lenses and also to invert the image (Sec. 6.4B).

PONDER

Would the devices of Fig. 2.56 work under water?

*B. Fiber optics

Another application of total internal reflection is ***fiber optics***—the use of thin flexible glass or plastic fibers as ***light pipes*** (Fig. 2.57a). You send light down the fiber, which has a larger index of refraction than its surroundings. If the light in the fiber hits the surface at an angle of incidence greater than the critical angle, it is totally internally reflected, and there are no losses due to light escaping. The light continues bouncing back and forth down the fiber, even if the fiber bends. As long as the fiber does not bend too sharply (so the incident angle is always greater than the critical angle), the light is gradually bent with the fiber.

You see this effect in fountains where the "fibers" are streams of water, in advertising displays, and in "decorative" lamps (Fig. 2.58). These light pipes are easily adjustable light sources for miniature photography, and are used as tiny window lights on spacecraft models in science fiction films. (The TRY IT gives another example.)

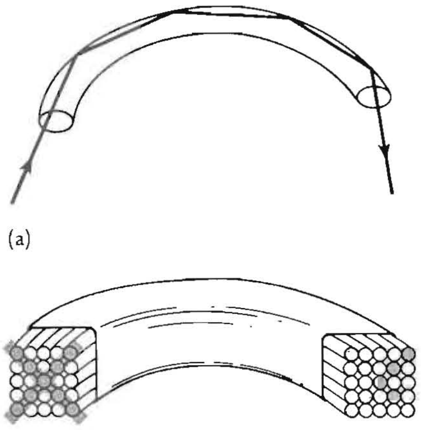


FIGURE 2.57

(a) A glass or plastic fiber can be used as a light pipe. **(b)** Many light pipes packed together can transmit an image.

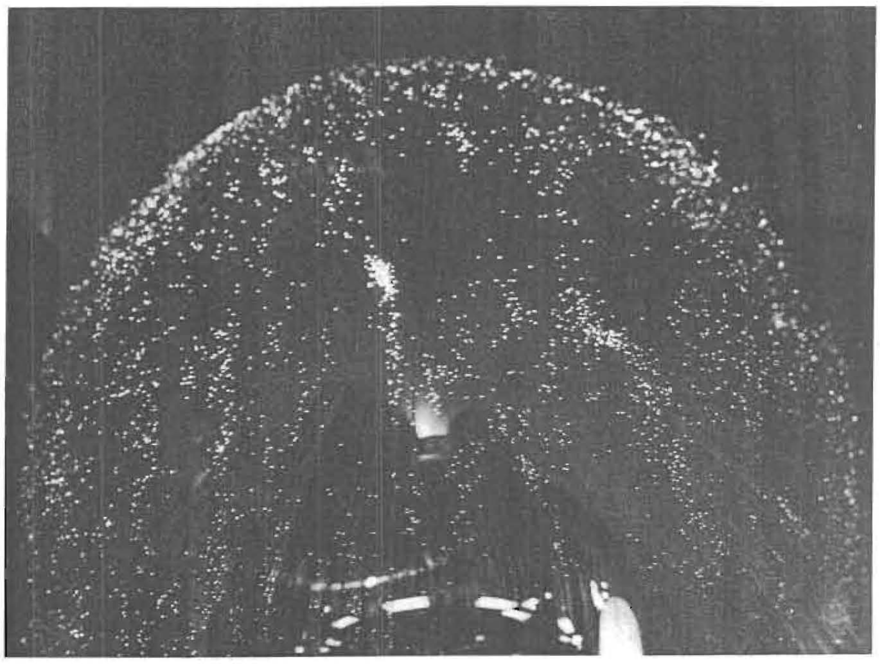


FIGURE 2.58

A fiber optic lamp.

A bunch of such fibers, suitably clad so the light does not leak from one fiber to the next, can be used to transmit *images*. At the original object, some fibers are exposed to light, and others are not. At the other end we get an array of fibers lit up in some places, dark in others, corresponding to the original image (Fig. 2.57b).

If the fiber is used to transmit a light signal consisting of a rapid series of pulses, it is important not only that all the light from a given pulse reach the other end of the fiber, but that it all arrive there at *one time*. Otherwise the pulse becomes spread out, and might blur with the next pulse. ***Graded fibers*** with varying index of refraction can be made so that the time for rays going straight down the center is the same as that for rays bouncing from side to side. Such fibers are replacing metal wire for telephone lines and for data transmission (Fig. 2.59). In general, the higher the frequency of a wave used for communication, the more information can be transmitted per second. Since the frequency of visible light is much higher than radio frequencies, these fibers can carry many more conversations at the same time. Further, they don't radiate signals, so they can't be bugged without directly connecting to them.

The ability to "bend" light and thus see around corners without the rigidity of a periscope allows doctors to see and photograph the inside of vital organs in living persons. Some of the fibers are used to get light inside the living organ, and the rest of the fiber bundle to get the image out.

Another example, at the opposite end of the scale of technological sophistication, is the "wet tee shirt" phenomenon. Here light pipes of water are formed through the cloth, allowing an image of something in contact with one side of the cloth to be transmitted to the other side. (The water also provides a more gradual change in index of refraction between the air and the cloth, so there is less reflection.) The principle works both ways—you can get a sunburn in a wet shirt.

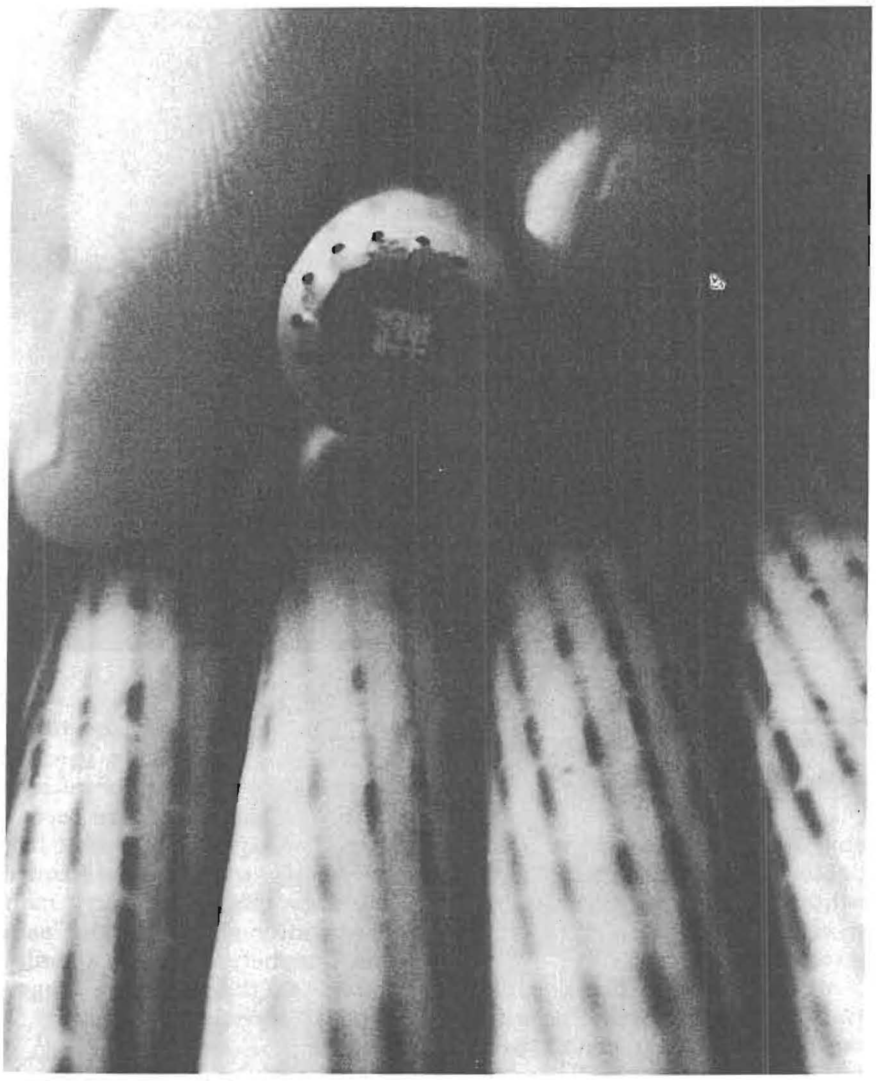


FIGURE 2.59

Photograph of a 12 × 12 array of light pipes, used to transmit laser light coded with telephone conversations. Such a cable can carry 40,000 voice circuits.

Nature also thought of this idea. Light is transmitted down the eyes of insects by total internal reflection through a bundle of light pipes, called *ommatidia*, which take the light incident on the eye and bring it to the light-sensitive cells. A similar process occurs in the cones of our eyes (see Sec. 5.3B). Bundles of fibers are even used in oats and corn, where they transmit light into the seedling plant to assist various photochemical processes.

TRY IT

FOR SECTION 2.5B

Light trapped in glass by total internal reflection

A piece of window glass can be used as a light pipe to demonstrate total internal reflection. Look at a small light source, such as a lightbulb, with the glass tilted between your eye and the bulb so that the light reflects from the glass to your eye. As you tilt the glass more, to make the reflection be almost at grazing incidence, the reflection should become quite good, indistinguishable from a reflection in a mirror. However, if you look at the edge of the glass toward you, you should also be able to see light emerging that has been trapped and reflected back and forth within the glass. This light didn't enter through the face of the glass, but rather from the opposite edge (the edge closest to the bulb). Check this by covering that edge with your finger, without blocking the surface reflection. If the light bulb is far enough away, you may also be able to see light emerging near the sides of the edge closest to you. This light will have been internally reflected from the side edges of the glass.

**C. Mirages and atmospheric distortion*

We have been saying that the index of refraction of air is 1. That is not exactly true—it is about 1.0003, and depends on the air's temperature, density, and other properties. (For example, the index changes from 1.0003 to 1.0002 when the air temperature is increased by 100°C.) These properties of air can change from place to place, so when light propagates through the atmosphere it usually experiences a continually varying index of refraction.

When the index of refraction of a medium varies smoothly (rather than abruptly as it does at the surface of glass or water), parts of each wavefront of light move faster than other parts (as in Fig. 2.48), so the wave bends. Here, however, the smooth variation in the index of refraction causes a smooth change in the wave direction (rather than the abrupt change of Fig. 2.48). In other words, the light ray bends in a curve.

This gradual bending of light occurs frequently. One example is that we see the sun after it has set—when it is beyond the horizon. The atmosphere bends the light rays, but we interpret things as if the light were coming in a straight line, so we "see" the sun above the horizon, even though it is actually below the horizon (Fig. 2.60).

You can see the effects of air temperature on index of refraction more dramatically where the air is shimmering above a hot jet engine or candle flame. Light passing through the rising and wiggling hot air is bent by varying amounts, depending on the nonuniform temperature of the air through which it passes. Frequently, a dark asphalt

FIGURE 2.60

We can see the sun after it has set below the geometrical horizon, even if we're not in love. The atmosphere is denser toward the bottom, less dense toward the top. The gradual change in density produces a gradual change in index of refraction, which bends the light.

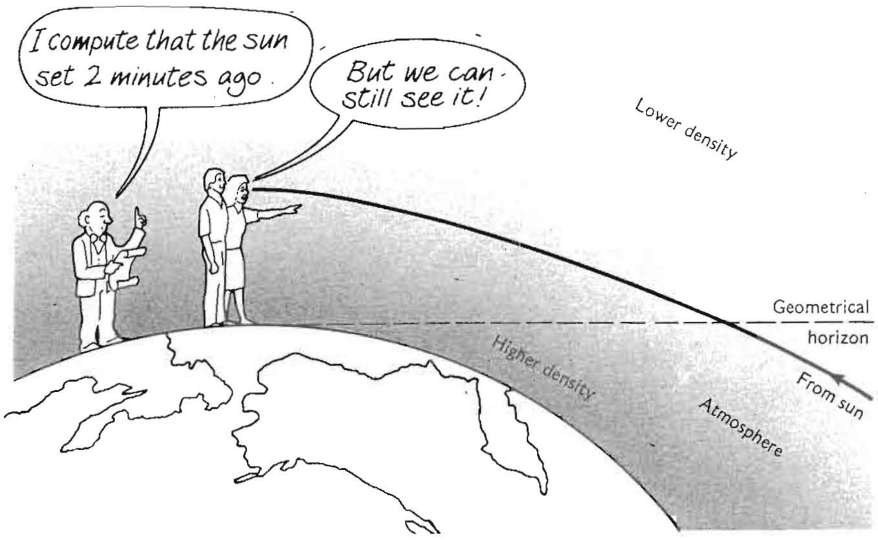
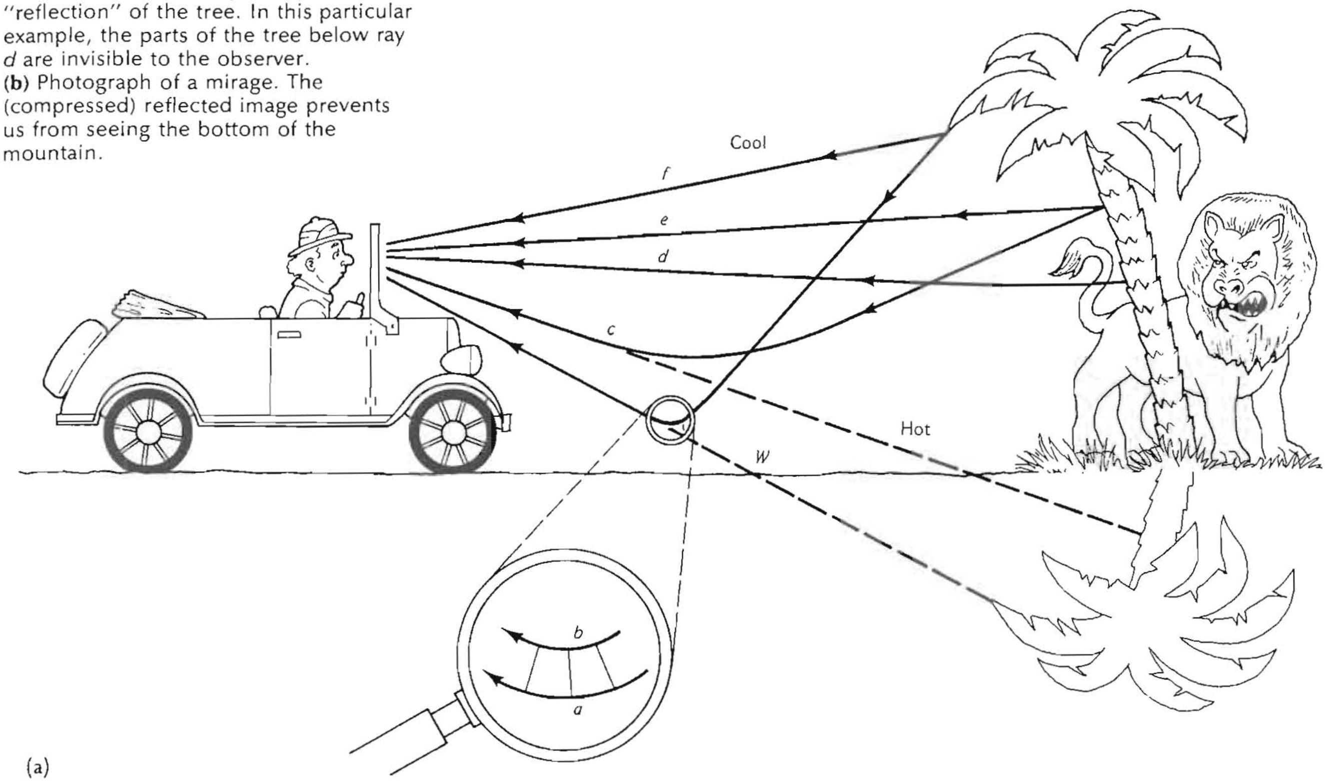
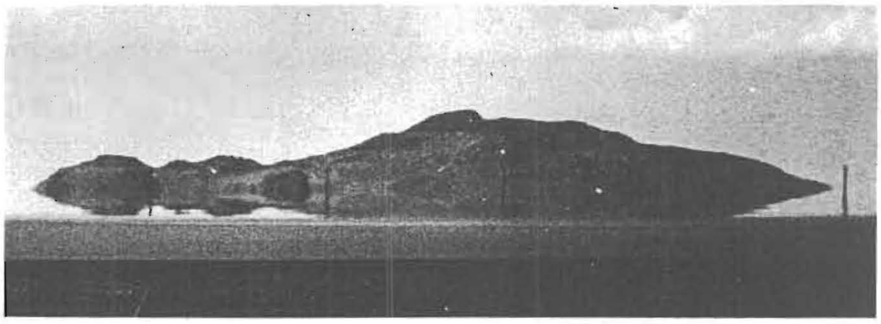


FIGURE 2.61

(a) A (rather extreme) mirage created by cool air above hot air. To analyze the mirage in detail, look through the magnifying glass; since ray *a* travels faster through the hotter air than ray *b*, *a* gets ahead of *b*, hence the beam gradually bends upward. The observer sees the tree both directly by means of rays *d*, *e*, *f*, through the cool air, and indirectly by means of the bent rays *a*, *b*, *c*. Thus, he sees a (somewhat compressed) "reflection" of the tree. In this particular example, the parts of the tree below ray *d* are invisible to the observer.
(b) Photograph of a mirage. The (compressed) reflected image prevents us from seeing the bottom of the mountain.



road will get very hot in the summer sun. When this happens, the air next to it may become much hotter than the higher air. This, in turn, can cause light to be bent so much that it appears reflected (Fig. 2.61)—a ***mirage.**** In a rough way, you can think of the effect as a gradual total internal reflection at



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*French *se mirer*, to be reflected.

FIGURE 2.62

Photograph of a mirage as frequently seen on a hot road. Note the snow in the background. You don't need a hot day for a mirage—only a temperature *difference*.

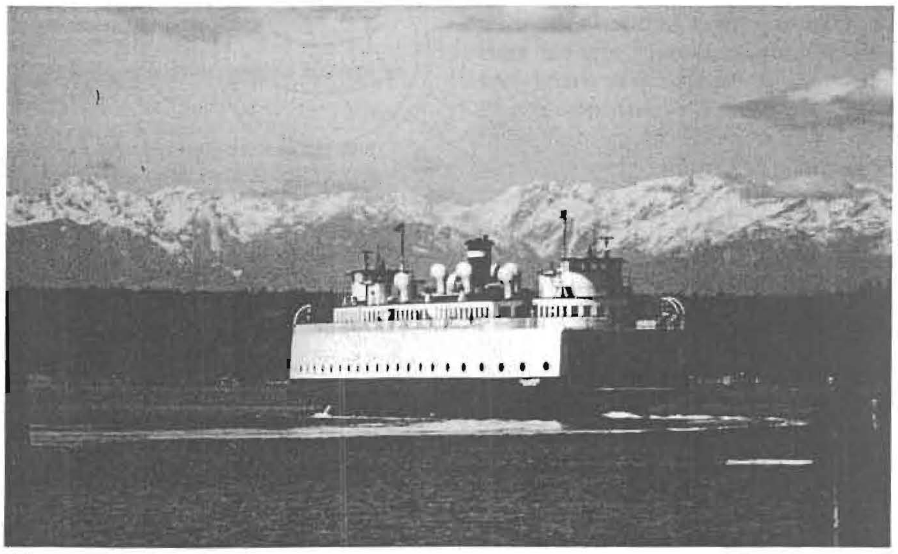
the boundary between hot and cool air. When you look at the road ahead, you see a reflection in it of whatever lies ahead. You also see the object directly. Since you associate such reflections with the surface of water (the most common source of reflections on roads), you get the impression that there is water on the road ahead of you (Fig. 2.62). (The water would seem to be at the place where the bent rays appear to cross the road surface, near the point *W* in Fig. 2.61.) Of course, the water never materializes as you get close. The closer you get, the more sharply the rays must be bent to reach your eyes. But, as the light is only bent by a certain amount, and not more, it misses your eyes when you get close and the "reflection" vanishes. This is rather disappointing if you happen to be thirsty in the desert.

The appearance of "water" under conditions of hot ground has suggested to Alistair B. Fraser an explanation for the parting of the Red Sea for Moses. The idea is that nobody was in the Red Sea, but rather out in the desert, and the waters were a mirage. The "waters" apparently receded at night, when the ground cooled and the mirage would retreat, and the Children of Israel "crossed." The next morning, as the ground became warmer, the "waters" returned and "swallowed" the pursuing Egyptians. This, at least, is the Jewish version (Exodus 14:20–28). The Egyptians, for their part, presumably saw the Children of Israel being drowned by the advancing "waters" and, figuring there was no need to chase them anymore, went home. Similarly, the British lost the Turks in a mirage during the First World War, and had to call off the battle.

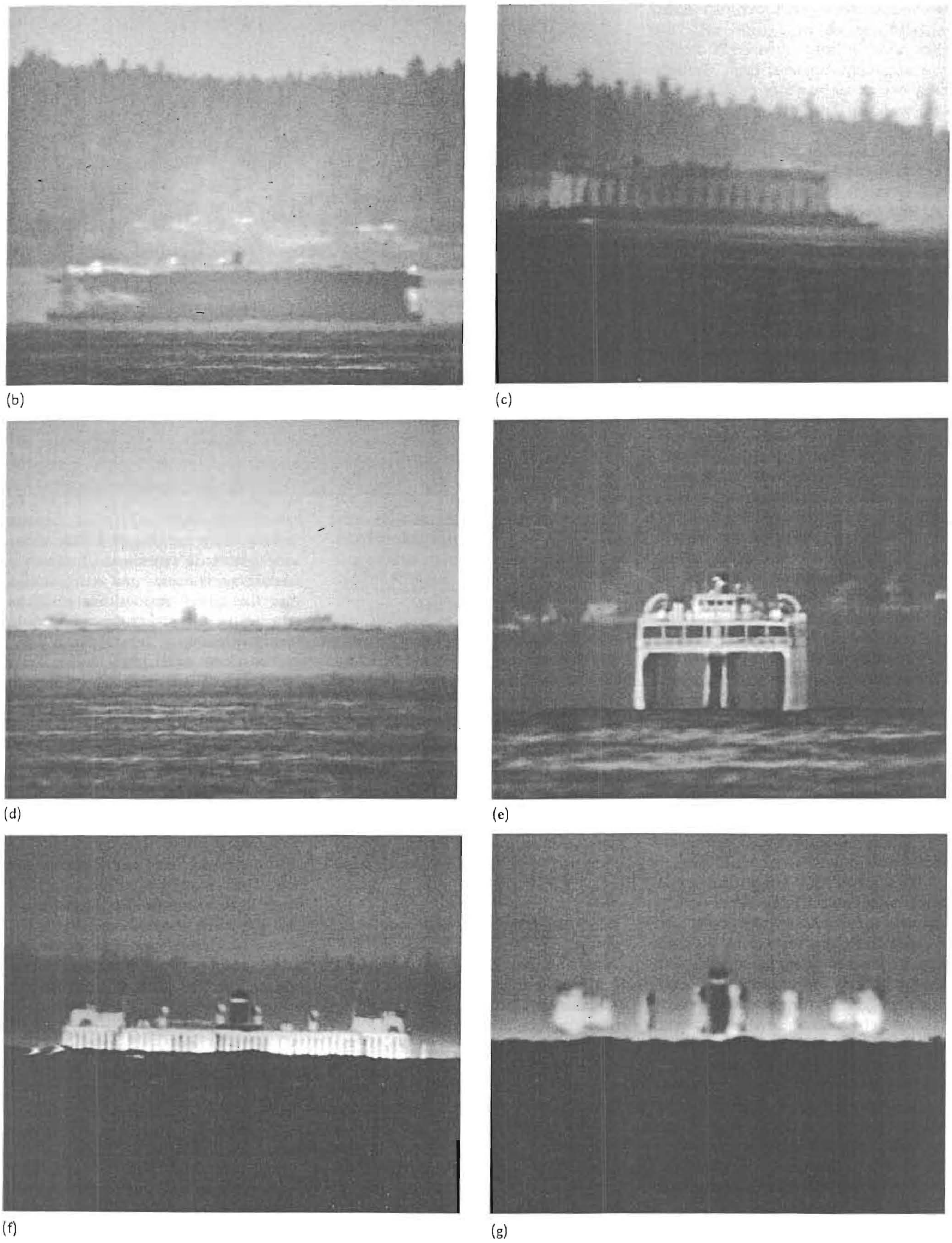
There are various types of mirages possible, some of them quite spectacular (Fig. 2.63). The details differ depending on whether the hot air is above or below the cool air, and on how sharp the transition is from cool to warm air. If warm air is below (say, at the surface of a

FIGURE 2.63

A series of photographs of a ferryboat.
(a) The undistorted boat.
(b) to **(g)** Various mirages distort the boat in different ways.



(a)



(b) (c) (d) (e) (f) (g)

warm lake on a cool morning), light is bent up, as in Figure 2.61, so that objects may disappear below the apparent horizon even though they are actually quite close. A person walking on a flat island in the lake may appear to be walking on water when the ground under his feet is depressed below the observer's horizon by such a mirage (cf. bottom part of tree in Fig. 2.61). If warm air is above cool air, you can see ships that are located "beyond" the horizon (like the sun in Fig. 2.60), seemingly floating in air, like that of the Flying Dutchman. (These effects are best seen if you look at the horizon with your eye very close to the water.) If the air temperature changes sufficiently gradually, the ground (or, more typically, the surface of the water) can appear to loom up like a wall, greatly magnified and sufficiently distorted so that it may look like mountains, or castles in the sky. This ***fata morgana*** mirage derives its name from Morgan le Fay, King Arthur's evil sister who could actually make castles in the sky.

More recently, in 1906, Commander Robert E. Perry discovered an area of peaks and valleys west of Greenland, which he named "Crocker Land." In 1913, Donald MacMillan confirmed the discovery, only then to watch Crocker Land disappear as he approached it. The warmer air over the colder air had produced a fata morgana, and the conditions were common enough in the Arctic that two explorers (and many Eskimos) had seen it. Indeed, so common is this effect that it has been suggested that various early discoveries, where explorers set off in just the right direction, such as the Celtic discovery of Iceland and Eric the Red's discovery of Greenland, might have been aided by such Arctic mirages, which would enable these people to see these places beyond the usual horizon.

Mirages in combination with the earth's curvature can distort the apparent shape of the sun; it can look flatter, have wiggles in its outline, or even have horizontal gaps in it (Fig. 2.64). Effects of the curvature of Venus are described, with considerable poetic license, in Tom Robbins' *Even Cowgirls Get the Blues:*

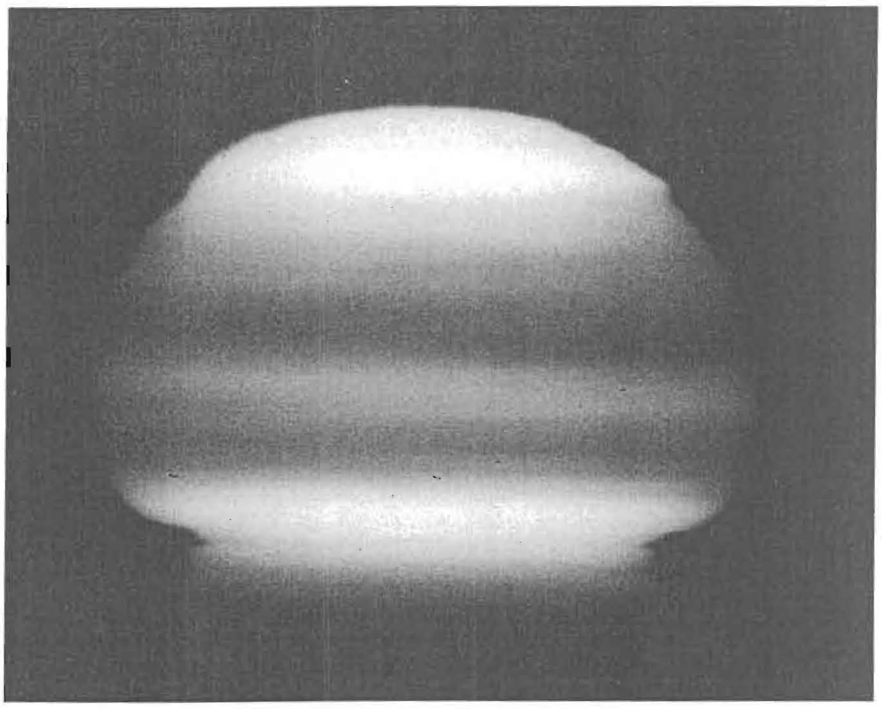


FIGURE 2.64

Photograph of setting sun shows both flattening and corrugations of solar limb due to uneven inversion layer.

> *On Venus, the atmosphere is so thick that light rays bend as if made of foam rubber. The bending of light is so extraordinary that it causes the horizon to tilt upward. Thus, if one were standing on Venus one could see the opposite side of the planet by looking directly overhead.*

2.6
DISPERSION

So far we have pretended that a substance like glass has one index of refraction for all kinds of light, no matter what the frequency. But the index of refraction, $n = c/v$, is specified by the speed of light in the glass, which depends, in turn, on the way the charges in the glass respond and radiate when they are wiggled. We have seen that the amplitude of the charges' motion depends upon the frequency with which they are wiggled; the closer one gets to a resonance frequency, the more the charges will oscillate for the same applied force. Thus, the index of refraction will depend on frequency.

For glass and many transparent substances the resonance frequencies are in the ultraviolet (UV). (You don't get suntanned sitting in the sun behind a window because glass is not transparent in the UV—see Fig. 1.25.) As the (visible) frequency with which we wiggle the charges gets closer to the resonance (UV) frequency, the charges oscillate more violently and radiate more. We can then expect a greater effect on the light beam in the glass, which is the combination of the initial beam and the radiated waves. So the speed of light in glass should differ more and more from its vacuum speed. That is, the speed should get smaller and the index of refraction larger as the light changes from red to blue toward the UV:

$1 < n_{red} < n_{blue} < n_{UV}$*

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*The notation A < B means A is less than B.

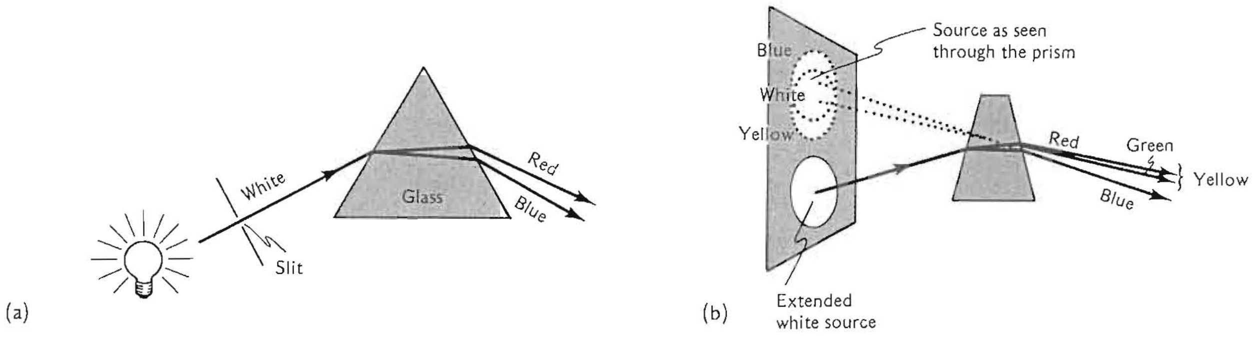

FIGURE 2.65

(a) Dispersion of colors by a prism—narrow source (also see Plate 2.1).
(b) View of *extended* white source through a prism.

The dependence of n on frequency is called ***dispersion.***† As expected, n is larger at higher frequencies for glass (as well as for diamond and water—Table 2.6). This difference of the index of refraction of a piece of glass for different frequency (or color) light means that the amount of bending that occurs when light hits a surface depends on the color of light—light is spread into the different colors when incident on a glass prism. Since n_{blue} is greater than n_{red}, the blue light is bent more than the red at each surface of the glass (Fig. 2.65a, Plate 2.1).

Newton showed that white light is made of all the different colors of the spectrum by breaking up white light with a prism and then recombining it into white light with another prism. However, if you look through a prism and expect to see everything in brilliant color, you will be disappointed. You will find that objects viewed through a prism acquire color only at their borders. For example, if you have a white area surrounded by black, and view it through a prism as in Figure 2.65b, you will see blue along the top edge of the white area, and mainly yellow with a little bit of red along the bottom edge. The reason is that the border in blue light is shifted away from the border in the other colors, because the glass has its greatest dispersion at the blue end of the visible. Thus, the top edge looks blue. The rest of the colors of the spectrum are usually not well separated; all these other colors together combine to give yellow at the bottom edge (Sec. 9.4B), with the red peeping out just a bit. Any point well within the white area still is seen in all colors, and hence looks white, even though that light comes from different points in the original white object. Similarly, most of the black area receives no light, and hence still looks black. In order to examine the spectrum of a light source, therefore, you should place a narrow slit in front of it, and view the slit through your prism.

*A. Diamonds

Diamonds have a number of properties that account for their high price (not the least of which is that their supply is controlled by the diamond industry). Diamonds, as gems, have a long history of intrigue, passion, and superstition, and many powers have been ascribed to them. Diamonds could prevent nightmares, loosen your teeth, provide strength in battle, and repel phantoms. But the properties that concern us here are the ones that are responsible for their "brilliance," "fire," and "flash."

Diamond has a large index of refraction, $n \simeq 2.4$, so that the critical angle for total internal reflection is about 24.5°—much smaller than that for glass. This accounts for the ***brilliance*** of diamonds. A diamond is cut (Fig. 2.66) so that almost any ray of light that hits it from the front strikes one of the rear surfaces at an angle greater than 24.5°, is internally reflected to another surface, and eventually is reflected back out the front. Viewed from the front, then, the diamond is bright—brilliant. However, if you look from the back through a cut diamond at a light source, the diamond will appear black—almost no light passes out the back.

The brilliance of diamond can be imitated by glass if the glass is backed with metallic foil or silvered like a mirror, as that will also return the light to the front. However, the glass still has a major visual shortcoming compared to diamond; diamond is highly dispersive—the index of refraction varies considerably with frequency (Table 2.6). The blue is bent *much* more than the red, so white light is spread out into a broader spectrum than in glass. This accounts for the ***fire*** of diamonds—the beautiful colors.

No matter how you look at a cut

TABLE 2.6 Index of refraction for various media

			Index of refraction			
Frequency (hertz)	*Wave-length (nm)*	*Color*	*Glass zinc crown*	*Glass light flint*	*Diamond*	*Water*
4.57×10^{14}	656	Red	1.514	1.571	2.410	1.331
5.09×10^{14}	589	Yellow	1.517	1.575	2.418	1.333
6.91×10^{14}	434	Deep blue	1.528	1.594	2.450	1.340

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†Latin *dispersio*, spreading out.

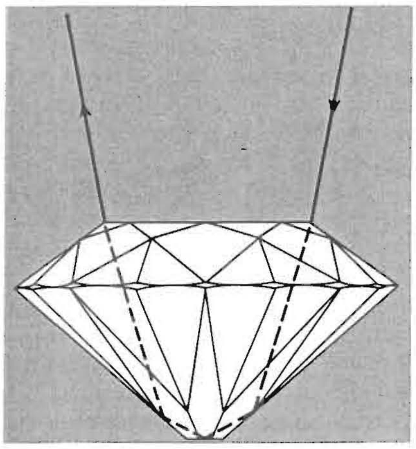


FIGURE 2.66

Most of the light entering a diamond is eventually reflected back out the front.

diamond, the chances are that you will get rays reflected from some source of light in the room, and it will be spread out into its spectral colors. As you move your eye slightly or rock the diamond, the view changes and you see some other rays that reach your eye by some other path (Plate 2.2). This causes the ***flash*** of diamonds; the motion causes the diamond to sparkle as the light reaches you from different sources. Diamonds are best shown off in rooms with lots of small lights or candles and mirrors, and must have been at their best when borne by the beauties in the ballroom of Versailles.

It is important to note that the bending of light and total internal reflection in diamond is due to the ratio of the index of refraction of diamond to that of *air*—the greater the ratio, the greater the refraction at a boundary. If the diamond is badly mounted it can get a film of oil ($n_{oil} \simeq 1.4$) on its back side, which lowers the ratio. Therefore, the critical angle for total internal reflection in an oily diamond is much higher than 24.5°. Since the diamonds are cut assuming $\theta_c = 24.5°$, the increased θ_c means a loss of light that should have been totally internally reflected—a loss of brilliance. The moral is: keep the back, as well as the front, of your diamond clean.

B. Rainbows

Because of the dispersion of water (Table 2.6), droplets of water can break up the sun's light into a spectrum, much as a prism does. This accounts for the ***rainbow*** (Fig. 2.67, Plate 2.3). Its formation involves not only dispersion of light on entering and emerging from the drop, but internal reflection as well. The result is that the drop reflects the sun's light, but the different colors emerge at different angles since, for example, blue light is bent more than red. If you are to see both red and blue light, these lights must come from different raindrops; those reflecting red light to your eye are at the higher angle, because red emerges more downward than blue. Thus, the rainbow is red at the top (outside) and blue at the bottom (inside), with the other colors in between.

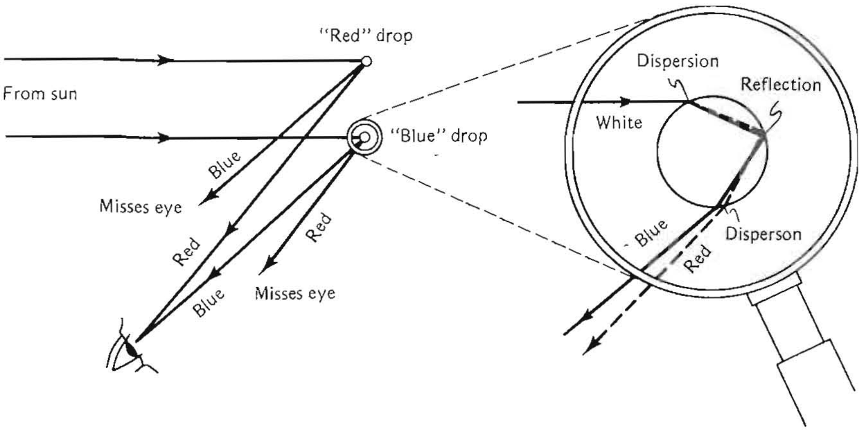
An entire arc of water drops will look red to your eye: all those drops lying on a cone from your eye of about 42° around the direction of the sun's rays (Fig. 2.68). Similarly, drops along arcs at smaller angles reflect other colors to your eye. The entire rainbow thus appears as an arc ranging between 40° and 42°. Since this angle is measured from your eye, it is your own private rainbow. The light from these particular water drops reaches *your* eye, and misses everyone else's. Others must see the light coming from different drops, those at the appropriate angles from *their* eyes. Indeed, as you move your eye, the rainbow moves with you to maintain the same angular relationship—it follows you as you move, and you never can get any closer to the pot of gold at its "end."

Notice in Plate 2.3 that outside the ***primary rainbow*** there is a ***secondary rainbow.*** This is caused by rays that undergo *two* reflections in the water drop (Fig. 2.69). Here, because of the extra reflection, the blue emerges more downward than the red. Hence in the secondary rainbow, the colors are reversed; the blue (at 54°) is outside the red (at 50°).

Look closely at Plate 2.3 and notice that the region between the two rainbows is darker than the region inside the primary bow. To explain this darker region (called Alexander's Dark Band), we must discuss some subtleties omitted in Figure 2.67. In that figure only one incident ray was drawn to each drop. Of course, there are many such rays hitting all over each drop (Fig. 2.70a). Each of these rays is refracted, dispersed, and reflected, as was the one drawn in Figure 2.67, though the one drawn there was a special ray. It was the ray that comes out at the steepest angle (e.g., 42° for red). All other rays of the same wavelength come out at shallower angles (Fig. 2.70a), but most of them come out *near* the steepest ray. Hence, the rays of a

FIGURE 2.67

A light beam is dispersed twice and reflected once by a raindrop, letting the eye see a rainbow. (The dispersion is exaggerated here.)



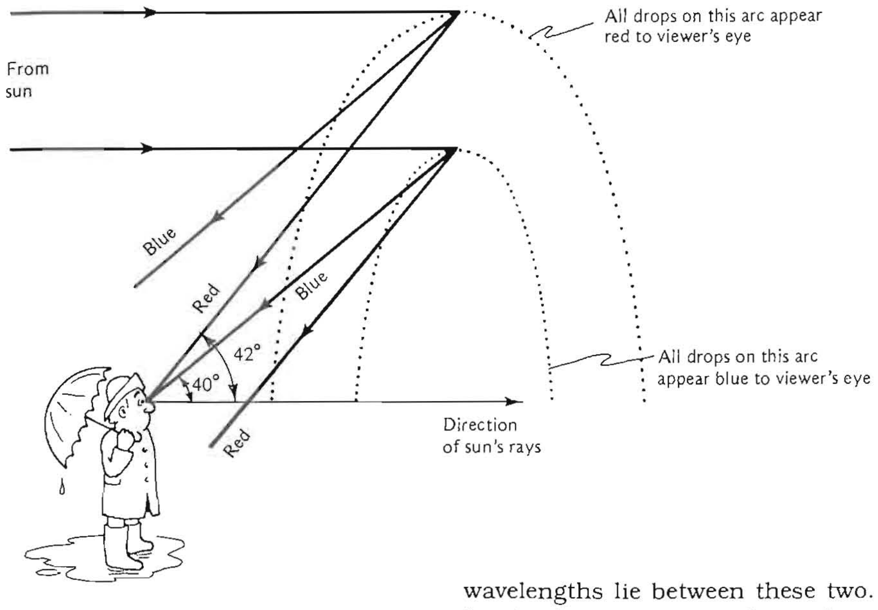


FIGURE 2.68

Only the drops along the rainbow's arc send light to a particular eye. All the drops on the upper arc appear red—the blue light they send out misses the eye. Similarly, all the drops on the lower arc appear blue.

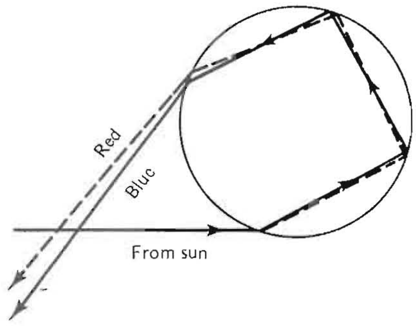


FIGURE 2.69

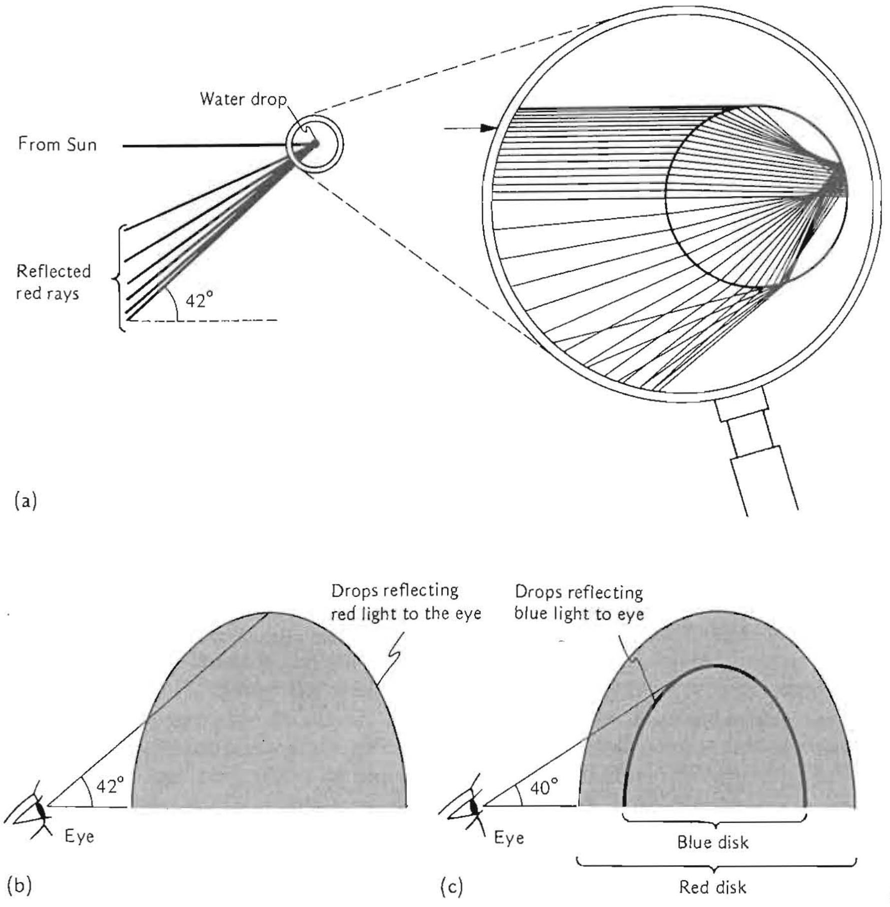
The secondary rainbow is caused by two reflections and two refractions in each raindrop. The extra reflection reverses the order of the colors.

given wavelength form more than an arc; they form a disk that is brightest at the rim and fades toward the middle (Fig. 2.70b). The disks from the different wavelengths overlap, with the red (being the widest angle disk) sticking out the most. For example, the blue disk overlaps the inner part of the red disk (Fig. 2.70c). All the other wavelengths lie between these two. Inside the primary rainbow, then, all the wavelength disks overlap giving white light, so it looks brighter there. (Because the secondary rainbow is reversed, the outside is its brighter side, but that is a weak effect.)

Finally, since the sun must be behind you and the rain in front, the presence of a primary rainbow tells you something about the weather. As old English folklore has it: "A rainbow at night, fair weather in sight; a rainbow at morn, fair weather all gorn." At night (evening) the sun is in the west, so the rain must be in the east. As the weather in the northern hemisphere travels from west to east, the rain has passed, the good weather is coming. A rainbow in the morning has rain to the west, heading toward you.

FIGURE 2.70

(a) Light from the sun falls on the raindrop at all levels. Most but not all of the red rays from this light are returned by the raindrop at about 42°. In the magnified view, the arrow indicates the special ray that is returned *at* 42°. **(b)** The eye sees a red disk, brightest at the rim. **(c)** Superimposed on the red disk is a smaller blue disk (as well as disks of intermediate colors).



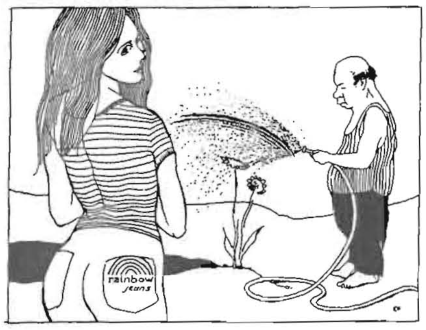


FIGURE 2.71

A private rainbow for each person.

*C. Sun dogs, 22° halos, and more

Hexagonal ice crystals in the atmosphere refract light more nearly like a prism than raindrops do. The same flat hexagonal plates floating horizontally that cause sub suns or sun pillars by *reflection* off their horizontal surfaces can also *refract* light entering their vertical surfaces (Fig. 2.72). The amount of bending depends on the angle at which the light enters the crystal, but no matter at what angle it enters the hexagonal ice crystal, the light will be bent by at least 22°, with most light bent by about 22°. (22° is about the angle between the tips of your thumb and little finger when you hold your hand at arm's length and spread out your fingers as much as possible.) So in addition to the sun itself, you may see two images of the sun, one on each side, about 22° away from the sun. Dispersion gives the images colors, but usually not as brilliant as those of a rainbow (more rays are bent by more than the minimum amount, so the different colors mix together and tend to wash out). Either of these two images is called a ***sun dog**** (Fig. 2.73). Sun dogs are visible only when the sun is close to the horizon, so that the rays can go through the horizontal, rather thin, plates of ice.

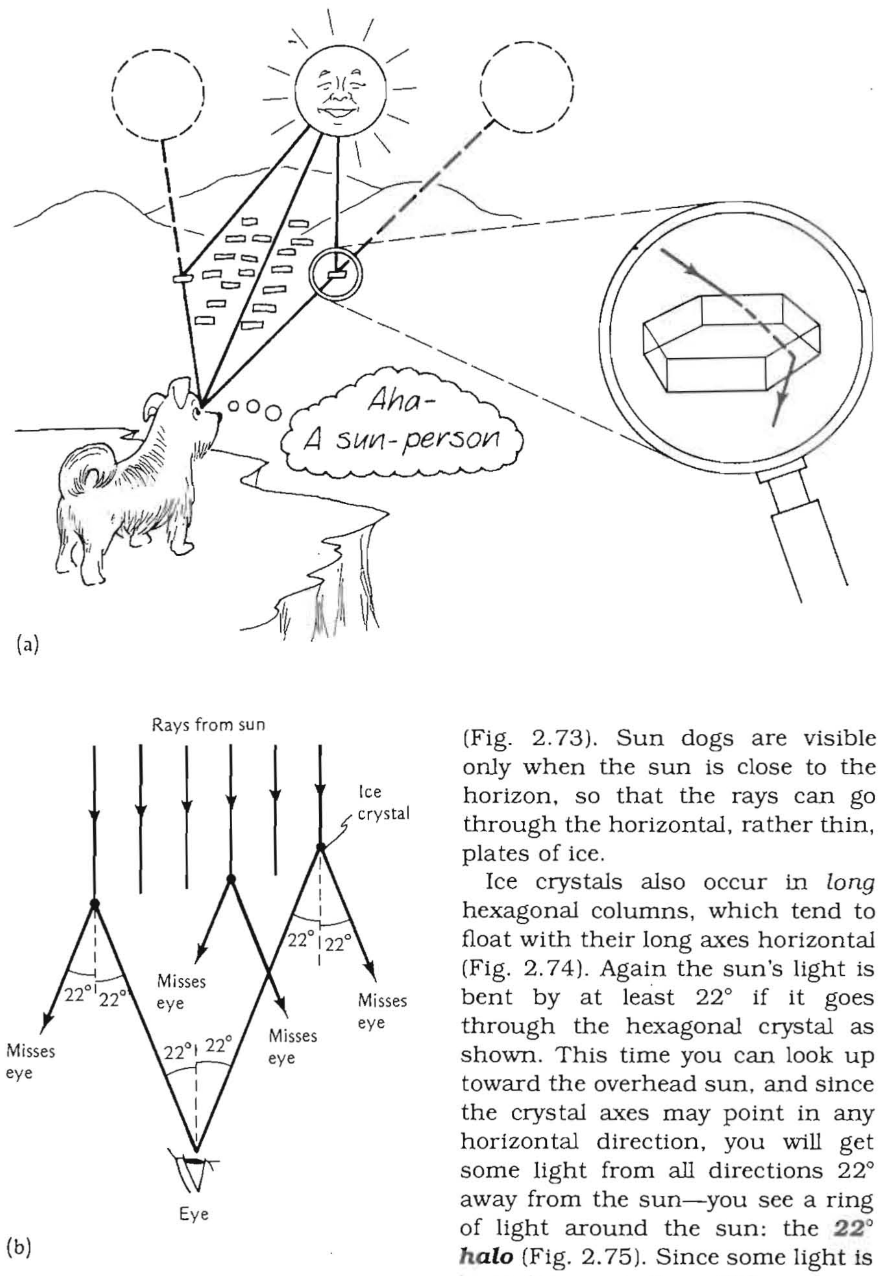


FIGURE 2.72

A sun dog is caused by refraction in hexagonal ice crystals. **(a)** As seen by an observer. **(b)** The path of a light ray through a crystal, as seen from above. Only those crystals that are at 22° from the eye send light to it. Light from all the other crystals misses this particular eye.

Ice crystals also occur in *long* hexagonal columns, which tend to float with their long axes horizontal (Fig. 2.74). Again the sun's light is bent by at least 22° if it goes through the hexagonal crystal as shown. This time you can look up toward the overhead sun, and since the crystal axes may point in any horizontal direction, you will get some light from all directions 22° away from the sun—you see a ring of light around the sun: the ***22° halo*** (Fig. 2.75). Since some light is bent by more than 22°, you see some light outside the circle, but since no light is bent less than 22°, it is dark just inside the circle—you see a halo that fades off outside the circle. The halo may be slightly reddish on the inside because of dispersion.

If the sun (or moon) is not exactly overhead, you may still see the 22° halo if the ice crystals are more randomly oriented. This occurs for somewhat shorter ice crystals, which tend to tumble as they fall. Other times you may see only a part of the halo. You may also observe

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*Probably from "to dog"—to follow. The sun dog is always near the sun. Sun dogs are also referred to as mock suns or parhelia. Presumably, the sun with two sun dogs (as in Fig. 2.73) is what Shakespeare refers to in *Henry VI, Part III* when Edward says, "Dazzle mine eyes, or do I see three suns?"

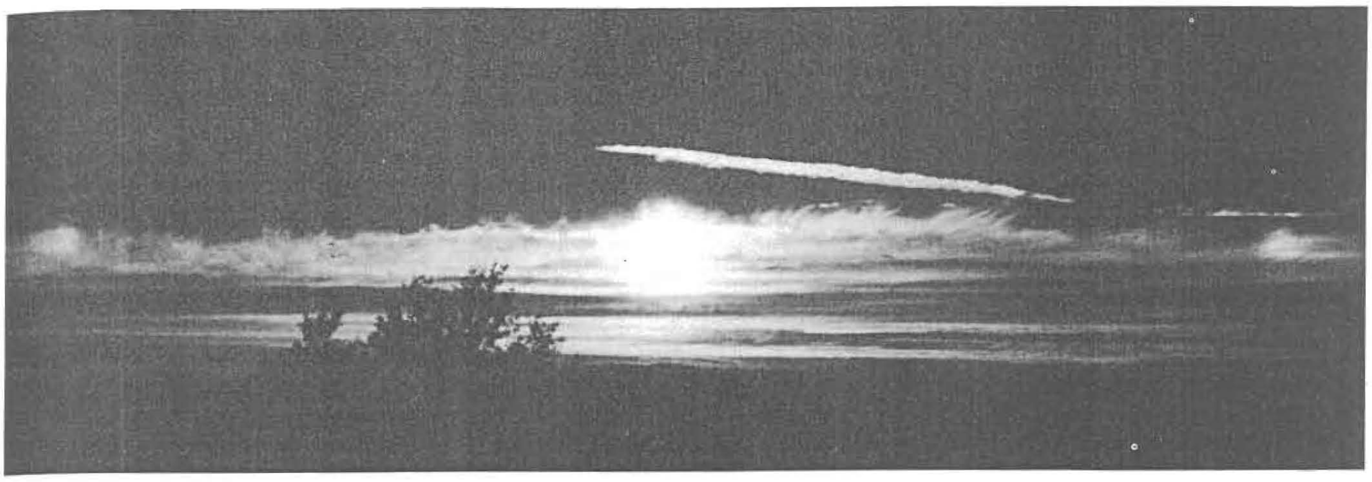
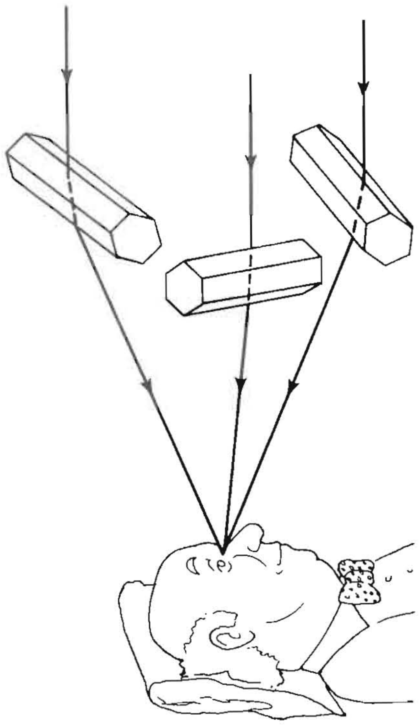


FIGURE 2.73

Photograph of the sun flanked by sun dogs on each side.

FIGURE 2.74

The 22° halo, caused by tiny pencil-shaped ice crystals. As in Figure 2.72b, only those crystals that are at 22° from the eye send light to it.



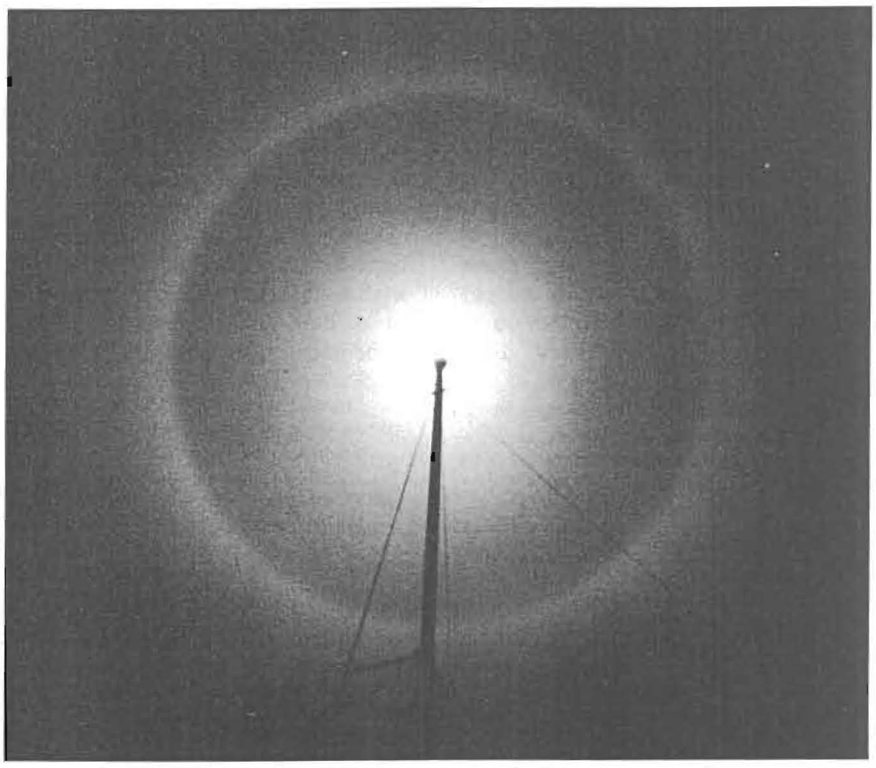


FIGURE 2.75

Photograph of a 22° halo.

strange arms leading off the 22° halo above and below the sun (or moon). They are due to other, less common, reflections and refractions off these crystals. For example, reflection off the *ends* of the long crystals of Figure 2.74, when the sun is low, produces a low arc in the sky called a parhelic circle.

These phenomena are relatively rare, but may be seen from time to time (more easily seen around the moon, against the dark night sky). To photograph them, try to get an obstacle (such as a tree branch) in front of the moon or sun, which would otherwise be far too bright and completely overexpose your film during the time it takes to properly expose the halo or arc. (You may also need a wide-angle lens—Sec. 4.3A.)

SUMMARY

As long as light encounters only obstacles much larger than its wavelength, ***geometrical optics*** may be used to describe the behavior of light ***rays.*** An obstacle in front of a ***point source*** casts a ***shadow,*** whose location can be determined by tracing the rays that just pass the edge of the obstacle. An ***extended source*** may give a region where all shadows overlap ***(umbra)*** and a region of partial shadow ***(penumbra),*** which receives light from some points on the source. ***Eclipses*** of the sun and moon are examples. From the penumbra we see a ***partial eclipse.*** By blocking all the light except that passing through a small hole, we make a ***pinhole camera.***

In addition to traveling in straight lines, light can be ***reflected*** by objects. The reflections are used in ***radar*** (or ***sonar*** with sound waves). ***Metals*** are excellent reflectors of light at frequencies below their ***plasma frequencies.*** (The ***ionosphere,*** with a low plasma frequency, reflects AM radio waves.) ***Mirrors*** are made by coating glass with silver metal. If the silver is thin enough, we have a ***half-silvered*** mirror, which appears transparent to someone on the dark side of the mirror, but appears reflecting to someone on the bright side. For light striking a reflector at an angle, the *angle of incidence equals the angle of reflection.* The reflection gets *stronger* at *grazing incidence.* ***Sub suns*** and ***sun pillars*** are reflections of the sun from flat plate, or long pencil-shaped, hexagonal ice crystals, falling almost *horizontally.* Light can be reflected more than once, as in a ***corner reflector,*** which ***retroreflects*** the rays. Reflections can be ***specular*** (obeying the law of reflection) or ***diffuse*** (reflections that go in all directions from a rough surface).

Light can also be ***refracted***—bent as it enters a new medium with a different ***index of refraction*** ($n = c/v$). ***Snell's law*** states that *light going from small* n *to large* n *is bent toward the normal* to the surface; *light going from large* n *to small* n *is bent away from the normal.* If the ***angle of incidence*** is greater than the ***critical angle,*** when light is traveling *in* the *slower medium,* the light will be ***totally internally reflected.*** This principle is used in ***fiber optics*** to guide light beams. The bending of light as the index of refraction changes is responsible for a variety of ***mirages,*** depending on how the air temperature changes. The amount of refraction depends on the *wavelength* of the light ***(dispersion).*** A diamond's ***brilliance, fire,*** and ***flash*** are due to its *large index of refraction (large critical angle), high dispersion,* and the *cut* of the diamond, chosen to take advantage of these. ***Rainbows*** result from *dispersion* of the light entering and leaving water droplets, and one (for the ***primary rainbow***) or two (for the ***secondary rainbow***) *reflections* inside the droplets. *Refraction* and *dispersion* in hexagonal ice crystals produce ***sun dogs,*** the ***22° halo,*** and other meteorological optical phenomena.

PROBLEMS

P1 Stand a pencil point up on a piece of paper and illuminate it, from one side only, with a broad light source (a window, a fluorescent tube, etc.). With another pencil, trace the shadow formed, indicating which regions are umbra and which penumbra.

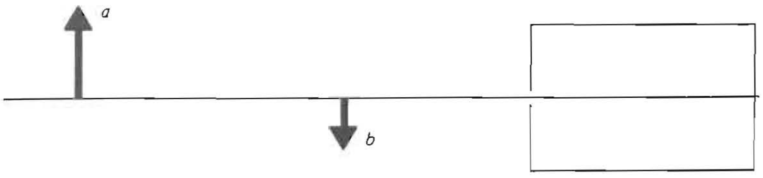
P2 Suppose the earth, in Figure 2.5, were farther from the moon, so that lines *a* and *b* crossed before reaching the earth. What would the eclipse look like when viewed from a point on the earth between those two (crossed) lines? Would that point be in a region of umbra or penumbra? (It may help to make a sketch and draw straight lines back from the viewing point to the sun to see which parts of the sun are visible, if any.)

P3 In "The Rime of the Ancient Mariner" Coleridge writes: "And on the bay the moon light lay,/And the shadow of the moon." (a) Under what conditions does the shadow of the moon touch the earth? (b) Is there any moonlight visible under these conditions? Why?

P4 Why did Etienne de Silhouette use a candle (point source) when making his silhouette tracings rather than a larger (and perhaps brighter) source?

P5 A camera using a lens has a much larger hole than the pinhole camera has. (a) Without knowing anything about lenses, why would you want to have a large hole rather than the tiny pinhole? (b) We've seen what happens to the image in the pinhole camera when the pinhole is made larger. What do you think the purpose of the lens is in a regular camera?

P6 Draw a 10-cm line across the top center of a piece of standard $8\frac{1}{2} \times 11$ inch paper. Toward the left end of the line, print the word "Pinhole," and toward the right end, print the word "Camera." Now, at the bottom of the page, draw a square, 15 cm on each edge, with a 5-mm hole in the center of the top side of the square. This will represent your pinhole camera. The bottom edge of the square will be the film. The words at the top of the page are your objects. (a) Using a good straight edge, carefully find the location of the image of the word "Pinhole" on the film. Repeat for the word "Camera." (b) Notice that the pinhole is fairly large (5 mm). This would give a blurred image—the image of a point would be a spread out blur patch. Construct the blur patch that is the image of the dot on the i in "Pinhole." (c) To decrease the blur, one can make the pinhole smaller. What would be the *major* drawback of a pinhole camera whose hole was small enough so that the blur were acceptably small?

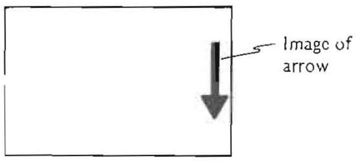


P7 The figure shows a pinhole camera photographing two arrows. (a) Which arrow will have the largest image? (b) Which arrow's image will be pointing up?

P8 The light of the pinhole image of the sun comes from the sun, reflects off the white screen, and gets to your eye. The eye, thus, still gets the sun's light. Why is the pinhole image less likely to hurt your eyes?

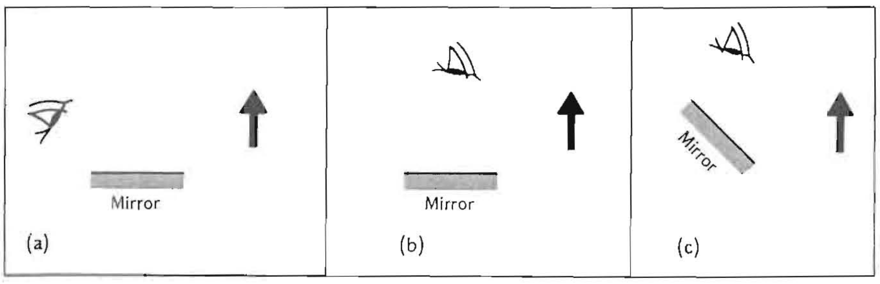
P9 If a cloud drifts over the sun from right to left, which way will it seem to drift over the sun's image in a pinhole camera?

P10 The figure shows a pinhole camera with the image of an arrow. The actual arrow is located somewhere outside the camera, is 2-cm long, and is vertical. Use a ruler to redraw the figure and construct suitable rays to (a) draw the position of the actual arrow that would result in the image drawn, and (b) draw two arrows, each of length 3 cm, that would result in the *same* image.

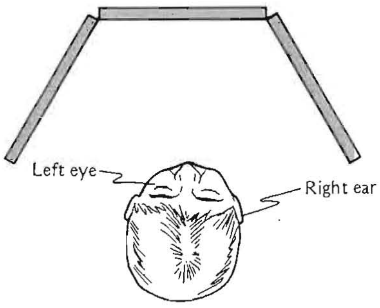


P11 Write down at least five words that still make sense when reflected by a plane mirror. The reflected word need not be the same as the original. The words may be written or printed, in small or capital letters.

P12 Three different setups of eye, mirror, and object are shown in the figure. In which of the setups can the eye see a reflection of the object? Redraw the setups with appropriate light rays to support your answer.



P13 The man in the figure is looking at himself in a triple mirror found in a clothing store. Redraw the figure and show two *different* rays of light that go from his right ear to his left eye. One ray should hit only one mirror, the other ray should hit the other two mirrors. At each reflection, draw the normal to the mirror, and label θ_i and θ_r.



P14 Briefly explain how an oceanographer might use sonar to measure the depth of the ocean.

P15 In some places, housepersons like to "spy" on what is going on in the street without leaning out the window. They attach a mirror to the window frame on a bracket so it sticks a little way into the street. (Look for these the next time you're in Europe.) (a) Draw a diagram showing how houseperson *H,* without getting up from his or her chair, can see the visitor *V* at the neighbor's home. Draw how the mirror must be placed, and show how light gets from *V* to *H*'s eye. (b) Suppose *H* has no mirror, but does have a large 45°-45°-90° prism. *H* mounts the long face of the prism in the same position as you have drawn the mirror, and obtains a good reflection of *V*. What phenomenon is *H* using to get this reflection? (c) Why can *H* hear *V* ringing the neighbor's bell when *H* cannot see *V* directly?

P16 At nighttime, we can often pick up radio stations from cities very far away. (a) Why? (b) Why does this occur for AM radio, but not FM radio?

P17 As you look at an ordinary bathroom mirror at large (grazing) angle, you can often see three, four, or more images of an object. Why?

P18 A student, who obviously didn't read this book, had a van whose window he had covered with half-silvered mirrors (actually aluminum on plastic). When he checked it out during the day, he assured himself that, while he could see out, no one could see in. That night he parked his van in a dark parking lot, climbed inside with his lantern and a friend, and enjoyed what he thought was the privacy of his own home. A passing police officer saw him through the half-silvered mirrors, and promptly busted him. Why could the police officer see in?

P19 When peering into a dark house from outside, you often get right up to the window and cup your hands around the side of your head. Why?

P20 In Figure 2.37, we have shown how light from the headlights of a car gets to the driver's eye. Redraw the figure for a very foggy day. (Fog consists of many small droplets of water. Draw some of these droplets.) Draw how the light now gets to the driver's eye, and explain why it is harder for her to see the road ahead.

P21 Blue light bends more than red light when entering glass from air because: (a) red light travels faster than blue light in glass, (b) blue light travels faster than red light in glass, (c) red light travels faster than blue light in air, or (d) blue light travels faster than red light in air. (Choose one.)

P22 (a) Relate the fire, flash, and brilliance of a diamond to physical properities of the diamond. (b) Why does a diamond look black when you attempt to look through it from the back at the only light in an otherwise dark room?

P23 (a) What is the critical angle? (b) Which pair of materials has the greater critical angle, air-water or air-diamond?

P24 Why does the sun appear above the horizon when it is actually below the geometric horizon during a sunset?

P25 James Morris, in *Heaven's Command,* describes a view from Grosse Isle in the Gulf of St. Lawrence as follows: "in the early morning sun the islands are inverted in mirage, and seem to hang there suspended between sky and water." In the early morning, the air gets warm faster than the water, so there is warmer air above the cooler air that is lying just above the

water. Sketch the air, water, and islands and show how the light rays bend, resulting in the mirage Morris describes. In particular, why do the islands seem "suspended?"

P26 In what direction can you see a rainbow early on a rainy morning (east, west, north, or south)? Why?

P27 In a riddle by the fifth-century Anglo-Saxon poet Aldhelm, the rainbow speaks of itself: "I am born red by the sun in the vicinity of a watery cloud. I shine here and there in the northern skies, but I do not climb through the southern skies." Explain the last sentence.

P28 Draw a sketch showing where you might see a rainbow in your lawn sprinkler in the late afternoon. Show the position of the sun, the sprinkler, and you.

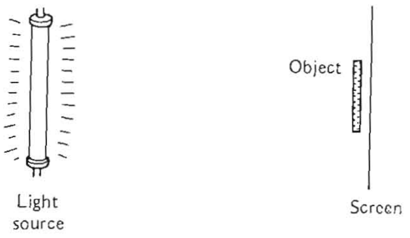
P29 (a) Draw a suitable sketch showing where the ice crystals would have to be if you were to see a "sun" pillar from a bright, distant streetlight. (b) Draw another sketch to show where the ice crystals would have to be if you were to see a "sun" dog from the same streetlight.

HARDER PROBLEMS

PH1 (a) Draw a diagram to explain a solar eclipse, showing regions of umbra and penumbra. (b) What do you see when you look up while standing in the umbra? The penumbra? (c) What would you see if you were standing on the moon, looking at the Earth? (d) During a solar eclipse, some regions see a total eclipse, some a partial eclipse, and some no eclipse at all, yet everyone sees a lunar eclipse the same way. Why?

PH2 (a) Use a diagram to explain a lunar eclipse. Label the umbra and penumbra. (b) What would you see looking up at the moon? (c) What would you see if you were standing on the moon looking up at the earth?

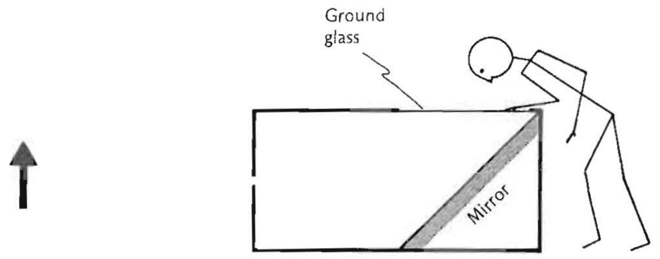
PH3 The figure shows an extended light source and a screen. You have an object 1 cm in height, which you are allowed to place in various positions. When the object is very close to the screen (as shown), it will throw an umbra about 1 cm in height, and a very small penumbra. Redraw the figure, and show where to place the object so that: (a) the penumbra will be larger than the umbra, and (b) there will be no umbra, only penumbra. Show the rays you use to construct the umbra and penumbra in the two cases, using different colors to keep the two cases apart.



PH4 If there were a radio station on Mars, would we be more likely to receive the Martian broadcast if it were at AM or FM frequencies? Why?

PH5 An eye E wants to see a bug B that has crawled down a long, narrow, dark tube. E can't use a light L to illuminate B directly because E and L would get in each others' way. Fortunately, a half-silvered mirror M is available. Draw where M should be placed to illuminate B allowing E to see it. Draw a ray from L to M to E, verifying that E can see B. (Photographers sometimes use this "coaxial illumination.")

PH6 The figure shows a pinhole camera with a slightly different design; it has a mirror mounted at 45° in the back, and a ground-glass screen for viewing is at the back top of the camera. (A ground-glass screen is a piece of glass with a rough surface.) An observer looks at the image of the usual object—an arrow. (a) Redraw the figure and trace the rays from the tip and from the tail of the arrow through the camera to the screen. (Be sure to use the law of reflection at the mirror.) Hence draw the image of the arrow on the screen. (b) Does the observer see the image upside down or rightside up? (c) Suppose the camera, pinhole, and glass are removed, but the object, mirror, and observer remain in place. In the same drawing, draw the image of the arrow in the mirror, where the observer now sees it, and label it "Image."



PH7 (a) Do you think a pedestrian would be more visible in heavy fog wearing dark or light clothing? Why? (See Problem P20.) (b) Should the motorist use the high- or low-beam headlights?

PH8 When a crack develops in glass, and the two edges of the crack separate by a small amount (as almost always happens), the crack becomes quite visible when viewed obliquely through the glass. (a) Explain why this happens—why the crack looks shiny. (b) If you rub some oil into the crack, it becomes much less visible. Explain. (The index of refraction of oil is close to that of glass.)

PH9 People walk into sliding glass doors because they see right through them. Birds fly into the same doors because they see the sky's reflection in them. Explain.

PH10 In *Tess of the D'Urbervilles*, Thomas Hardy writes: "As the looking glass was only large enough to reflect a very small portion of Tess's person at one time, Mrs. Durbeyfield hung a black cloak outside the [window] casement, and so made a large reflection of the panes, as it is the wont of bedecking cottages to do." How does Mrs. Durbeyfield's homemade mirror work?

PH11 Some sunglasses have half-silvered mirrors on them. (a) Why can the wearer see through them, but no one else can? (b) If you take them off, the reflection looks brighter and sharper from the front than from the back. Why? (Remember, glass absorbs some light.)

PH12 (a) A vengeful underwater fish is planning to poke a stick at a fisherman standing on the shore. Should the fish push the stick above, below, or directly toward where he sees the fisherman? Explain your answer, using a diagram. (b) The fish now uses a powerful laser as a weapon. Should he aim the laser above, below, or directly toward where he sees the fisherman? Explain.

PH13 Place a coin in the bottom of an empty teacup that is sitting on a table. Position yourself so that the coin is just barely hidden by the near rim of the cup (that is, you cannot see the coin). Hold your head in the same place while you fill the cup with water. Now you can see the coin. Explain, using a diagram. What physical principle is responsible?

PH14 Even *before* you've drunk your whiskey, a swizzle stick placed in it may appear bent or broken when viewed from the side. Draw a diagram of the glass, as seen from above, to explain this phenomenon. Draw the glass as a circle, and the (vertical) stick as a heavy black dot somewhere inside (but not at the center of) the circle. Draw a ray from the bottom half of the stick to your eye outside the glass (a ray that passes through the whiskey) and one from the top half of the stick, which is not in the whiskey. Show where the bottom and top halves of the stick would appear to be.

PH15 Draw a glass rectangle with a ray of light incident, at an angle, on one of the long sides. Continue the ray through the glass and let it enter an eye somewhere beyond the glass. How does the apparent position of the source of the ray compare with its actual position?

PH16 (a) Describe the appearance of a simple mirage. What atmospheric conditions are necessary for one? (b) What conditions, on the other hand, might allow Erik the Red to see Greenland hundreds of miles from Iceland, where he was staying?

PH17 You lie on your back in a bathtub, without any water in it, and look at a cup resting on a soap dish. Now somebody fills up the tub with water, so the water level is over your head, exactly at the top of the soap dish and the bottom of the cup. Sketch what your view of the cup and soap dish is after the water is added and before you drown. To help you do this, you should draw a side view, showing how the rays from the cup and soap dish reach your eye.

PH18 Explain, using a diagram, how reflection off the ends of the long crystals of Figure 2.74, when the sun is low, produces a horizontal line in the sky—a parhelic circle.

PH19 Use a top-view diagram to explain Figure 2.52. Draw a ray from a point *P* on the outside edge of the beer (inside the glass) to a viewer's eye. Of course, the ray must bend when it gets to the outer surface of the glass. Draw the normal to this surface at the appropriate place, and make sure the ray bends in the correct direction. Show the apparent position, *P'*, of the point *P*, as seen by the eye.

PH20 A glass beaker has a small round hole in its side. A laser beam shines horizontally through the beaker, out the hole, and strikes a screen. The hole is now plugged up and the beaker is filled with water. When the plug is removed, the water streams out the hole. The laser beam still shines through the hole, but no longer strikes the screen. Rather, there is a region of light on the floor where the streaming water is landing. Explain what has happened to the light, drawing a diagram and showing a light ray's path.

MATHEMATICAL PROBLEMS

PM1 Suppose you are 2 m tall and are standing halfway between a candle on the floor and a white vertical wall. (a) How high is your shadow on the wall? (b) If you shake a rope up and down at a frequency of 2 Hz and amplitude $\frac{1}{2}$ m, with what frequency and amplitude does your shadow shake the shadow rope?

PM2 (a) The speed of light in a certain glass is 200,000 km/sec. What is the index of refraction of this glass? (b) The index of refraction of a certain type of diamond is 2.5. What is the speed of light in this diamond?

PM3 Explain how you could use Snell's law to measure the speed of light in diamond.

PM4 A beam of white light in air is incident on a diamond, with an angle of incidence of 45°. (a) Using Table 2.6, calculate the angle at which red light is transmitted in the diamond. (b) Do the same for deep blue light. (c) What is the angle, from red to deep blue, into which the white is spread?

PM5 Use Table 2.4 to calculate the critical angle for light going from: (a) water toward air, (b) glass toward air, (c) diamond toward air, and (d) glass toward water.