage, has any shape you like—the image of the aperture shape.

*D. *Off-axis aberrations*

Even if a lens or mirror makes a perfect image on its axis, as a parabolic or ellipsoidal reflector does, other aberrations generally occur if the object does not lie on the axis. One such ***off-axis aberration*** is called ***curvature of field.*** Here the image of a flat object does not lie in a plane, but rather on a curved surface (Fig. 3.39). If you try to focus the image on a flat screen, the center will be in focus while the extremes are blurred, or vice versa. For this reason, large screens such as one finds in drive-in theaters or projection TV systems are usually curved.

TRY IT

FOR SECTIONS 3.5B AND C
Spherical aberration in lenses and mirrors

To see a pattern of spherical aberration produced by a lens, you need a candle and a clear glass filled with water. Arrange them as shown in Figure 3.38a, turn off all other lights, and look down on the table at the light transmitted by the glass of water, which serves as a lens. (This is a cylindrical lens, not a spherical lens, but the spherical aberration pattern you'll see is the same.) You may have to move the candle around a little before you get a good spherical aberration pattern. Dust on the table will scatter the light from the pattern to your eye. The pattern will be most visible against a dark table.

To see the pattern of spherical aberration produced by a mirror, you'll need a china cup partially filled with dark coffee or tea and a nice sunny day (Fig. 3.38b). Allow the parallel light from the distant sun to reflect off the inside rim of the cup, which will serve as the "spherical" mirror. The pattern should be visible on the surface of the dark liquid in the cup, and should look like Figure 3.35.

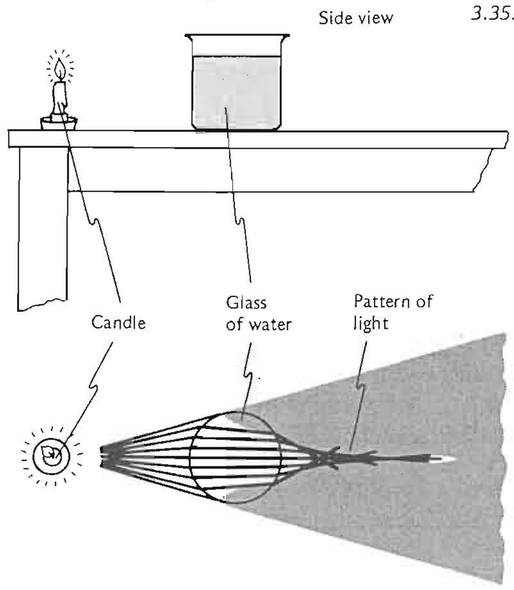


FIGURE 3.38

(a) Spherical aberration in a water glass lens (side and top views). (b) Photograph of spherical aberration pattern in a teacup reflector.

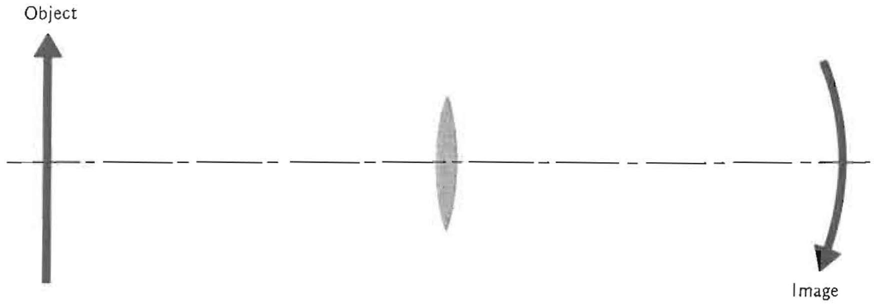


FIGURE 3.39

Curvature of field of a converging lens.

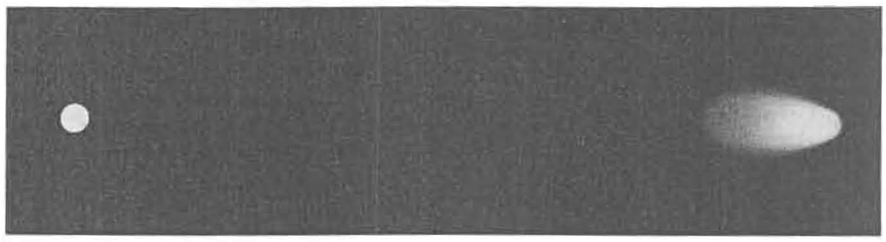


FIGURE 3.40

A small circular spot of light *on the axis* is projected by a lens to form the faithful image on the left (no coma). An identical *off-axis* spot of light produces the image with coma, on the right.

Coma, another off-axis aberration, is a modification of spherical aberration for off-axis objects. For a point source, the blur due to spherical aberration that surrounds an on-axis image shifts to one side when the image is off-axis. The result is that the image of a point source has a flaired tail, something like a comet* (for which the aberration is named) (Fig. 3.40).

Astigmatism is a difference in focal length for rays coming in different planes from an off-axis object. For example, the rays that we have drawn in Figure 3.26 all lie in the plane of the paper. If there is astigmatism, rays from Q that *leave* the plane of the paper and hit the lens are brought to a slightly different focus than Q'. The sharpest image of the point Q is then a short line, perpendicular to the paper at Q', or a short line in the plane of the paper at the other focus. Somewhere between these two points, the image of Q is smeared into a small circle (Fig. 3.41). (It is unfortunate that a lens defect of the eye, caused

*Greek *kometes*, long-haired.

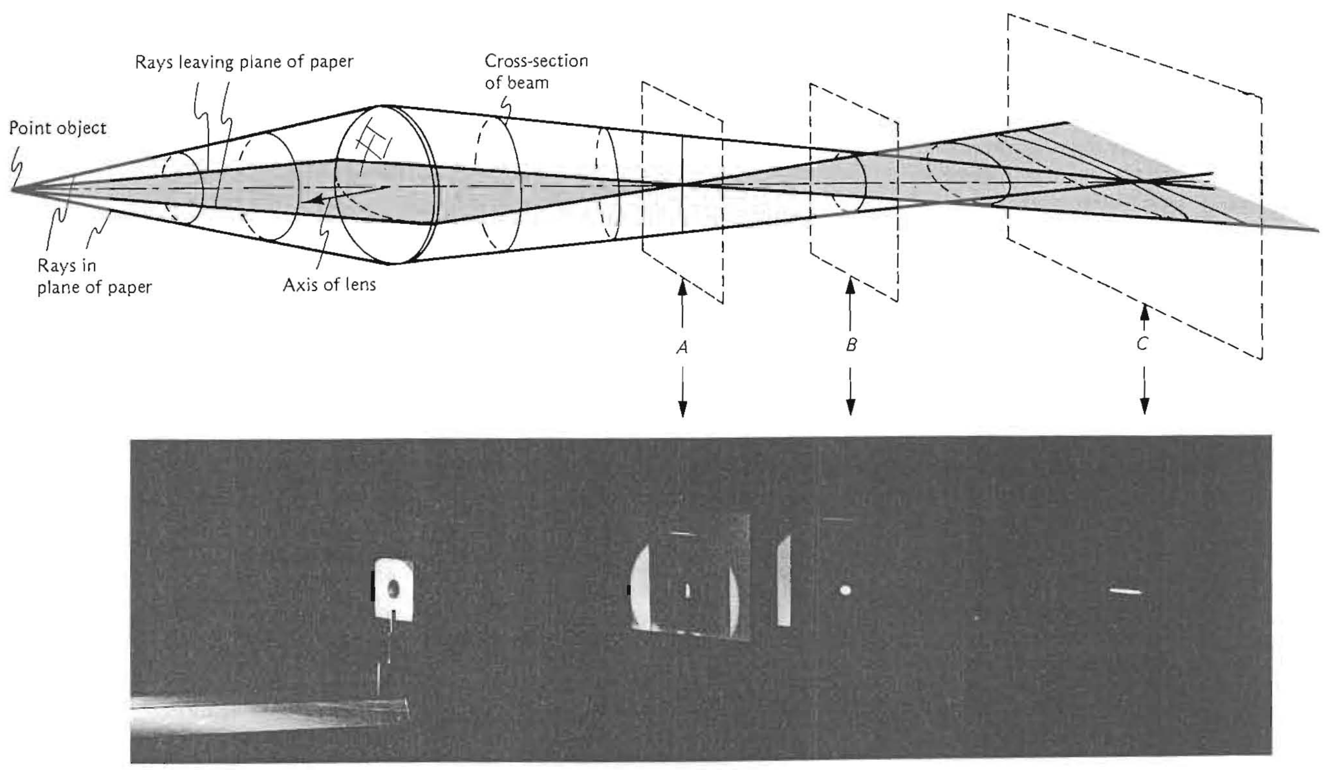


FIGURE 3.41

The astigmatic image of a point object in three planes, perpendicular to the axis. When the screen is at *A*, the image is a vertical line, and it is a horizontal line when the screen is at *C*. A screen at *B* gives the smallest circular image. (The photograph was taken by triple exposure, changing the position of a single screen between exposures.)

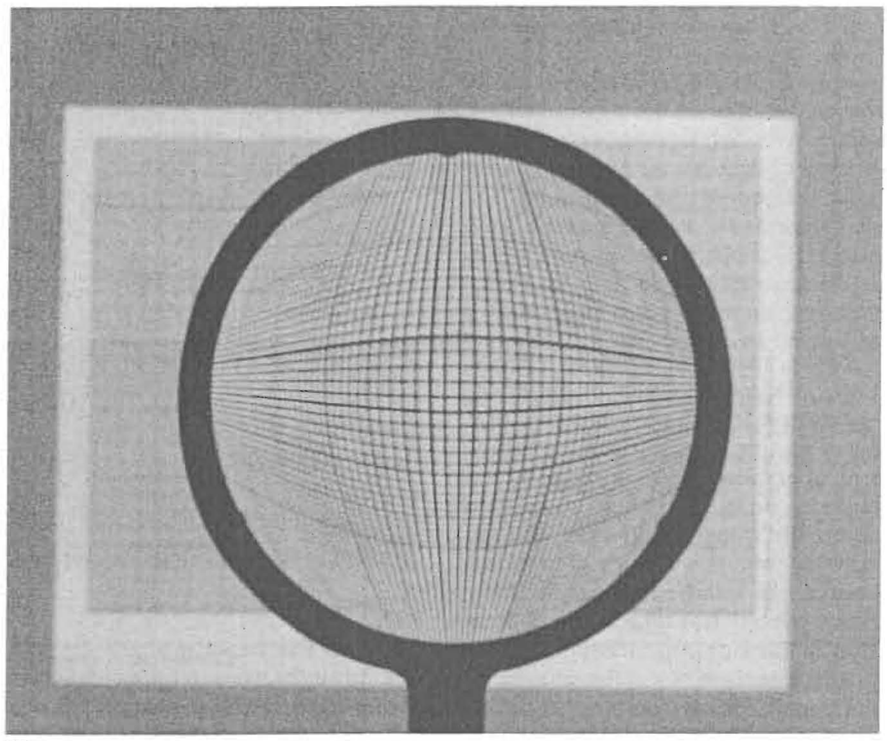
by nonspherical curvature of the lens, is also called astigmatism—see Sec. 6.2B.)

The final off-axis aberration is **distortion.** The image of an object that extends off-axis is pulled out of shape, as if first drawn on rubber and then stretched. Figure 3.42 shows a rectangular grid under two types of distortion.

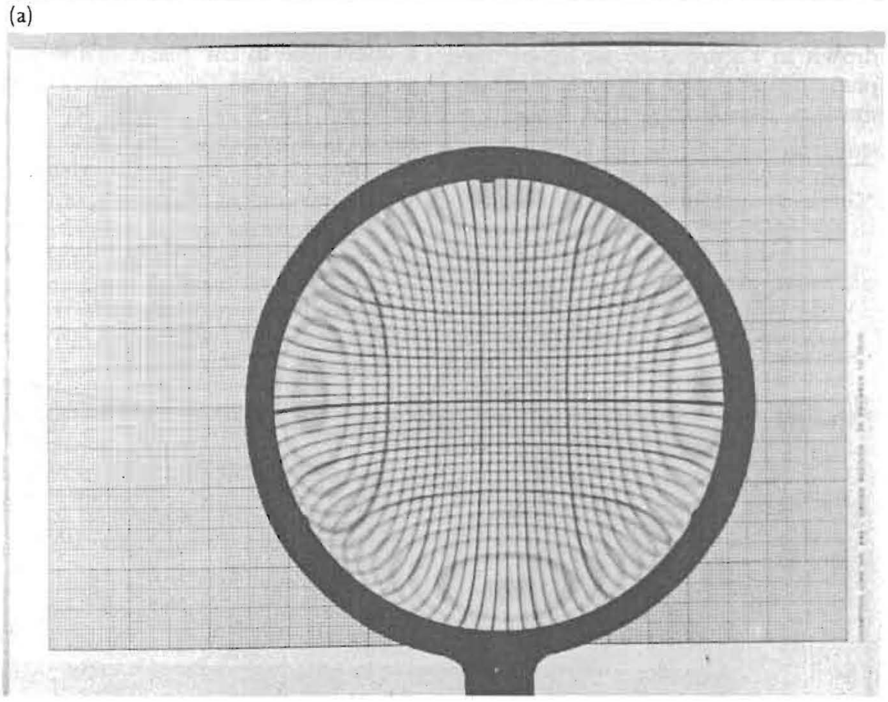
The amount and nature of aberrations depend on where the ***stops*** in the optical system are located, that is, how *the dimensions of the beam are limited*—for example, by an aperture or by the outer edge of a lens. The location of the stop determines which parts of the object can send rays to the periphery of the lens, where they will be bent too much. In particular, distortion changes from *barrel distortion* to *pincushion distortion* as the stop is moved from in *front* to *behind* the lens. The diaphragm in a camera, as well as your eye's iris, which serves as a stop, is placed *between* the lenses. The pincushion distortion of the front lens is then balanced by the barrel distortion of the rear lens, giving a relatively undistorted image.

These aberrations often are minor effects, but for precise work, as required in modern optical instruments, they become significant. Lens designers try to reduce these effects, usually by adding additional lens elements, but also by choosing lens *shapes* carefully. For example, a focusing lens with a given focal length may have both sides bulging outward (a ***double-convex lens***), or only one side bulging outward and the other flat (a ***plano-convex lens***) or even curving inward slightly (a ***meniscus lens***)—just so long as the middle of the lens is thicker than the perimeter. However, their aberrations will be different. Indeed, simply turning a plano-convex lens around changes the aberrations it produces. The size of these aberrations also depends on the location of the object, so the design must depend on the use of the lens.

Many of these aberrations may be seen with a magnifying glass and the help of the TRY IT.



(a)



(b)

FIGURE 3.42

Graph paper with square rulings as seen through a lens that exhibits distortion. (Photographs taken by method described in the TRY IT.) **(a) Barrel distortion. (b) Pincushion distortion.**

PONDER

Convince yourself, using a similar construction to that of Figure 3.21a, that a plano-convex lens, facing either way, is a focusing lens.

TRY IT

FOR SECTION 3.5D
Aberrations of a magnifying glass

You will need a focusing lens, such as a magnifying glass, preferably of large diameter. Image the grid of a window with sunlight shining through onto a piece of paper. Can you get every point of the window grid in focus at once, or does one position of the paper provide a better focus for some parts, and other positions for others? What aberration is this? Do straight lines of the grid image as straight lines? If not, what type of distortion do you have? Does twisting the lens slightly affect the aberrations? Why should it?

Make a point source by covering a light source with aluminum foil that has a pinhole in it. Image this source on a piece of paper. How sharp an image can you make? Is there spherical aberration? Can you see any colors around the image? What happens to the image when you twist the lens? What kind of aberration can you see?

Look through the lens at the lines on a piece of ruled paper. The lens should be just far enough from the paper (somewhat beyond a focal length) so that the image is inverted. Your eye must be far enough from the lens so that the image is in focus for you. What kind of distortion do you see? Now move the lens closer to the paper (a little less than a focal length away) and examine the distortion again (also see Fig. 3.42).

How can there be distortion without any stop evident? The answer is that the pupil of your eye is usually the stop when you are looking through the lens. Because of this stop, a lens sometimes gives a better image when you look through it than when you use it to project an image. You can see this effect best with a large lens. Try it!

SUMMARY

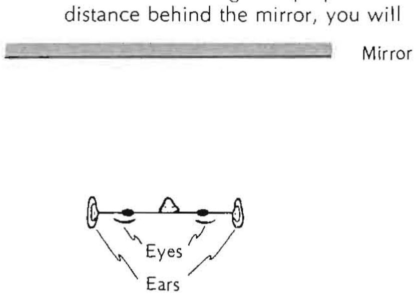
Lenses and ***mirrors*** form ***images*** depending on the relative location of the object. The images are located using the *rules* of ***ray tracing.*** Light actually passes through ***real images,*** but only seems to come from ***virtual images.*** A plane mirror produces virtual images behind its surface. *Multiple reflections* may produce many images, for example, in a ***kaleidoscope.***

Both *spherical* lenses and mirrors are characterized by their ***focal lengths*** *(f)*. A spherical *mirror* has only one ***focal point*** *(F)*; it is midway between the mirror and its *center of curvature (C). Lenses* have two *focal points (F,F′)*. For a *thin lens* they are on opposite sides and equidistant from the lens. *Positive* focal length lenses are ***converging*** lenses, while *negative* focal length lenses are ***diverging*** lenses. Converging lenses can be used to produce a *retroreflector* ***(heiligenschein). Fresnel lenses*** are made thin by eliminating much of the glass (or plastic). The ***power*** (in ***diopters*** = $1/f$, f in meters) of a compound lens, made of two or more thin lenses in *contact,* is equal to the *sum of the powers* of its component lenses.

Aberrations are imperfections of the image produced by a lens or mirror. ***Chromatic aberration*** in lenses results from dispersion and can be corrected using an ***achromatic doublet. Spherical aberration*** can be corrected by using a lens or a mirror of nonspherical shape, or by allowing, through the use of ***stops,*** only ***paraxial*** rays (near the axis and nearly parallel to it) to be focused by the lens or mirror. A *parabolic mirror* has no aberrations for distant on-axis sources. ***Off-axis aberrations*** in lenses ***(curvature of field, coma, distortion*** such as ***pincushion*** or ***barrel distortion)*** can be reduced by proper placement of stops. They depend on the lens shape (***double-convex, plano-convex, meniscus,*** etc.).

PROBLEMS

P1 Copy the figure, which shows a top view of a (flat) face looking in a mirror. (a) By drawing rays, show where the image of the right ear is. To locate the image the proper distance behind the mirror, you will need two rays from the right ear that obey the law of reflection.
(b) Complete the image and use it to draw the ray from each ear to the right eye. (c) The person with this flat face wants to mark the outline of her face on the steamed-up mirror. She closes her left eye and, using only her right eye, writes an *R* where she sees her right ear and an *L* where she sees her left ear. Clearly indicate where she writes those letters on the mirror.

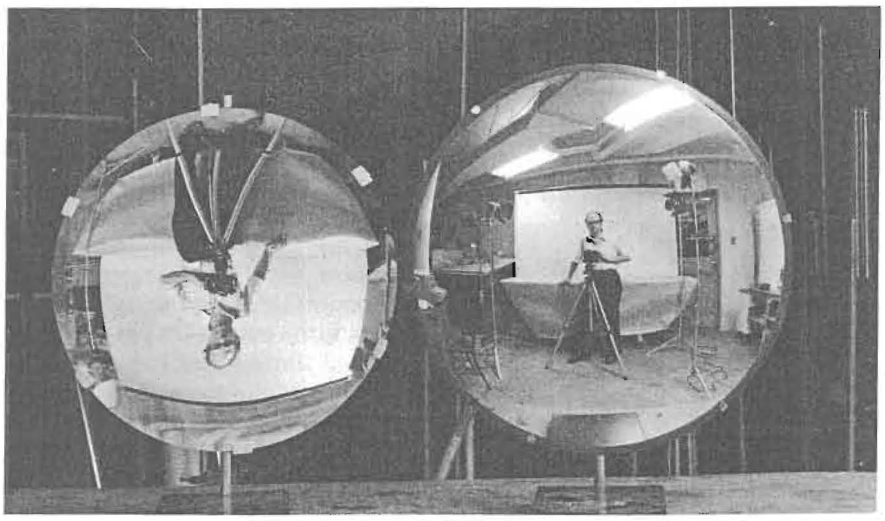


P2 An arrow is viewed through a kaleidoscope that has an angle of 45° between its mirrors. Draw two lines meeting at a 45° angle to represent the mirrors, and, between them, an arrow pointing at one of them. Sketch all the images of the arrow.

P3 (a) Use ray tracing to find the image of a 2-cm arrow in a concave mirror of focal length 2 cm, when the arrow is 6 cm in front of the mirror and perpendicular to the axis. (b) Is the image real or virtual? Upright or inverted? Larger or smaller than the object?

P4 Repeat Problem P3, but with the object a 1-cm arrow, only 1 cm in front of the mirror.

P5 Repeat Problem P3 for a convex mirror of focal length 2 cm, with the object 2 cm in front of the mirror.



P6 In the photograph, identify which mirror is convex and which is concave.

P7 (a) When you use a lens as a burning glass, it forms a small bright spot. Is the lens then forming an image, and if so, of what? (b) Why does it burn? (c) What would the bright spot look like during a partial solar eclipse?

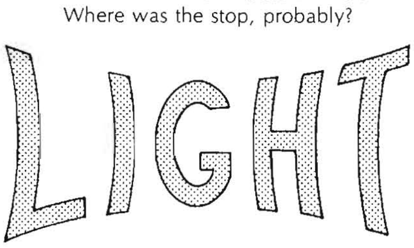
P8 Here is a problem that occurs almost every day. You are stranded on a sunny desert isle with only a standard flashlight but no batteries. How could you use some part of the flashlight to ignite a small dry leaf, and thus start a fire?

P9 (a) Use ray tracing to find the image of a 2-cm arrow due to a converging lens of focal length 5 cm, when the arrow is 10 cm in front of the lens and perpendicular to the axis. (b) Is the image real or virtual? Upright or inverted? Larger, smaller, or the same size as the object?

P10 Repeat Problem P9, but with the object only 2.5 cm in front of the lens.

P11 Repeat Problem P9, but for a diverging lens of focal length −5 cm, with the object 10 cm in front of the lens.

P12 The word "LIGHT" appears as shown in the figure, when viewed through a lens. What kind of distortion is this? Where was the stop, probably?



P13 Do front surface mirrors have chromatic aberration?

P14 Images in a very thick glass mirror may be slightly colored by dispersion. Draw two vertical lines, separated by about 2 cm, to represent the front (glass) surface and the rear (silver) surface of the mirror. About 2 cm in front of the mirror, draw the object—a white *point* source. (a) Carefully draw red and blue rays to represent the red and blue light. Show that there are separate red and blue images, and indicate their locations. (Hint: To locate each image you'll need two rays. Choose as one the ray perpendicular to the mirror.) (b) Where would the image be if the glass were removed, but the silver and everything else remained in the same positions? Would the image be colored in this case?

P15 A certain thin lens has an extremely large amount of chromatic aberration. Its focal length is 3 cm for blue light and 4 cm for red light. A 2-cm arrow is 8 cm in front of the lens and is illuminated with white light. (a) By ray tracing construct the images for red and blue light. (b) Draw a sketch indicating what you would see on a screen that is placed at the red image.

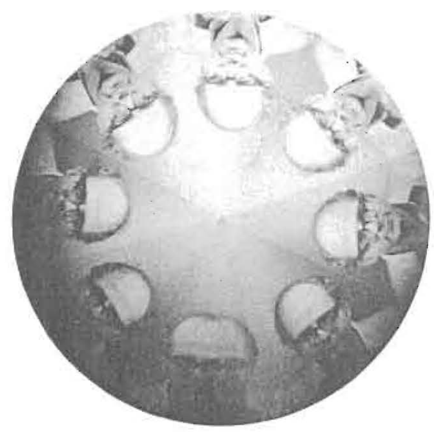
P16 An object (the usual arrow) is located at the focal plane of a thin lens of focal length *f*. On the other side of the lens, at a distance $\frac{1}{2}f$ from the lens and parallel to it, is a plane mirror. (a) Use ray tracing to find two rays leaving the lens. (b) Use the law of reflection to determine how these rays are reflected. (c) Use ray tracing again to follow the reflected rays back through the lens, and thus find the final image.

P17 In *Tess of the D'Urbervilles*, Thomas Hardy writes:
Then these children of the open air, whom every excess of alcohol could scarce injure permanently, betook themselves to the field-path; and as they went there moved onward with them, around the shadow of each one's head, a circle of opalized light, formed by the moon's rays upon the glistening sheet of dew. Each pedestrian could see no halo but his or her own, which never deserted the head shadow, whatever its vulgar unsteadiness might be; but adhered to it; and persistently beautified it, till the erratic motions seemed an inherent part of the irradiation, and the fumes of their breathing a component of the night's mist; and the spirit of the scene, and of the moonlight, and of Nature, seemed harmoniously to mingle with the spirit of the wine.
Explain the phenomenon Hardy describes.

HARDER PROBLEMS

PH1 (a) What is the maximum number of images of himself that can be seen by the man in Problem P13 of Chapter 2? (b) If the man raises his right hand, which of these images will raise their right hands, and which their left?

PH2 How was this photo taken? Discuss in detail how many mirrors were used, at what angles, the unequal brightness of the images, which image was formed by direct light (without mirror reflection), and the reason for the double image at the bottom.



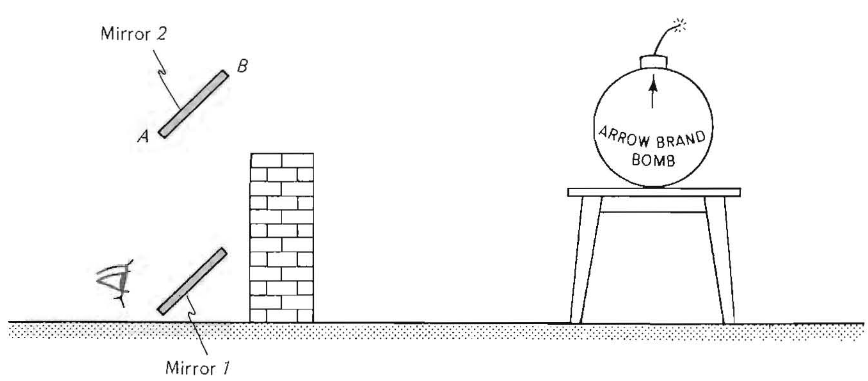
PH3 A shaving or makeup mirror is a large concave mirror. In terms of its focal length *f*, what is the *range* of distances from the mirror at which you can put your face and get an enlarged image in the mirror?

PH4 Will an underwater air pocket in the shape of a plano-convex lens act like a converging or like a diverging lens?

PH5 When you wear goggles underwater, objects usually appear larger than normal. Trace rays from the top and bottom of an object *PQ* to an eye to verify this statement. Draw a vertical line to represent the front of the goggles, with the eye on one side of the line, and water and the object on the other. Use Snell's law to trace the rays at the front of the goggles.

PH6 (a) Without goggles, things appear blurred underwater to you. Why? (b) The goggles form a plano-concave pocket of air in front of your eye—the flat part is formed by the glass (or plastic) plate, while the concave part is formed by your cornea. Use your answer to problem PH4 to explain why diving masks allow you to see clearly underwater.

PH7 The figure shows a periscope used to view a bomb being defused from a safe position behind a thick wall. (a) Find the image of mirror *2* seen by the eye, by locating the images of points *A* and *B* in mirror *1*. (b) Find the final image of the arrow as seen by the eye. If you do this by ray tracing, you must draw the rays *very* accurately. Preferably do it by first finding the intermediate image of the arrow in mirror *2*, and then finding the final image in mirror *1*. Before starting to draw, think about where the intermediate image will be, so you leave plenty of room for it. Make sure the distance from the eye to the final image is equal to the distance that the light actually travels from the object to mirror *2* to mirror *1* and then to the eye. (c) Draw the rays that define the field of view of the eye—that is, the rays that just hit the edges *A* and *B* of mirror *2*. What happens to this field of view if the periscope is made longer to see over a higher wall?



PH8 On a page turned sideways draw a horizontal line to represent the axis. Near the middle of this line, draw a 3-cm arrow to represent an *image* created by some lens not yet drawn. At a distance 2 cm to the right of this image, draw a 2-cm arrow to represent the *object* that produced this image. (Both arrows should point up.) Your job is to figure out what kind of lens will produce this image, and where it must be placed. Do this in steps as follows: (a) Use ray *2* to figure out the only place the lens can be placed to produce the image. (b) Now that you have located the lens, use ray *1* to locate the focal point *F′*. (c) Similarly, use ray *3* to locate the focal point *F*. (d) Since you know where the image is, you know how any ray from the object behaves. Draw one ray from the point of the object arrow to an eye located on the axis 5 cm to the right of the lens. (e) Does the image look larger or smaller to the eye than the object did without the lens? Is the image real or virtual? Is the lens converging or diverging?

PH9 A spotlight consists of a light source (say an incandescent bulb), a concave mirror, and a converging lens. The bulb is located at the center of the mirror, and the lens is a distance equal to its focal length in front of the bulb. Draw a sketch of such a spotlight. (a) Where is the image of the bulb in the mirror? (b) Where is the final image of the bulb after the light has passed through the lens? (c) Draw a typical ray that goes directly from the bulb through the lens. (d) Draw a typical ray that goes from the bulb via the mirror through the lens. (e) Explain why each of the elements is used in a spotlight.

PH10 A *beam expander*, used to widen a laser beam, consists of two lenses. A laser sends parallel light into the expander. In order to be able to see the effect, draw a beam of parallel light 2-cm wide to represent the laser beam. The first lens is a diverging lens of focal length −3 cm. (a) Draw the two rays that represent the edges of the beam after they pass through the first lens. (b) A second lens is located 6 cm after the first lens. It makes the rays parallel again. Should the second lens be a diverging or converging lens? Draw a suitable lens and the final beam. (c) What should be the focal length of the second lens?

PH11 A 1-cm arrow is located 1 cm in front of a converging lens of focal length 2 cm. On the other side of the lens, a distance 4 cm from it, is a second converging lens of focal length 6 cm. Use ray tracing to locate the final image produced by the two lenses. First find the image produced by the first lens alone, then use that image as a new object for the second lens, as if the first lens were no longer there.

PH12 An object (the arrow, as usual) is located at a distance $2f$ in front of a converging lens. (a) Use ray tracing to find the image. (b) Now draw a plane mirror at 45° to the axis, crossing the axis at a distance $4f$ behind the lens, and locate the final reflected image. (c) Repeat parts (a) and (b), but with the mirror at a distance f behind the lens.

MATHEMATICAL PROBLEMS

PM1 By finding two suitable congruent triangles in Figure 3.2a, prove that the distances $\overline{QO}$ and $\overline{Q'O}$ are equal.

PM2 Show that the central ray through a thin lens is undeviated, as follows. Consider the center of the lens as a slab of glass with *parallel* sides. By applying Snell's law (Eq. B5) twice, show that any ray entering one side of such a slab will emerge from the other side parallel to its original direction. The displacement between these parallel beams will be small for a thin lens.

PM3 The focal length of lens *1* is −25 cm, and the focal length of lens *2* is +75 cm. What are the powers (in diopters) of the two lenses? When the two lenses are brought together to form one compound lens, what is the resulting power? What is the focal length of the compound lens?

PM4 (a) Repeat Problem P3a without ray tracing, by using the mirror equation. (b) What is the magnification?

PM5 (a) Repeat Problem P4a without ray tracing, by using the mirror equation. (b) What is the magnification?

PM6 (a) Repeat Problem P5a without ray tracing, by using the mirror equation. (b) What is the magnification?

PM7 (a) Repeat Problem P9a without ray tracing, by using the lens equation. (b) What is the magnification?

PM8 (a) Repeat Problem P10a without ray tracing, by using the lens equation. (b) What is the magnification?

PM9 (a) Repeat Problem P11a without ray tracing, by using the lens equation. (b) What is the magnification?

PM10 Verify your result of Problem PH8 by seeing if it satisfies the lens equation.

PM11 Repeat Problem PH11 by using the lens equation twice, for the same two steps that you used when doing the problem by ray tracing.

Solar power

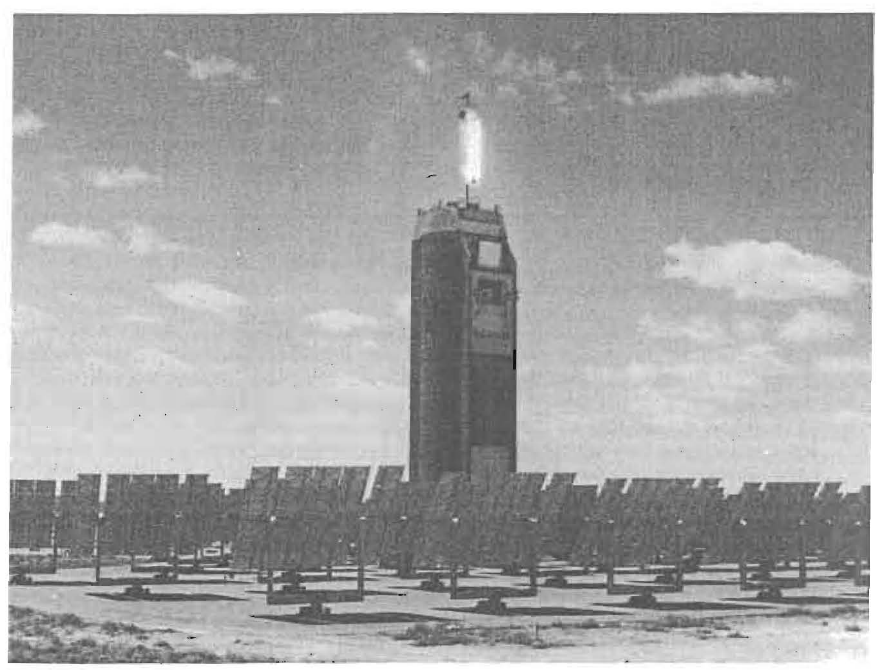
To produce solar power, you must collect energy from the sun, and extract that energy for a useful purpose, such as heating your home or making electricity. At best, any solar ***collector*** can absorb all the energy in the sunlight that it intercepts. Although the energy flux from sunlight that arrives at the ground varies greatly with weather conditions, it seldom exceeds a kilowatt per square meter. Thus, to yield an output of, say, 5 kilowatts, the collector surface must be at least 5 m^2. For maximum efficiency, of course, the collector should be oriented perpendicular to the sunlight's direction of incidence.

Some of today's large-scale collectors use a technique not much different from Archimedes' (Sec. 3.3D). In the ***solar tower*** (Fig. FO.2), a large flat area is covered with mirrors, each reflecting a beam from the sun onto a boiler on top of a tall tower, where water is heated to steam that drives generators of electricity. Instead of Archimedes' soldiers, automatic clock-driven mechanisms, called ***heliostats,*** are used to make each of the hundreds or thousands of mirrors follow the sun. (The many synchronized heliostats constitute a significant cost of these systems.)

Another solar collector that concentrates in a small area all the energy that falls on a large surface consists of a spherical or parabolic

FIGURE FO.2

Photograph of the Central Receiver Test Facility near Albuquerque, NM. Over 200 mirrors (some of which are shown in the foreground) are used to direct up to five million watts of sunlight toward the top of the tower, where different types of collectors can be tested.



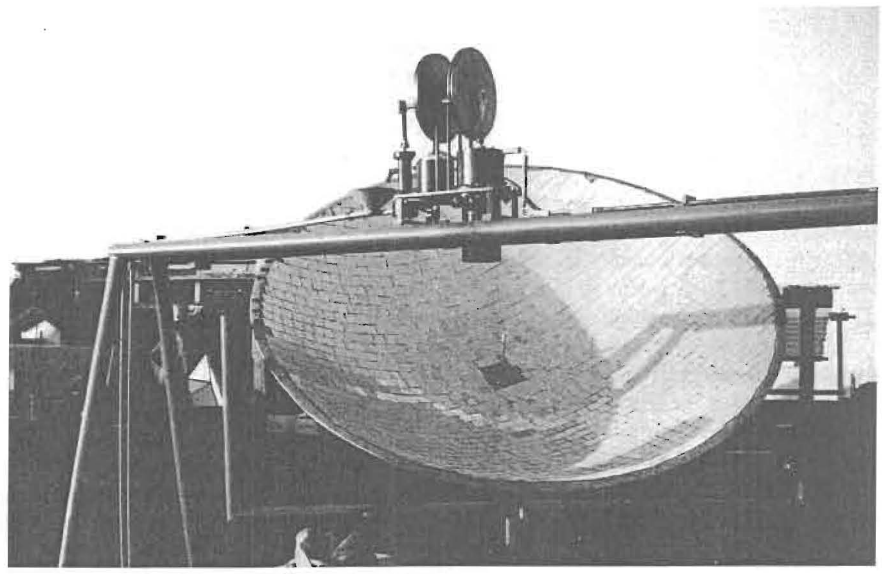


FIGURE FO.3

This parabolic reflector, in Bhavnagar, India, is made of many small plane mirrors. The sunlight is concentrated on a Sterling engine (similar to a steam engine, but it uses no water) that pumps water from a well.

reflector (Sec. 3.5C). The boiler, or ***absorber,*** is at the focus of the reflector. To make a large reflector surface sufficiently cheaply, it is often made of individual pieces of plane mirrors (Fig. FO.3). As in the case of the solar tower, a heliostat is needed to keep the reflector-absorber combination pointed at the sun.

If not as much concentration is needed, one uses ***cylindrical*** collectors. These may consist either of a cylindrical mirror or a cylindrical Fresnel lens (Sec. 3.4D) to concentrate the light onto a cylindrical absorber. If the cylinder axis is oriented in the east-west direction, these collectors need not be moved to follow the sun. A simpler type of solar collector that does not concentrate the energy at all involves just an absorbing ***flat plate.*** Such collectors are used on rooftops to heat up water flowing through them.

The reasons for using concentrating, rather than flat-plate, collectors involve questions of heat transfer and energy conversion. If solar energy is to be used for heating, the heat can be developed at low temperature; warm air from a solar greenhouse, or warm water from a flat-plate collector on your roof, is quite adequate for most home requirements. However, suppose that the *output* is to be electrical energy, but that the solar energy is to be *absorbed* as heat (rather than by direct conversion—Sec. 15.2B). This heat must then be developed at high temperature in order to be efficiently convertible to electricity, so concentrators are used. There is a limit, however, to how high a temperature can be developed by concentrating the sun's rays, for the following reason. A solar parabolic concentrator forms an image of the sun, which acts as a black body at the sun's surface temperature, 5500°C, at best. Anything placed at the focus of any concentrator, therefore, can be heated to a temperature no higher than this. In practice, the limit is much lower due to heat losses and imperfection of the image. For example, if the heated surface is large, it is likely to lose heat rapidly. Also, if the temperature of the heated surface is high, it is likely to lose heat much more rapidly than if it is kept relatively cool (e.g., by rapid circulation of the water in a flat-plate collector).

Heat losses are classified into three types: ***conduction, convection,*** and ***radiation.*** Conductive loss is due to heat transmission in matter, such as the collector supports, where there is no motion of the matter itself. Convective loss is due to heat carried away by some moving material, usually air. Radiative loss is due to electromagnetic radiation, which carries away some of the energy. Conductive losses are minimized in flat-plate collectors by backing them with insulation material, such as styrofoam. Convective losses are reduced by covering the collector with one or several sheets of glass at a distance of a few centimeters from the collector, which prevents rapid motion of the air—even at a larger distance, as in a greenhouse, a piece of glass or plastic can still aid significantly in reducing convective losses. (However, each glass-air surface reflects at least 4%, and more at oblique incidence. A comparable amount may also be absorbed by the glass.) Radiative losses can also be controlled by the glass cover's greenhouse effect (see the FOCUS ON Light, Life, and the Atmosphere). For more efficiency in combating radiative losses, the collector can be given a ***selective surface***—one that does *not* behave like a black body. This surface should absorb in the visible, where the energy of sunlight is greatest. However, it should be a poor radiator in the infrared so it doesn't radiate away its energy as it heats up. Such a surface can be made by coating a metal (which radiates poorly in the visible and infrared) with a layer of semiconductor (which absorbs in the visible, but is both transparent and does not radiate in the infrared).

Clearly, the surfaces of solar collectors are very important. Keeping them clean, free from dust, and protected from oxidation, corrosion, and hailstorms is a major problem in any large-area collection scheme.