Principles of Geometrical Optics

CHAPTER 2

2.1 *INTRODUCTION*

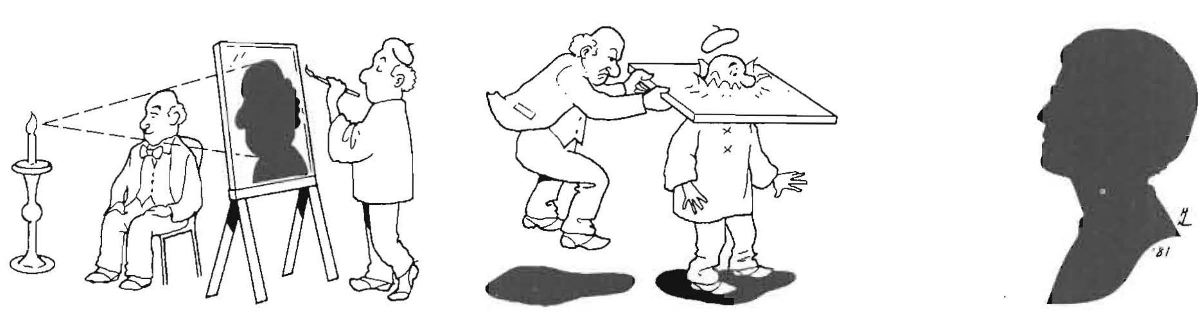
How do we so easily decide where to place a beach umbrella to keep the sun out of our eyes, without worrying about the electric field, the wavefront, the wavelength, and the frequency of the light? The answer is that, for many simple problems it is sufficient to concentrate on the light *rays* (Fig. 1.17), the lines that describe in a simple geometric way the path of light propagation. ***Geometrical optics*** is the study of those phenomena that can be understood by a consideration of the light rays only. Geometrical optics is useful as long as the objects with which the light interacts are much larger than the wavelength of the light. As our beach umbrella is about a million times larger than the wavelength of visible light, geometrical optics is a very good approximation for this and most other everyday objects. For smaller objects the beam will not propagate in only one direction, but rather spread out in all directions—much as sound waves, with wavelength of about a meter, spread out around obstacles in the street.

In geometrical optics, then, light does not bend around corners. We think of the light as traveling in straight lines as long as it is left alone (Fig. 1.4). This straight-line propagation enables us to locate the beach umbrella so its shadow falls on our eyes.

2.2 *SHADOWS*

To cast shadows you need light from a fairly concentrated source, such as the sun. The best shadows are cast by light that comes from just one point: a ***point source,*** which is an idealization, like a ray, that can only be approximated, for example, by a small light bulb or candle. (Even a source as large as the sun or a giant star can approximate a point source if it is far enough away.) You also need a screen, such as a flat, white surface, which redirects incident light into all directions, so that you can see the shadow (the light from the surrounding area must enter your eye).

If an obstacle blocks some of the light rays headed for the screen, the light rays that are *not* blocked still reach the screen and make that part of the screen bright. Those that *are* blocked don't reach the screen, so the places where they would have hit are dark—a ***shadow.*** We can figure out the shadow's location by drawing straight lines from the point source to the edge of the obstacle and continuing them to the screen. These lines separate the regions where the rays reach the screen from the region where they are blocked. The resulting shadow resembles the obstacle, but it is of course only a flat (two-dimensional) representation of the object's outline. Nonetheless, we can easily recognize simple shapes, such as a person's profile. Before photography, it was popular to trace people's shadows as silhouette portraits (Fig. 2.1a). Today's x-ray pictures are just shadows in x-ray



(a) (b) (c)

FIGURE 2.1

(a), (b) Various uses of shadows. **(c)** Silhouette of one of the authors. Etienne de Silhouette (1709–1767), France's finance minister for a year, was deposed because of his stinginess over court salaries. He used cheap black paper cutouts in place of conventional decorations in his home and invented a technique for making paper cutout shadow portraits to raise money. When he died, he was penniless and destitute, a shadow of his former self.

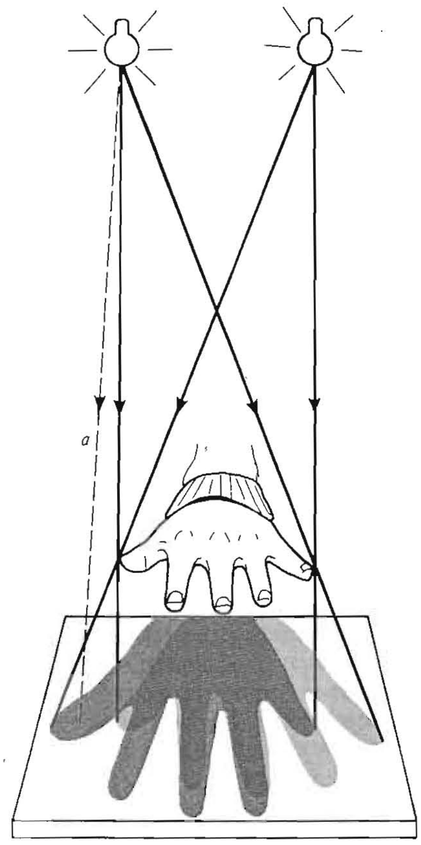


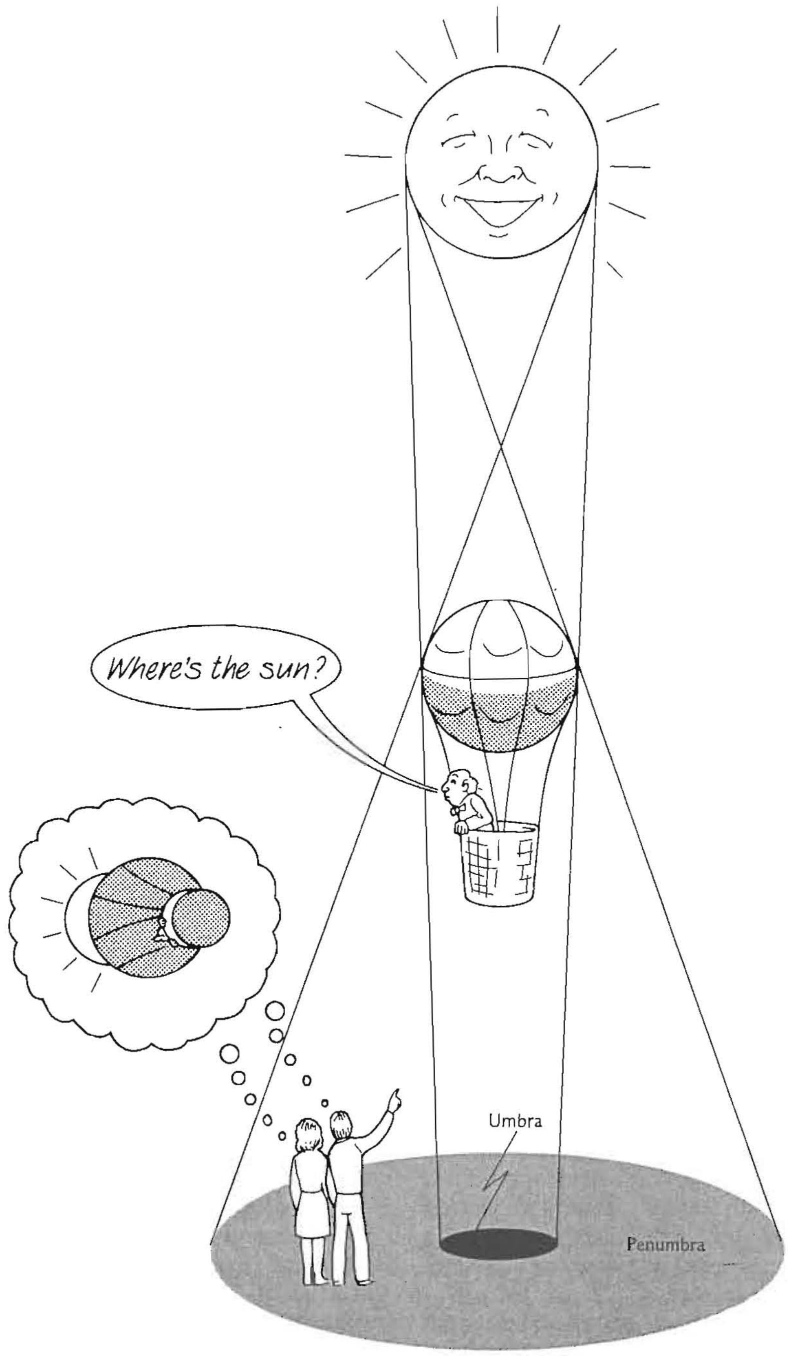
FIGURE 2.2

Two light sources throw two shadows. Their overlap, reached by no ray from either source, is the umbra. The penumbra is illuminated by rays, such as ray *a*, from only one of the sources.

"light," made visible by a fluorescent screen or photographic film.

In these examples, the shadow is seen on an object *different* from the obstacle blocking the light. An object can cast a shadow on *itself* as well. For example, the earth prevents the sun's rays from reaching the other (night) side of the earth. When you watch a sunrise, your back is in such a shadow: your own shadow.

Shadows help artists represent objects more realistically. For example, an unsupported object in mid-air is portrayed detached from its shadow (Fig. 2.1b), long shadows create a twilight (or dawn) mood, shadows of a face on itself give a more three-dimensional appearance, and so on.



What happens if we have *two* point sources? We get two shadows. If the two shadows overlap, the only completely dark region is this overlap. Where they don't overlap, the shadows are only partly dark because they still are struck by rays from the other source. The parts not in any shadow, of course, are struck by rays from both sources (Fig. 2.2).

FIGURE 2.3

Looking back at the source from places in the penumbra and umbra.

The story is similar if we have three or more point sources. The only completely dark region is the overlap of all shadows. Other regions on the screen will not be as dark, because fewer shadows from the point sources overlap there. An ***extended source*** such as a long fluorescent tube can be thought of as a collection of many point sources, each of which casts a shadow. A region where *all* the shadows overlap will be completely dark. It is called the ***umbra.**** Regions where the shadows don't all overlap will be partially dark because they still receive light from some parts of the extended source. Such a region is called the ***penumbra.***† You can figure out the darkness of the shadow by supposing you are at a point on the screen and looking back toward the source. Where you are, it will be darker in proportion to the amount of the extended source that is blocked from your view. If you stand on the screen in the umbra you cannot see the source at all, but while standing in the penumbra you can see part of the source. Of course you would need a large screen, obstacle, and source in order actually to stand in the umbra or penumbra (Fig. 2.3).

A. Eclipses

We can actually do the experiment just described, by using the sun for the source, the moon for the obstacle, and the earth for the screen, whenever they all lie along a straight line. If the moon's shadow falls on the earth and we stand in the umbra, we cannot see the sun at all, and are enveloped in darkness. This is called a *total* ***eclipse***‡ of the sun or total ***solar eclipse*** (Fig. 2.4). In the penumbra we can still see part of the sun, and we are therefore in a *partial eclipse* of the sun.

(This is the first of many optical phenomena associated with the sun that we'll discuss. We must therefore provide you with this important consumer protection:

*Latin for shade (cf. umbrella).

†Latin *paene* plus *umbra*, almost a shade.

‡Greek *ekleipsis*, failure to appear.

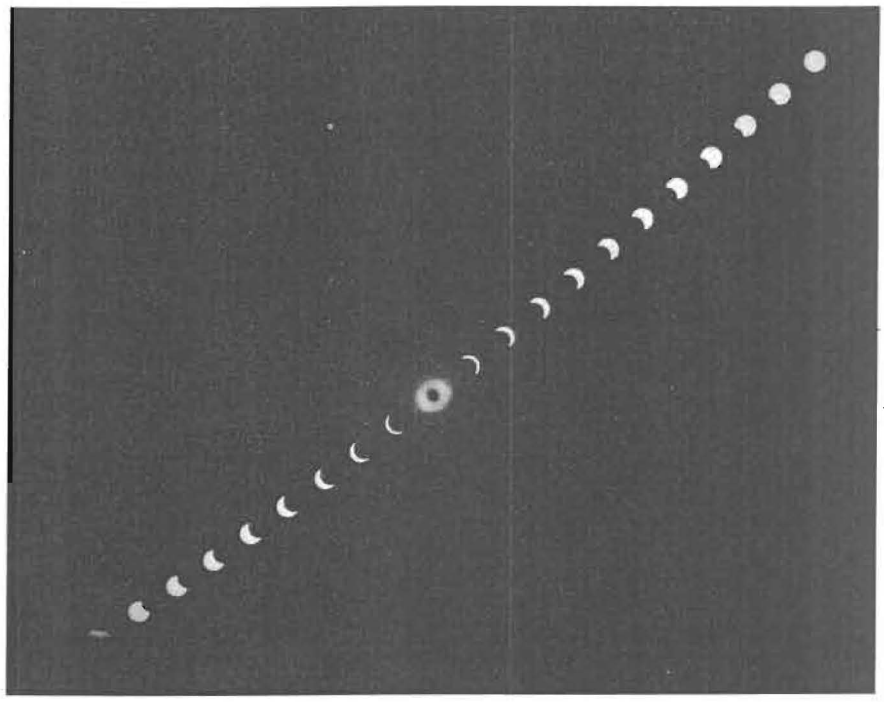


FIGURE 2.4

A time sequence of photographs of a solar eclipse.

WARNING

DO NOT STARE AT THE SUN.
Failure to heed this warning can cause serious damage to your eyes.

After every solar eclipse, eye clinics have a rash of patients with eclipse blindness (Plate 5.1). However, it is not the eclipse that does the damage, it is the sun. Normally we don't stare at the sun, except fleetingly, nor focus on it particularly carefully, other than at sunset or sunrise when our eyes are protected by the longer path through the atmosphere that the sun's rays then traverse (Fig. 2.60). However, an interest in optical phenomena might tempt you to look more fixedly at the sun, and you would run the risk of permanently damaging your eyes. Constantine VII, the East Roman emperor, apparently went blind looking at an eclipse. Galileo injured his eyes by looking at the sun through his telescope. Should you damage your eyes by staring at the sun, you would then be in good company, but that would be scant recompense indeed. The best way to examine a solar eclipse is with a pinhole device discussed in Section 2.2B. Because the moon is nowhere near as bright as the sun, you may stare directly at a lunar eclipse to your heart's content. You may also look at other phenomena we'll describe that are *near* the sun.)

We can determine the region of the umbra in an eclipse by drawing two straight lines (Fig. 2.5): one from the top edge of the sun, past the top edge of the moon, and on to the earth *(a)*, and a similar line along the bottom edges *(b)*. Any region behind the moon that lies between these two lines (before they cross) will have the sun completely obscured by the moon, and thus lie in the umbra. The penumbra is bounded by straight lines that start at one edge of the sun and cross to the *opposite* edge of the moon *(c* and *d)*. That is, lines *a* and *c* give the boundary of the shadow of the moon produced by the source on the sun at *A*. Lines *b* and *d* give the shadow produced by the source at *B*. The shadows produced by all other sources between *A* and *B* lie between these two. Hence, all these

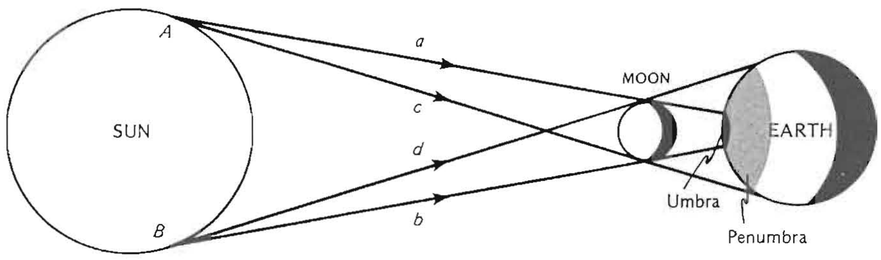


FIGURE 2.5

The umbra and penumbra of the moon projected on the earth, in exaggerated scale—the umbra is only about 200 km wide, which explains why total solar eclipses are rare at any given location.

shadows must overlap between *a* and *b*, and none can overlap outside of *c* and *d*.

An eclipse of the moon (a ***lunar eclipse***) is similar, but now the earth is the obstacle and the moon behind it is the screen. We don't stand on the moon, but rather stand on the dark side of the earth and look at the light reflected by the moon.

PONDER

What would a lunar eclipse look like if viewed from the moon?

The Chinese poet Lu T'ung wrote a poem describing an eclipse of the moon that occurred in the year 810:

The glittering silver dish rose from the bottom of the sea,
. . . . There was something eating its way inside the rim.
The rim was as though a strong man hacked off pieces with an axe, . . .
Ring and disc crumbled as I watched
Darkness smeared the whole sky like soot,
Rubbing out in an instant the last tracks,
And then it seemed that for thousands of ages the sky would never open.
Who would guess that a thing so magical
Could be so discomfited?. . . .
I know how the school of Yin and Yang explains it:
'When the mid-month sun devours the moon the moonlight is quenched,
When the new moon covers the sun the sunlight fails.'

(Of course, it is the earth that "devours the moon," not the sun.)

An understanding of eclipses has occasionally proven useful. During the winter of 1503–1504, Columbus and his men were stranded on Jamaica. The Indians, who had been giving him food, decided to stop. Using his knowledge of an impending lunar eclipse, Columbus informed the Indians that his god wished them to supply food, and, to demonstrate this, would give them a sign from heaven that night. The eclipse began at moonrise and the Indians soon begged Columbus to intercede for them. This he did, when he judged the total phase of the eclipse to be over. The Indians resumed supplying him. (They apparently never asked why his god didn't simply teach Columbus to get his own food, rather than mess around with the moon.) This same theme was used by Mark Twain in *A Connecticut Yankee in King Arthur's Court.*

B. Pinhole camera

Using the principle of looking back from the screen at the partially blocked source, we could construct the umbra and penumbra of any complicated object in the light of an extended source of any shape—but in general the result is complicated and not very enlightening. However, in one particular case the result is again simple, even though the source may be an arbitrary, extended shape. This happens when the obstacle is entirely opaque except for one small hole, a ***pinhole.*** In Figure 2.6 we have drawn an arrow as the source. (The arrow could represent a standing person, a tree, a building, or whatever. We use an arrow because it is easy to draw and you can tell its direction.) If we follow the rays in the usual way, we see that light from the head *a* of the arrow arrives at only one point on the screen, *A*; all other rays from *a* are blocked by the obstacle. Similarly for the tail *b*, and all points in between. So there will be light on the screen from *A* to *B*, and this light will have the same shape as our arrow. The screen will have an inverted (upside-down) ***image**** of the arrow.

Such a device is known as a ***pinhole camera.*** Though we have much better cameras nowadays, pinhole cameras are still useful occasionally. For example, since only a small amount of light gets through the tiny pinhole, looking at the image of the sun on the screen won't hurt your eyes. Industrial spies have been known to use small cardboard-box pinhole cameras, fitted with film instead of the screen.

*Latin, *imago*, imitation, copy.

FIGURE 2.6

A pinhole forms an inverted image.

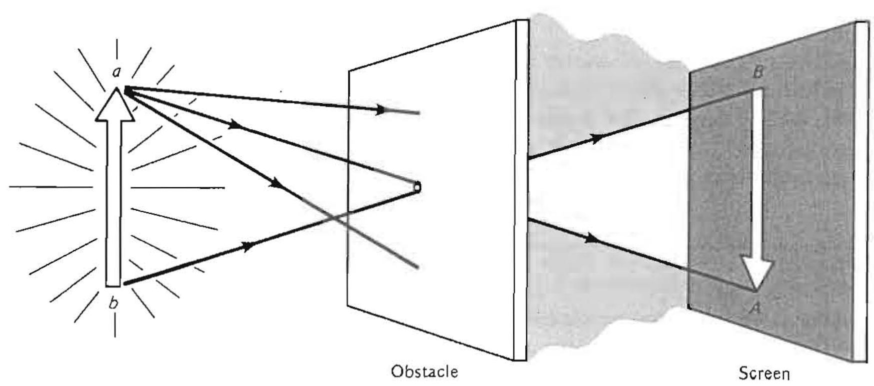
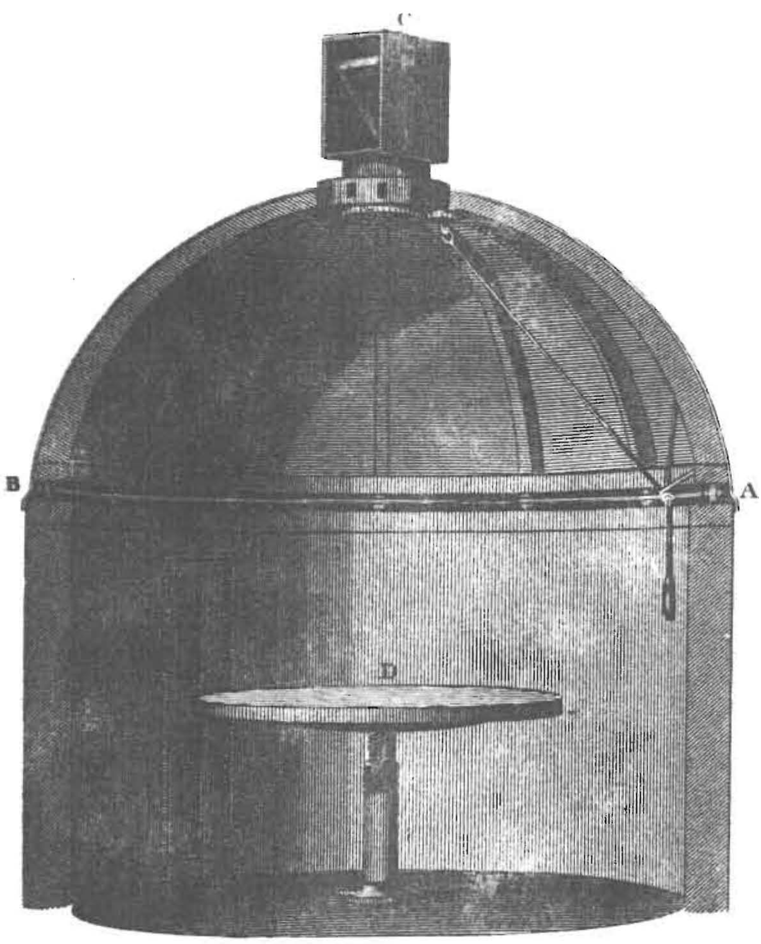


FIGURE 2.7

An elaborate camera obscura. To spare the viewer the need for standing on his head, a 45° mirror (Sec. 2.4) is used to bend the light so that the image is projected vertically down on a round table. The viewer can then walk around the table until he sees the image upright. The mirror can be rotated to provide a change of view.



They leave these innocuous looking boxes lying around for a day and then return to pick them up. (Since the pinhole admits so little light, a day's exposure may be just right.) Pinhole cameras are also used where a great depth of field (Chapter 4) is needed, and as cameras on satellites for detecting high frequency x-rays.

A pinhole camera is an example of a ***camera obscura,**** named by its fifteen-year-old inventor in the sixteenth century, Giambattista della Porta (later accused of witchcraft). He had a completely dark room except for a small hole and could then see, on the wall opposite the hole, an inverted image of the outside scene (Fig. 2.7). Much earlier Aristotle noticed round images in rooms that had a small irregular hole in the wall, no matter what the shape of the hole. Aristotle was, of course, looking at the images of the round sun. You can sometimes also see this on the ground under a shade tree, from light coming through the small spaces between the overlapping leaves. Though the small spaces are not usually round, the image is round because the object (the sun) is round (but not always, see Fig. 2.8).

*Latin, dark room. The name also refers to later devices with lenses—cameras without film.

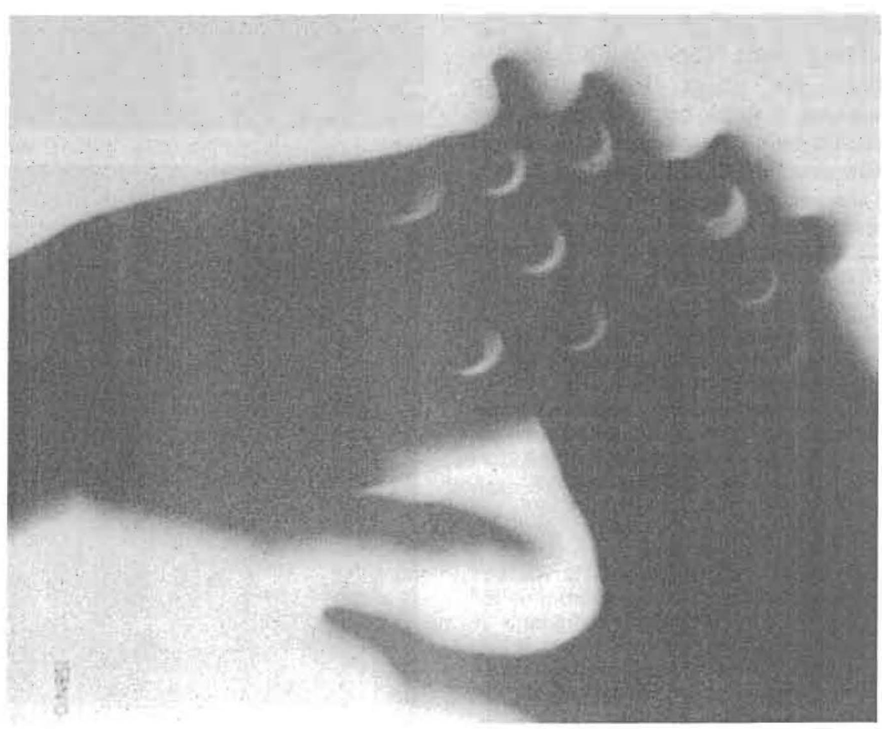


FIGURE 2.8

Images of a partially eclipsed sun. The images are produced by the "pinholes" formed in the small spaces between overlapping fingers, one image for each "pinhole." Notice the crescent shape of each image, due to the partial eclipse. This is a safe way to watch an eclipse.

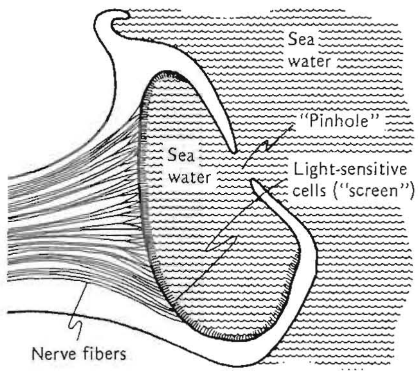


FIGURE 2.9

The pinhole eye of the *Nautilus*.

Even before Aristotle, Nature used the pinhole camera in the design of the eye of the mollusk *Nautilus* (Fig. 2.9). This simplest of eyes is clearly poor for gathering light, so Nature quietly gave up on it: such an eye is found in no other living organism.

An obvious way to cure the problem of the dim image is to increase the size of the pinhole. However if the hole is too big the image becomes blurred—rays from each point on the object pass through different parts of the hole and reach different parts of the screen, thus spreading out (blurring) the image of that point. Too small a hole also blurs the image (in addition to making it dim), because as the size of the hole gets close to the size of the wavelength of light, light ceases

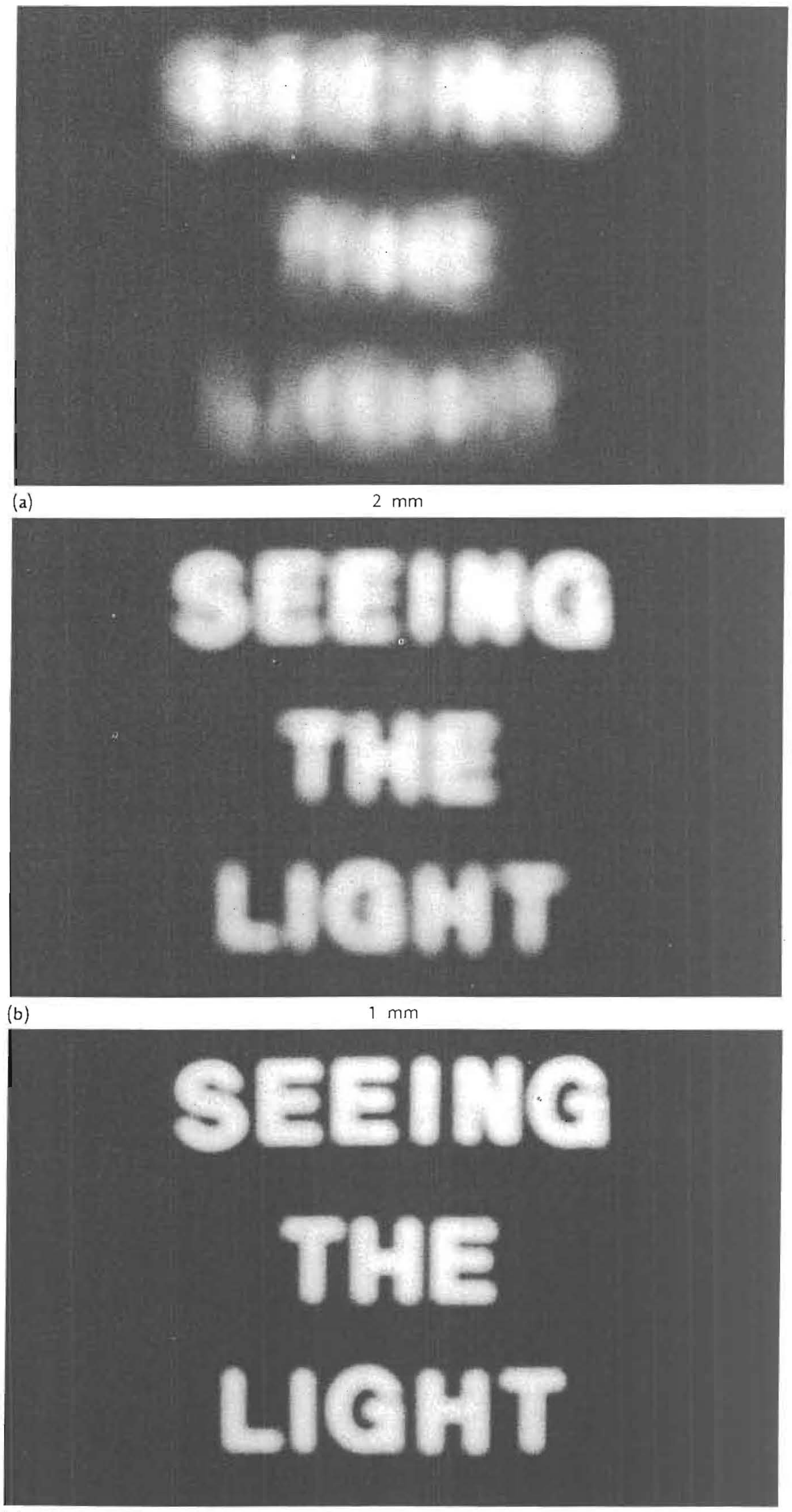
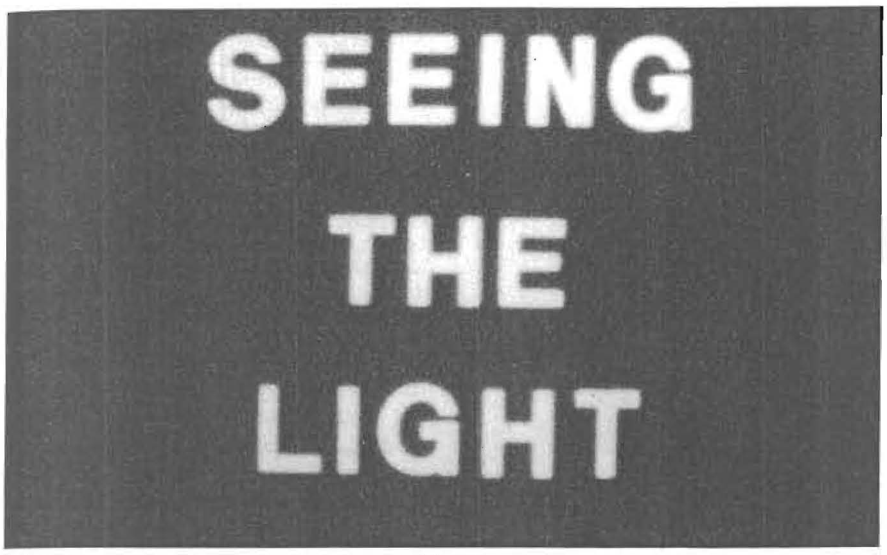
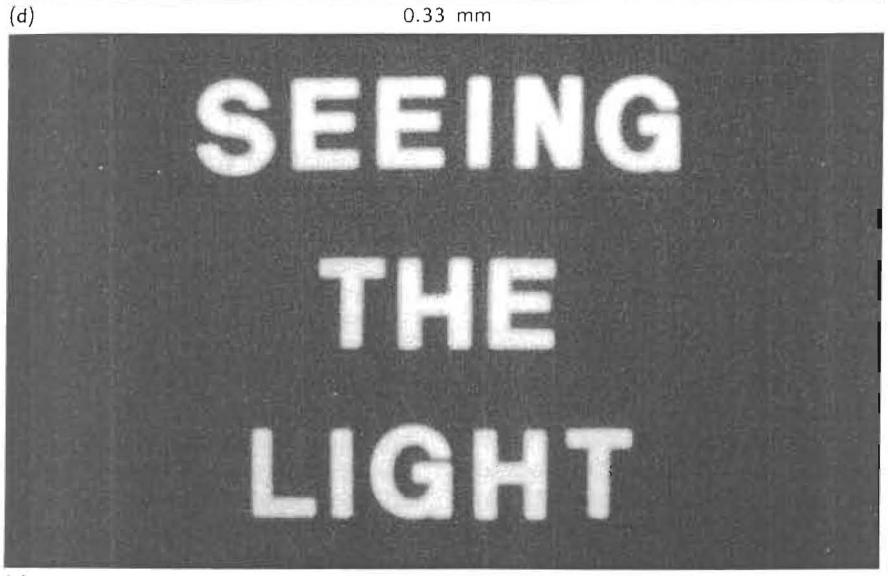


FIGURE 2.10

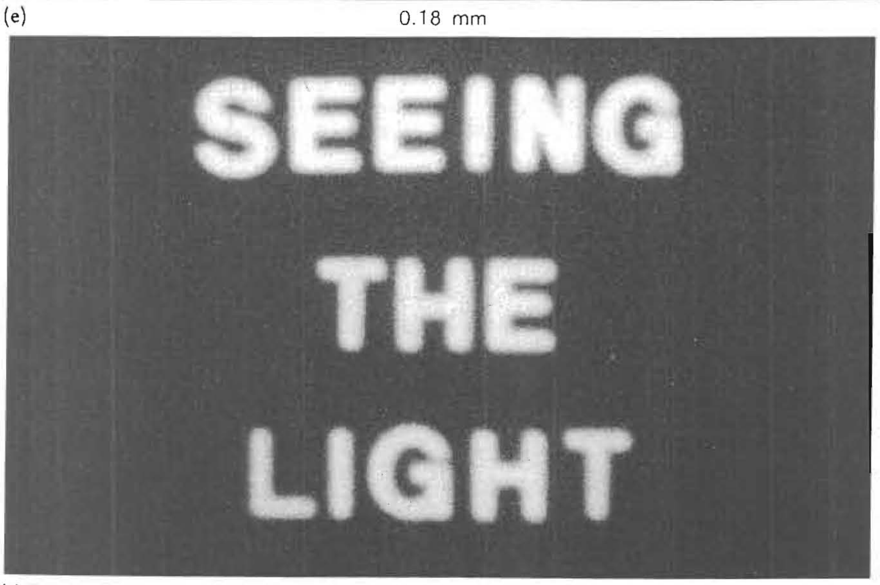
Pinhole camera photographs of the same object but with different pinhole sizes. Pinhole diameter is largest at **(a)** and decreases, as labeled, for each successive picture. The images in **(a)** and **(b)** are blurred because the pinhole is too large. At **(c)** the smaller pinhole gives a sharper image. At **(d)** and **(e)** the pinhole has about the optimum size. Image **(f)** is blurred because the pinhole is so small that the wave properties of light become important (Sec. 12.5C). Exposure time was increased from **(a)** to **(e)** in order to make each photograph equally bright. The optimum pinhole diameter is about twice the square root of the product of the wavelength and the pinhole-to-screen distance.



(d) 0.33 mm



(e) 0.18 mm



(f) 0.10 mm

to propagate along straight lines (Chapter 12). The optimum size is somewhere in between (Fig. 2.10). For a pinhole camera the size of a shoebox (about $\frac{1}{3}$ meter), the optimum pinhole size is about $\frac{3}{4}$ mm, just about the diameter of a common pin. For a room-sized camera obscura, the best size is 2 or 3 mm. The larger the camera, the sharper and more detailed, but also the dimmer, will be the image of a distant object. To make a pinhole camera and pinhole photographs, see the TRY IT.

TRY IT

FOR SECTION 2.2B

Pinhole cameras, cheap and expensive

You can make a pinhole camera out of any cardboard box, such as a shoebox. Make a small pinhole at the center of one end of the box. To look at the image, cut a large square hole in the opposite end and cover it with tissue paper. The tissue paper serves as a translucent screen so you can see the image on it from the outside. Throwing a black cloth or coat over both the camera back and your head will cut down stray light. First look for the image of some bright object, such as the sun or a lamp bulb. (The image of the sun is easily seen but may be disappointing because of its small size). Using a lamp bulb as the object, note the orientation of the image and its change in size as you move away from the bulb. Then see what happens when you slightly increase the pinhole size. Observing from a shaded area you should be able to see a fairly good image of any brightly lit scene. Next make the hole quite big and watch the image become brighter, but more blurred. Then tape a piece of aluminum foil over the large hole, make several pinholes in the foil, and examine the lamp bulb again.

For the image of the sun to be of appreciable size, use a long camera, for example, one made from a mailing tube coated inside with black spray paint. Even more simply, for the sun you can dispense with the box and simply project its pinhole image on a white screen by using a large cardboard with a pinhole in its center (the shadow of the cardboard is sufficiently dark to make the sun's pinhole image visible). This is the recommended way to observe a solar eclipse.

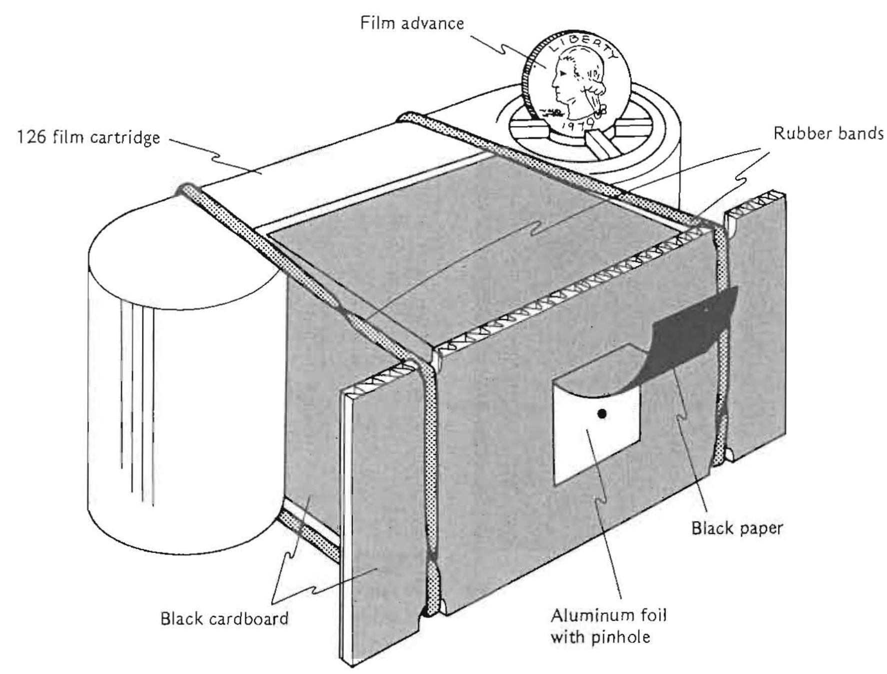


FIGURE 2.11

Design for a cartridge pinhole camera. The box should have a square cross section, $1\frac{7}{16}$ inches on each side so it just fits into the cartridge, and a length of about 2 inches or more, depending on whether you want a "normal" or "telephoto" pinhole camera (Sec. 4.3A). All edges and corners should be taped with black tape to prevent stray light from entering. The box can be held tightly in the cartridge with rubber bands. Note that you need a black paper flap over the pinhole to serve as a shutter.

If you want to photograph a pinhole image, you can use a 126 film cartridge in place of the viewing screen. Figure 2.11 shows a possible design, and Table 2.1 gives approximate exposure times. Be sure to have the camera on a steady support during the exposure. To advance the film, insert a coin in the round opening on top of the film cartridge, slowly turn counterclockwise, and watch the frame number through the small window in the back of the cartridge. (Stop turning when the third and fourth in each series of frame numbers both appear in the window.)

If you have a camera with interchangeable lenses, you can remove the lens and cover the opening tightly with a piece of aluminum foil. Make one or more pinholes in the foil, and you can take pinhole pictures the same way you usually take pictures with your camera (using a tripod for these longer exposures). Figure 2.12 shows some pictures taken this way.

***TABLE 2.1** Suggested exposure times for cartridge pinhole camera (Assumes pinhole diameter ≃ $\frac{1}{3}$ mm)*

ISO index of film	*Film type (example)*	*Exposure time: Bright sun*	*Exposure time: Cloudy bright*
400/27°	Tri-X	$\frac{1}{2}$ sec	2 sec
125/22°	Verichrome	2 sec	9 sec
100/21°	Kodacolor	3 sec	15 sec
64/19°	Kodachrome	4 sec	20 sec

Notes:
1. Also try half and double the suggested time. If the picture improves with one of these changes, continue halving or doubling until the optimum exposure time is found.
2. To measure the proper pinhole camera exposure with a light meter, set the meter at f/11 and find the exposure time it indicates. Multiply that time by 300 to obtain the suggested pinhole camera exposure time.

FIGURE 2.12

Pictures taken with pinhole camera on 35-mm film. **(a)** One pinhole. **(b)** Two pinholes.

(a) (b)

2.3 REFLECTION

We are familiar with reflected light, just as we are familiar with a ball reflecting (bouncing) off a wall. This common phenomenon, ***reflection***, can be treated by geometrical optics. Light is traveling, say in air, in a straight line when it hits a different medium, say a mirror. Its direction then changes suddenly, the old ray stops and a new one starts.

If we want to know *why* a ball is reflected from a wall, we have to look into the structure of the ball—how it is compressed when it first hits the wall, how the resulting elastic forces push it the other way, and so on. Similarly, to understand *why* light is reflected by some substances, we must look at the nature of light. So we must digress a bit from geometrical optics and return to waves.

In Figure 2.13 there are two hands holding a rope. *A* has just started a wave moving to the right. What will happen next? If *B* keeps tension in the rope but doesn't pull up or down on it, the wave *C* will arrive there, shake *B*'s hand up and down, and we simply have a long-distance hand-shaking device. But now suppose *B* decides to pump his hand *down* just at the moment the wave *C* gets there. If *B* pushes his hand down just as hard as the rope is pulling it up, his hand won't move at all. However, because he has exerted a downward force on the rope, he has generated a wave *D* that propagates to the left as a downward wave (Fig. 2.14). But since *B*'s hand never moved, even though it exerted a force, we might as well replace him by a brick wall *E* on which the rope is firmly and immovably attached. So our story from *A* to *E* teaches us that when we send a wave down a rope with a *fixed end*, an *opposite wave* comes back. We say that the wave, *C*, is *reflected* by the wall, because *D* looks just the same as *C* turned upside down (and backward, since the motion is also reversed). In the fancier language of Section 1.2B, we say the wave has undergone a 180° phase change (up to down).

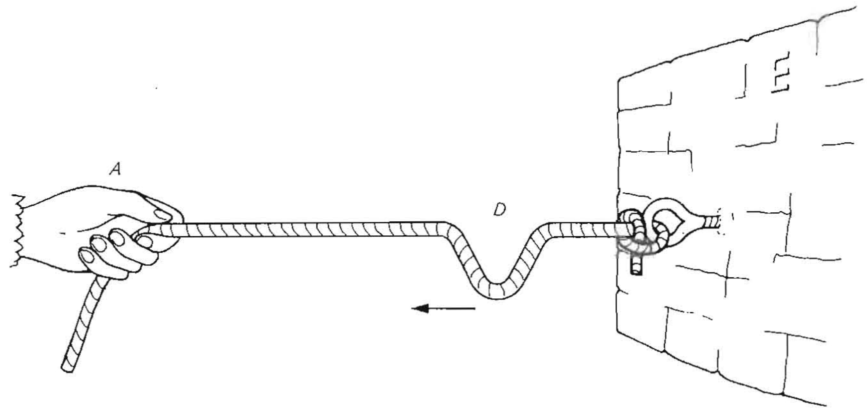


FIGURE 2.14

B, stonewalling the lady's gesture, refused to let his hand be shaken. Consequently, an upside-down wave returns to *A*.

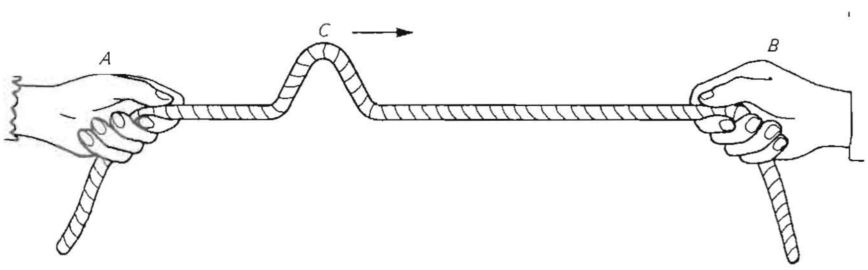
If the rope were not tied to a brick wall, but instead to a much heavier rope that is nearly as difficult to move as the wall, the junction point would then hardly move when *C* arrives, and we would still get a reflected wave. Because the end of the very heavy rope moves, albeit very slightly, a small wave continues on in the heavy rope. We call this the ***transmitted*** wave. Such reflections, which occur when the medium of propagation (the rope) changes (rather than stops), and which result in a reflected wave that is the negative (180° phase change) of the incident wave, are called *hard reflections*.

The opposite extreme are *soft reflections*. An example is a rope tied to a very thin thread. The thread has hardly any mass and only serves to keep the rope under tension. Under these conditions the junction also reflects waves, but *C* comes back as an *upward* wave, no phase change: because the thread is so light, it is easily pulled up by *C*. It is as if *B* had pulled up on the rope's end, sending an upward wave back toward *A*. Again, part of the wave will be transmitted while the rest is reflected.

Reflections of any kind of wave (including light) occur whenever the medium of propagation changes abruptly. What counts is the change in the wave's speed of propagation. If it changes a lot, a lot will be reflected. If it changes only a little, most of the wave will be transmitted, and the reflected wave will be weak. The reflection is hard if the incident wave is trying to go from a faster medium (say light traveling in air) to a slower medium (say light traveling in glass), and soft otherwise. If the two media have the same wave speed there is no reflection.

FIGURE 2.13

The lady, *A*, keeping her distance, has started a wave propagating toward *B*.



*A. Radar

Reflected waves can be quite useful. If we know how fast the wave travels in the first medium, we need only send out a wave pulse and measure the time it takes to be reflected back to us in order to determine the distance to the reflecting boundary. This echo principle is the basis of ***radar.**** The radar antenna is both emitter and receiver. It first sends out a pulse of electromagnetic radiation. If the pulse comes back, then the radar operator knows there is some reflecting object out there. From the echo's time delay, she can determine the distance to that object. Typically radar uses electromagnetic radiation of about a billion hertz.

Bats (as well as the oil birds of Venezuela, Borneo swiftlets, dolphins, and seals) use a similar system, but with ultrasonic *sound* waves (up to 100,000 hertz, well above the maximum frequency of 20,000 hertz that humans can hear). Although folklore refers to them as blind, bats actually "see" very well by means of these short-wavelength sound waves (about $\frac{1}{2}$ mm). This process, called ***sonar,***† is also used by humans; for example, to detect underwater submarines, to measure the depth of water, and in certain cameras to determine the distance to the subject (in air).

B. Metals

Let us return to electromagnetic waves and consider reflections more closely. Suppose an electromagnetic wave is traveling in a vacuum and comes to some material, say glass. The electric field in the wave then causes the charges in the glass to oscillate, as we've seen, and these oscillating charges radiate, making a new wave. Some of this new radiation goes backward, becoming the reflected wave. Some goes forward and combines with the incident wave (which is also going forward) to make the transmitted wave (Sec. 1.3). The way these forward-going waves combine, either adding to each other or tending to cancel each other, determines the intensity of the transmitted wave, that is, how transparent the glass is.

But suppose the material is a metal, rather than glass. Metals are good conductors of electricity. That means that they have lots of charges (electrons) that aren't attached to individual atoms, but rather are free to move around in the metal. There are many free electrons in a metal, and, being charged, they set up their own electric field. In response to an electromagnetic wave hitting the metal, the electrons move until the field they set up exactly cancels the electric field of the incident wave. (After all, if there were any electric field left over, more electrons would move until it was completely canceled.) Once the field is canceled, there is no further force to move any more electrons. But if there is no electric field inside the metal, there is no electromagnetic wave there—no transmitted wave. Metals are thus ***opaque.*** Almost all the energy in the incident wave goes into the reflected wave. That is why metals make very good reflectors.

All of this is only true up to a point. If the *frequency* of the electromagnetic wave is *too high,* then the electrons in the metal cannot move fast enough to keep up with the incident wave and cancel it. The frequency at which this begins to happen is called the ***plasma frequency.*** For yet higher frequencies, more and more of the wave is transmitted—the metal becomes more transparent and less reflecting.

The plasma frequency depends on the particular metal. In silver, the plasma frequency is a little higher than the frequency of visible light, so silver is an excellent reflector of all visible light and is consequently used for mirrors (Table 2.2). Gold is similar to silver, but its plasma frequency is a little lower—in the blue region of the spectrum. Thus in gold all visible light except blue is reflected. White light with the blue removed looks yellow (Chapter 9), so gold looks yellow. Copper, also similar, has a still lower plasma frequency, so some of the green, as well as the blue, is not reflected. This causes copper to have its characteristic reddish color.

*C. The ionosphere

The same effect occurs in the ***ionosphere***—that layer of our atmosphere in which the ultraviolet light from the sun has stripped electrons off the atoms, producing lots of free charged particles, just like in a metal. Because of the low density of the ionosphere, the plasma frequency is much lower than in a metal—much lower even than the visible (Table 2.3). Thus we can see through the ionosphere; it transmits the visible. However, the ionosphere plasma frequency is well *above* the frequencies of AM radio waves, which are consequently reflected by the ionosphere. That is why AM radio signals carry so far. They can be reflected from the ionosphere, and sent to points on the earth beyond the line of sight (Fig. 2.15). (This doesn't work for FM radio and TV because their higher fre-

***TABLE 2.3** Selected frequencies*

Visible	10^{14} hertz
Ionosphere plasma frequency	10^{8} hertz
TV, FM	10^{8} hertz
AM radio	10^{6} hertz

***TABLE 2.2** Plasma frequencies of noble metals*

Metal	*Plasma frequency*	*Color of metal*
Silver	Above the visible	White—good for mirrors
Gold	In the blue	Yellow
Copper	In the blue-green	Reddish

*From *radio detection and ranging.*

†From *sound navigation and ranging.*

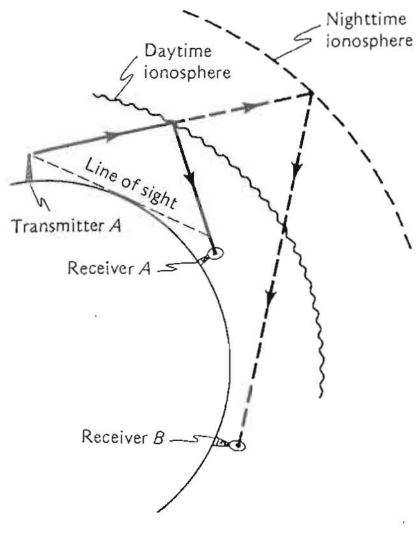


FIGURE 2.15

Reflection of radio signals from the ionosphere makes distant reception possible. The ionosphere rises at night, increasing the range of reception. (Exaggerated for clarity.)

quencies are comparable with the ionosphere plasma frequency.)

Consider now what happens at night. Less solar UV strikes the atmosphere after sunset, so the electrons and positively charged particles in the ionosphere can recombine into neutral atoms. This process occurs primarily in the lower, denser levels of the ionosphere. The result is that the ionosphere rises at night—the charged particles remain free only at the higher altitudes (Fig. 2.15). Thus, at night your AM radio can pick up stations from distant cities, reflected from the ionosphere. As some of these stations come in almost at the same place on the dial as local stations, AM reception may get lousy at night.

D. Mirrors

Returning to geometrical optics, we expect to have reflections whenever light traveling in one medium encounters another medium. Thus we get the familiar reflections from water surfaces or from panes of glass. Glass is, however, not a very good reflector. When light strikes perpendicularly, only about 4% of the intensity is reflected. For this reason glass is usually used to transmit light, rather than to reflect it.

Silver, we've seen, is an excellent reflector in the visible (close to 100%), 25 times better than glass. But exposed silver tarnishes, and tarnished silver is not a good mirror because the tarnished surface *absorbs* (rather than reflects) the light, and thus looks black. The trick to prevent this is to plate silver on the back of glass (Fig. 2.16). The glass protects the silver surface and provides mechanical strength so that there is no need for a lot of expensive silver. The silver reflects the light. Of course, there is some reflection from the front of the glass (you can sometimes see a weak extra reflection, see Sec. 2.4C), but most of the reflection, by far, comes from the silver. (Aluminum is also used.) High-class mirrors, for use in optical systems, have silver in front in order to eliminate this unwanted reflection. These front-surface mirrors will, of course, eventually tarnish or scratch and therefore must be handled very carefully.

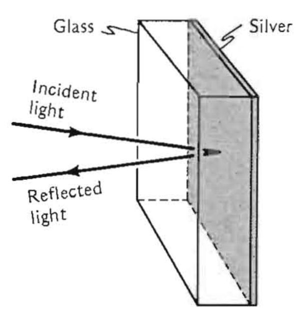


FIGURE 2.16

An ordinary mirror consists of a piece of glass, coated on its back surface with a layer of silver.

E. Half-silvered mirrors

Silver is such a good reflector, we argued, because the electrons inside it move around, and their electric fields then cancel those of the incident light wave. But the electrons need some space in which to do this, hence the light actually penetrates the silver a little (about $\frac{1}{50}$ the light's wavelength—10^{-5} mm) before it is reflected. For most purposes this penetration can be ignored. However, if the silver is made thin enough, some of the light will penetrate through to the opposite side, and thus some light will be transmitted. That is, you can see through a thin enough layer of silver (or other metal).

One can make mirrors (called ***half-silvered***) with glass coated by a layer of silver that is sufficiently thin to reflect half and transmit the other half of the incident light intensity. Such mirrors are useful in a variety of optical devices, such as a rangefinder in a camera (Sec. 4.2D). They are also used in "reflecting" sunglasses, and for "one-way mirrors" (also called "two-way mirrors" or "mirror pane"). These two-way mirrors do not work quite the way they show them in the detective stories on TV—light travels through them in *both* directions. You need the victim, *V*, to be well lit, whereas the spy, *S*, sits in the dark (Fig. 2.17). On the light side, *V* sees plenty of light reflected, because there is lots of light to *be* reflected from the light side. But he sees very little light transmitted from the dark side, because there *is* only a little light on the dark side to be transmitted. So *V* says, "It's a mirror." Even though the half-silvered mirror does not reflect as much light as a regular mirror would under the same conditions this difference is difficult to perceive (see Sec. 7.3C). On the other side, *S* sees only a little reflected light because there's not much to be reflected from his side, but lots of transmitted light from *V*'s side, so *S* says, "It's transparent." If *S* turns on a light or lights a cigarette, *V* can see him and the jig is up. If you turn the light on *S* and off on *V*, then *V* sees *S*, but *S* sees only himself. In that case, the mirror is "one way" in the other direction.

This last effect is used by magicians to make things appear and disappear, by lighting up one or the other side of such a two-way mirror. Or suppose that in Figure 2.17

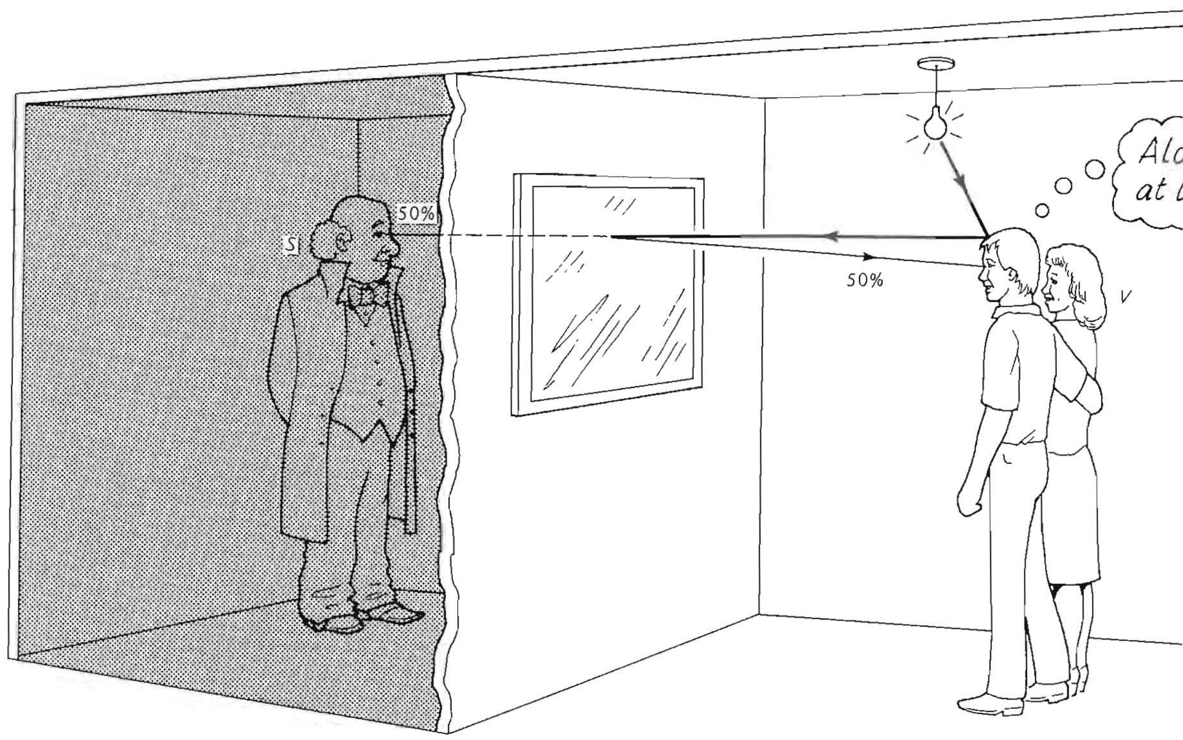


FIGURE 2.17

Illuminated innocents become visible victims of spookish spy behind harmless half-silvered mirror.

there is a weak spotlight on S's face. Then V would see a faint, disembodied face next to his own reflection in the mirror. Thus "ghosts" can be made to appear in the mirror—a "reflection" without an object (the opposite of vampires, which are well known not to reflect in mirrors).

Such mirrors are also used in modern architecture. A building may be covered with glass that has a thin layer of metal on it. During the day, most of the light is outside (daylight), so the people in the building can see out, but passersby see a mirror-covered building. At night, the only lights are from the electric lights inside, so the passersby can see in, while the occupants see their own reflections.

PONDER

What do occupants and passersby see at dusk when the light intensity outside is equal to that inside?

The same principle works with semi-transparent gauze curtains often used on windows. Here scattering takes the place of reflection. During the day the bright light is outside, so insiders can see out, but outsiders can't see in. At night the situation is reversed. This idea is also used in the theater. If the light is on the back part of the stage, you see through the screen (called a scrim). If not, it is apparently opaque and can be used as background or for projections. If there is some light on both sides you can get such effects as Siegfried walking through the magic fire. One often sees scrims or half-silvered mirrors (or, more likely, "half-aluminized") on the rear windows of vans driving down the highway.

2.4 REFLECTION AT OBLIQUE INCIDENCE

When light traveling in a given direction hits a smooth surface obliquely,* it is reflected in some different direction. What determines the direction of the reflected light? We describe this direction by the ***angle of reflection***, θ_r (θ is the Greek letter "theta"). It is the angle measured from the perpendicular (***normal****) of the surface to the ray of the reflected light. Similarly we call the ***angle of incidence***, θ_i, the angle between the normal and the incident ray. The ***law of reflection*** states that the two angles are equal,

$$\theta_r = \theta_i$$

(That is, they are equal in magnitude but on opposite sides of the normal, as shown in Fig. 2.18.)

The law of reflection describes how a mirror forms images. We see images the same way we see any object. That is, a ray of red light may be entering your eye in a certain direction, but the conscious information from your eye is not "a red ray is coming at me from 3° left of front." Instead you say "there is a red apple." In recognizing the apple, your brain sorts out the various rays that come into your eye and reaches a conclusion about their *or-*

*Latin *obliquus*, not at right angles.

*Latin *norma*, rule, carpenter's square.

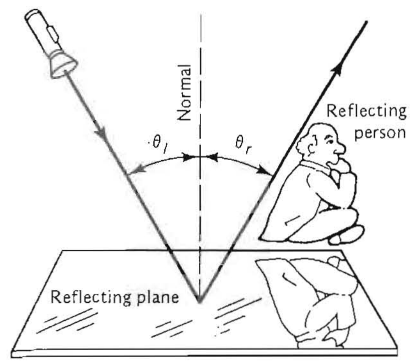


FIGURE 2.18

The law of reflection: $\theta_r = \theta_i$.

igin (the apple) rather than just about the direction from which they entered your eye. Because light usually travels in straight lines, your brain assumes that the light had traveled in the same direction in which it entered the eye (Fig. 2.19). How this processing is done need not concern us here.

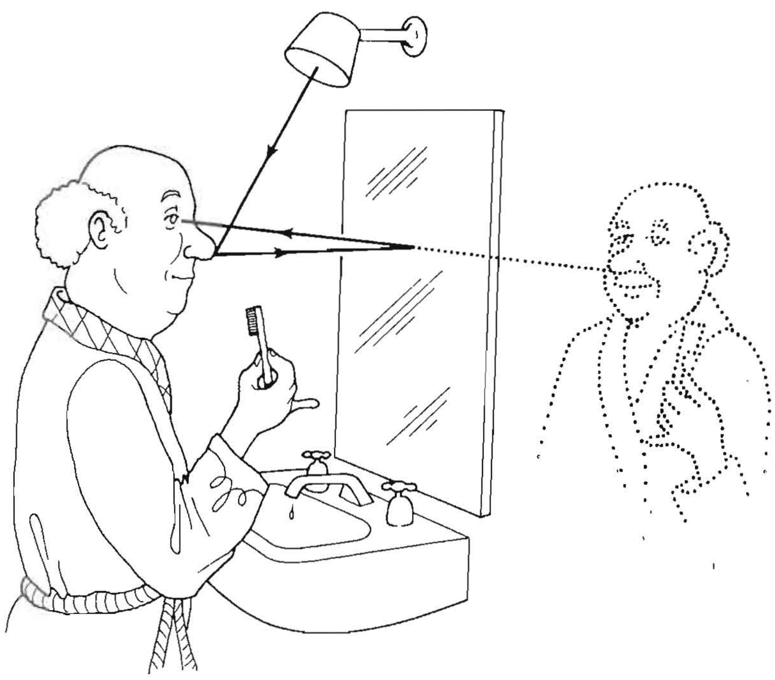
Reflections can look extremely realistic (Fig. 2.20), and magicians make good use of this (the TRY IT gives various examples). Similarly in a snapshot of a perfectly smooth lake you can sometimes hardly tell which side is up, that is, which is the real landscape and which is the reflection (Fig. 2.21). However,

FIGURE 2.19 (below)

The reflected ray does not originate where the eye thinks it came from. The light scattered from the observer's nose (the object) really takes a sharply bent path to his eye, since it is reflected by the mirror. But the observer's brain interprets the light as if it had come in a straight line from a part of the image behind the mirror. This is because the reflected ray comes from the same direction *as if* it came from an object behind the mirror: the image, which is as far from the mirror as the object, but on the other side.

FIGURE 2.20

Scene from the Marx Brothers' "Duck Soup." Groucho is unable to tell if he sees his own reflection or Harpo dressed up to look like him.



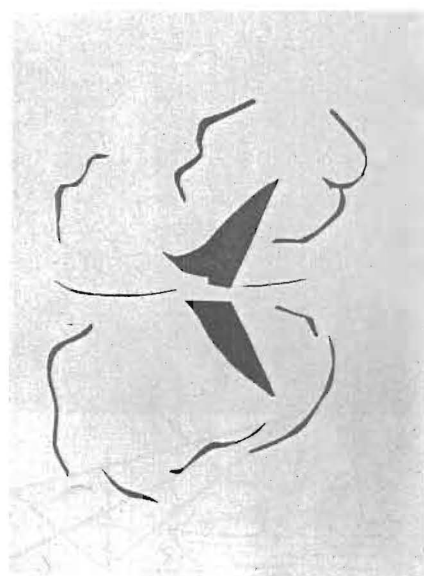


FIGURE 2.21

Henri Matisse, "La Voile." How can you tell it is printed right-side up? (Indeed, the Museum of Modern Art in New York hung this upside down for 47 days, thereby earning mention in the *Guinness Book of World Records*.)

side, so the next thing you see is the reflected, inverted, image of the tree (ray *c*). In this case, the reflection does not appear symmetrical.

The law of reflection tells us about the *direction* of the reflected light. The *amount* of reflected light depends on the materials involved and on the angle at which the light hits the interface. If you look straight down into a lake you can often see the bottom—there is little reflection for light incident perpendicular to the surface (only about 2%). But if you look at a point farther away on the lake, you see mainly the reflected light from the sun and sky. So the amount of reflection increases the further the incident ray is from the perpendicular—there is *enhanced reflection* at *grazing incidence.* Manufacturers of wax for cars, floors, and furniture often take advantage of this effect. Advertisement photographs of a newly waxed floor, for example, are usually taken at a large angle, to reflect well on the wax.

The same effect occurs in a puddle. When viewed from straight above, it may appear as an ugly puddle, but when viewed at a distance so as to catch the light coming off at grazing incidence, it is quite reflective. As the artist Ruskin wrote, "It is not the brown, muddy, dull thing we suppose it to be; it has a heart like ourselves, and in the bottom of that there are the boughs of the tall trees and the blades of the shaking grass, and all manner of hues of variable pleasant light out of the sky" (Fig. 2.24).

If the surface of the lake is no longer perfectly smooth but has small ripples, the reflections change and can provide the water with a whole spectrum of colors that vary with the surrounding light. The explanation involves the law of reflection and the increasing reflectivity with increasing angle of incidence. For example, in Figure 2.25 the light from the back of the wave *(a)* is reflected at a glancing angle and

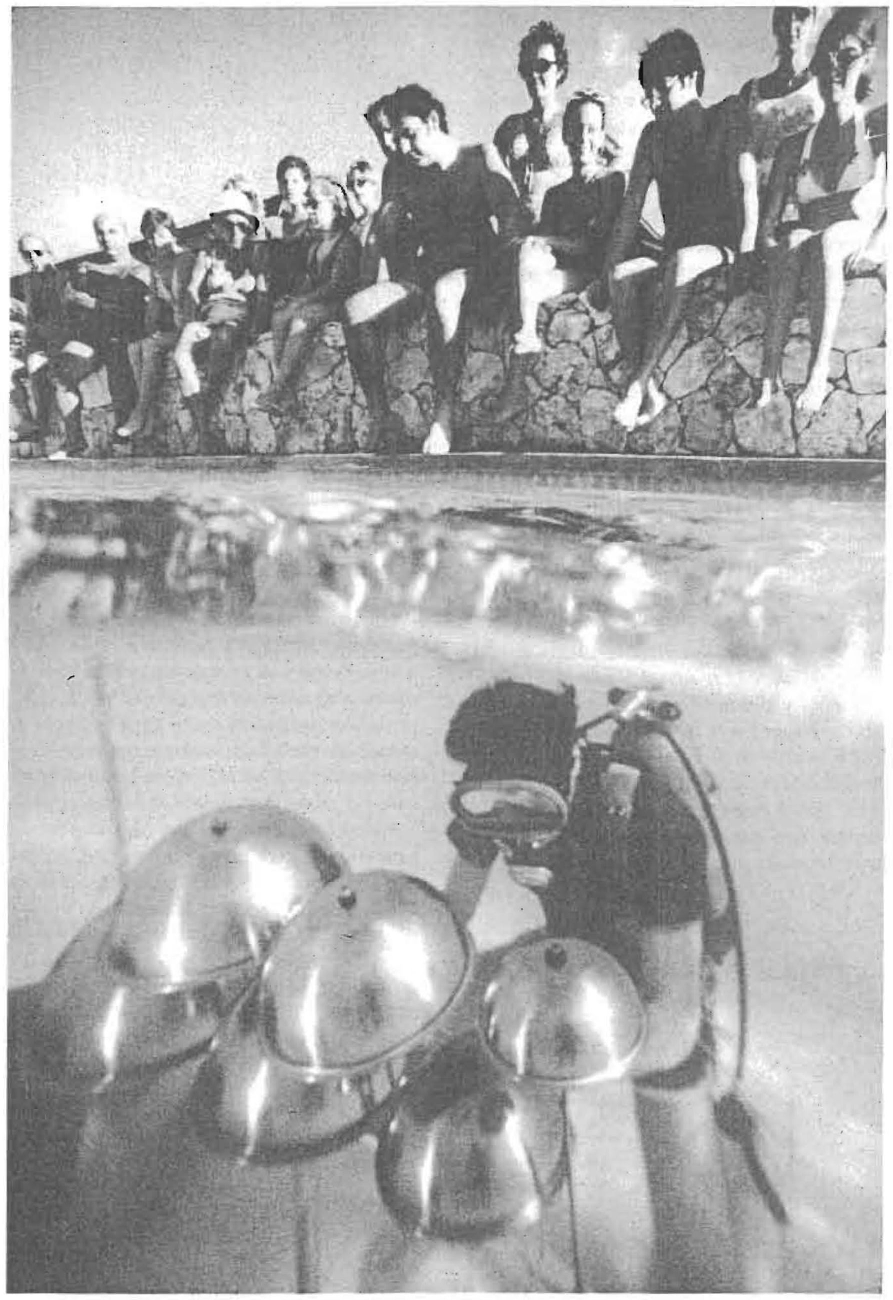


FIGURE 2.24

The nearby water surface is viewed at small angles of incidence and seems transparent, showing a man playing an underwater musical instrument; the more distant water reflects because the light reaching the camera grazes the water.

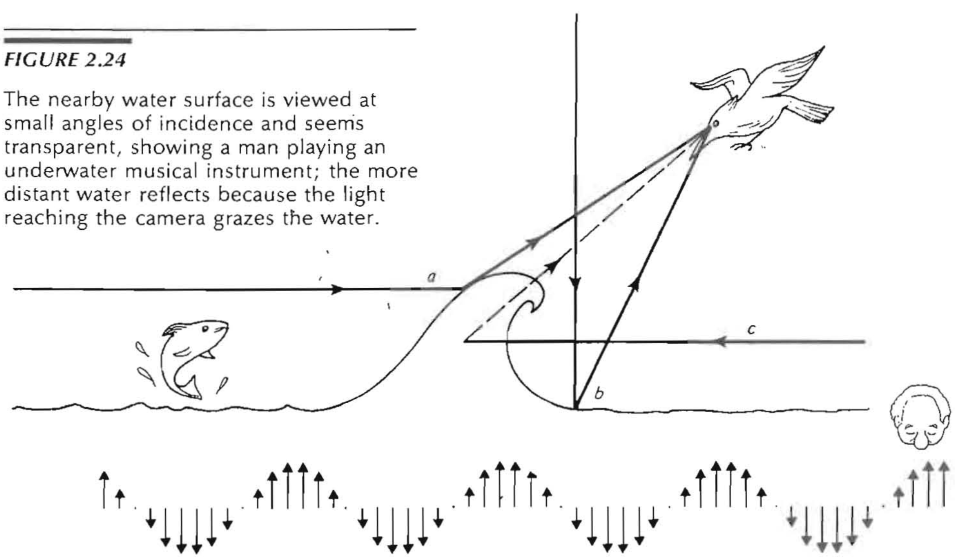


FIGURE 2.25

Reflections of light from an ocean wave.