The interference of waves

In physics, **interference** is the addition (superposition) of two or more waves that results in a new wave pattern. The displacements of the waves **add algebraically.**

Consider two waves that are in phase, sharing the same frequency and with amplitudes $A1$ and $A2$. Their troughs and peaks line up and the resultant wave will have amplitude $A = A1 + A2$. This is known as *constructive interference*.

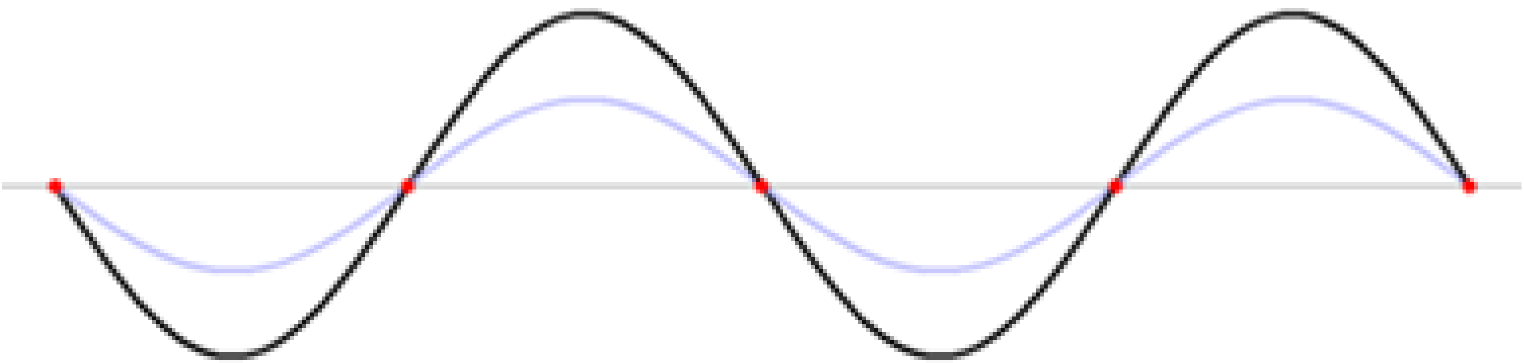
If the two waves are π radians, or 180°, out of phase, then one wave's crests will coincide with another wave's troughs and so will tend to cancel out. The resultant amplitude is $A = |A1 - A2|$. If $A1 = A2$, the resultant amplitude will be zero. This is known as *destructive interference*.

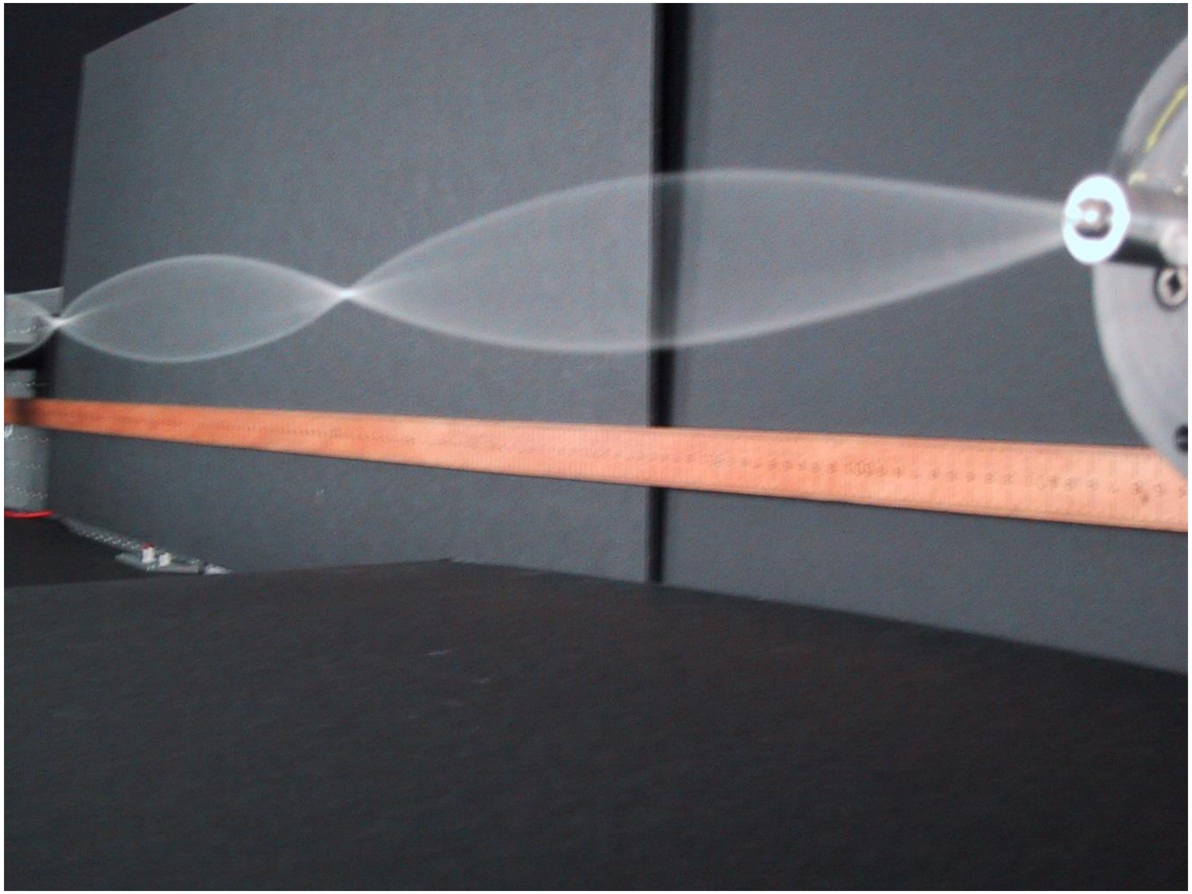
Interference (superposition) of simple one-dimentional waves – some important effects:

Standing waves – when two waves of the same frequency travel in opposite direction (e.g., when a wave hits a wall, a "back-reflected" wave is created, and the incident and the back-reflected wave interfere.

"Beats" – when two waves of slightly different frequencies interfere.

A link to a Web simulation and animation of standing waves and beats.

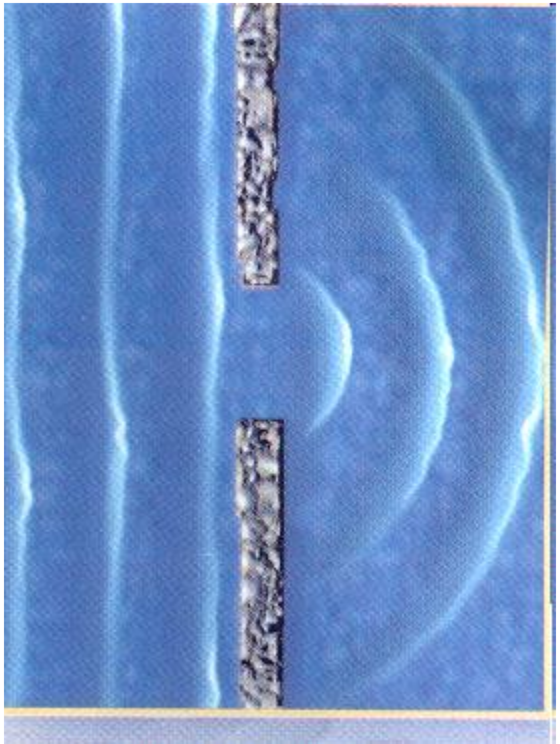


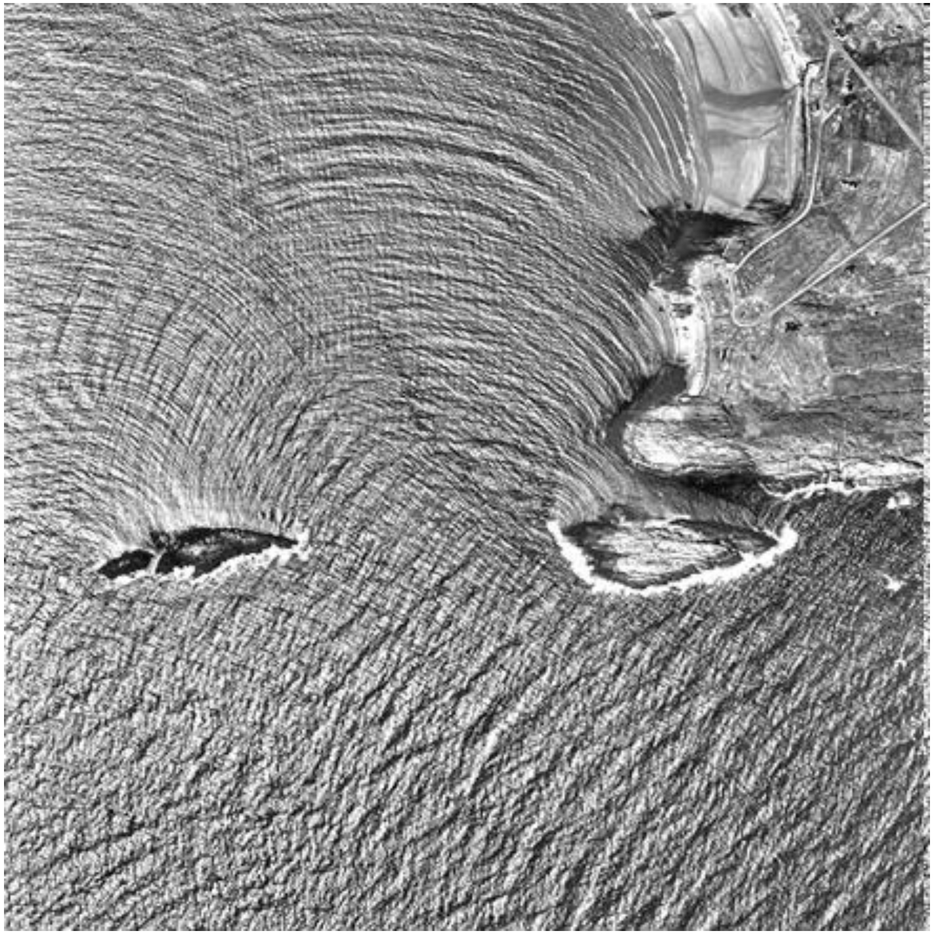


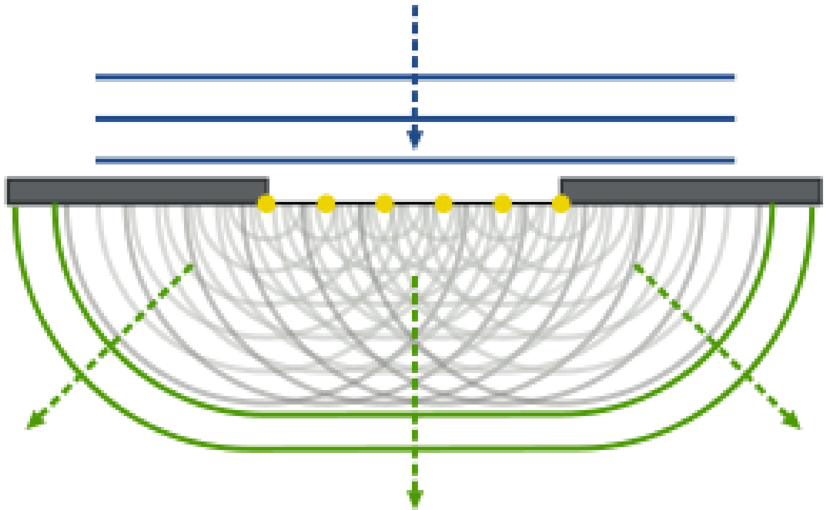
One of the many YouTube clips demonstrating standing waves on strings: [Please click here!]()

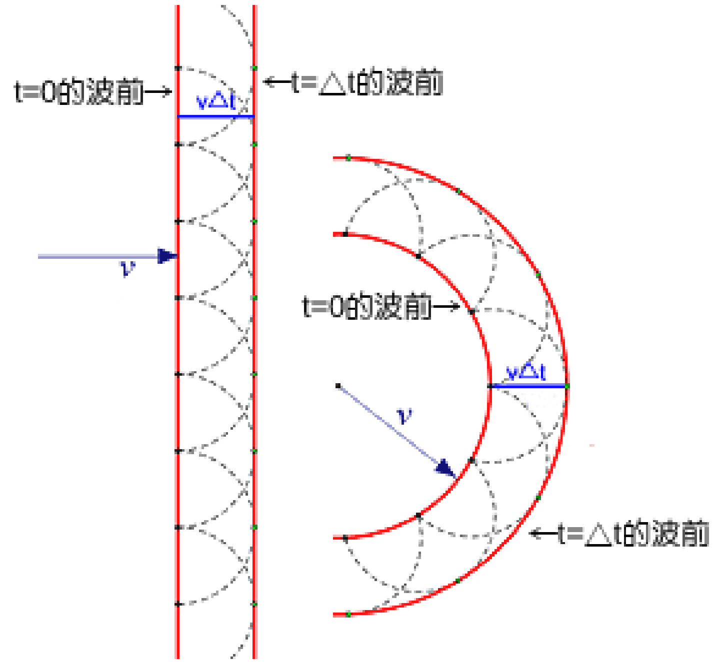
Interference filters – a practical demonstration only

Christian Huygens (XVII century)
Based on his observations of waves on water, he formulated a very important law, known as the "Huygens Principle".





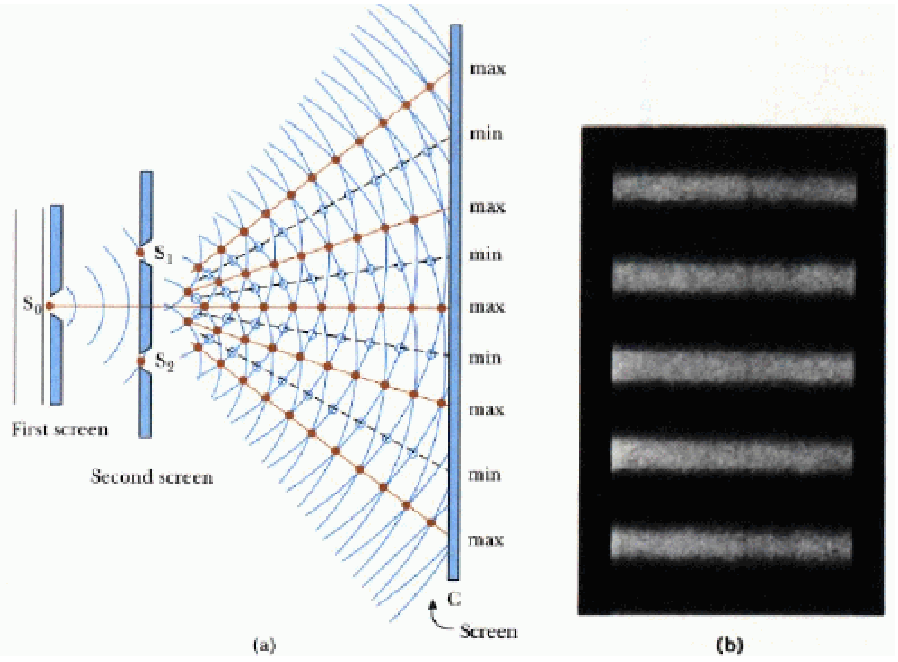


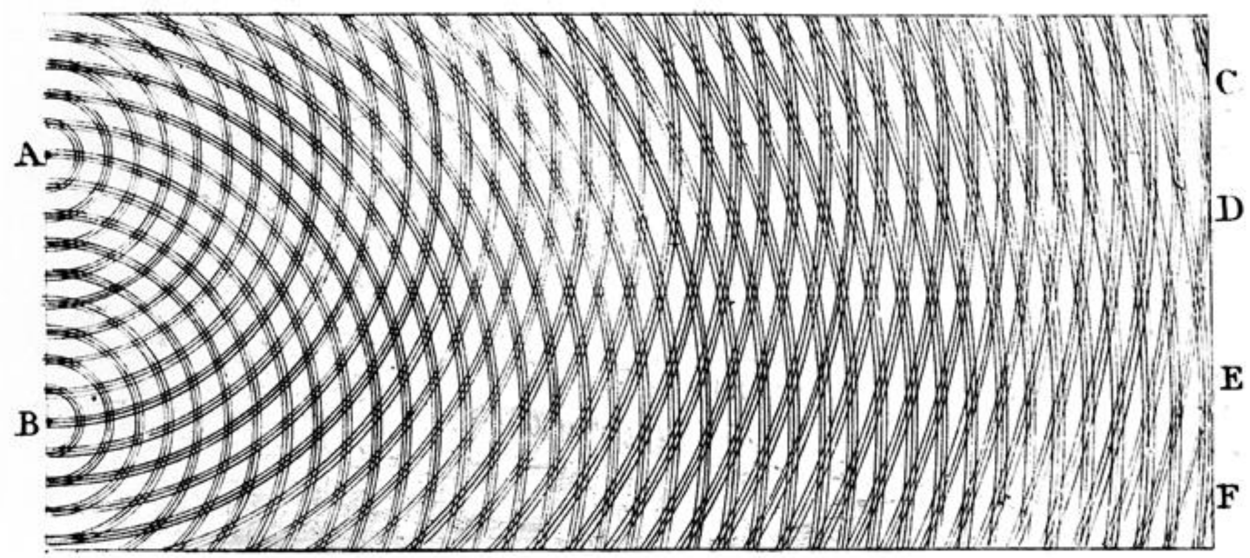


Huygens Principle: ***Any wavefront of a traveling wave can be replaced, as far as effects further along the propagation direction are concerned, by a large number of "point sources" located uniformly all over the wavefront, radiating in phase.***

Thomas Young's 1805 experiment: according to the Huygens Principle, a narrow slit becomes a "point-like" source af a circular wave.

Young demonstrated that light waves radiating from two narrow slits interfere, giving rise to a pattern of alternating bright and dark "stripes" on a screen.



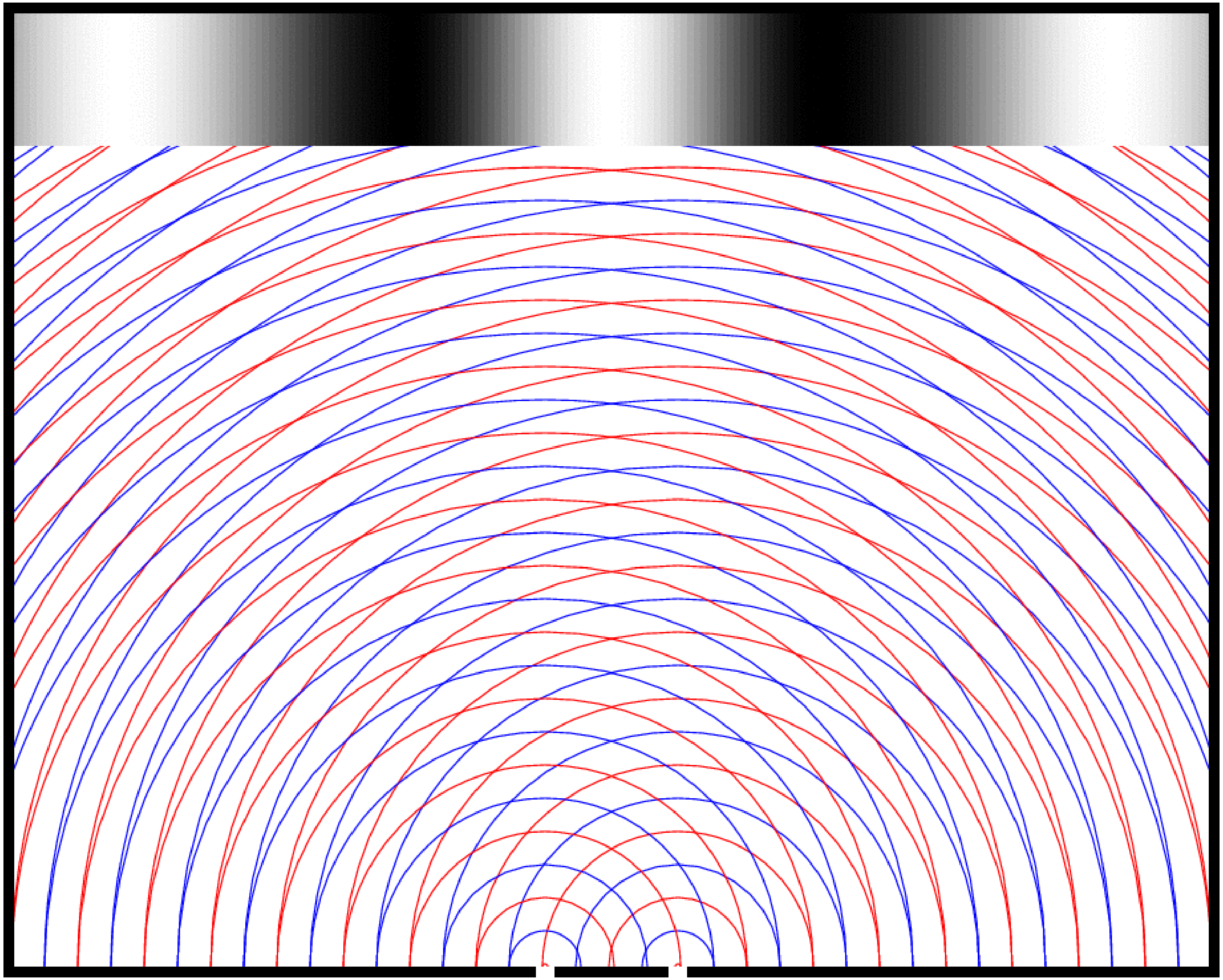


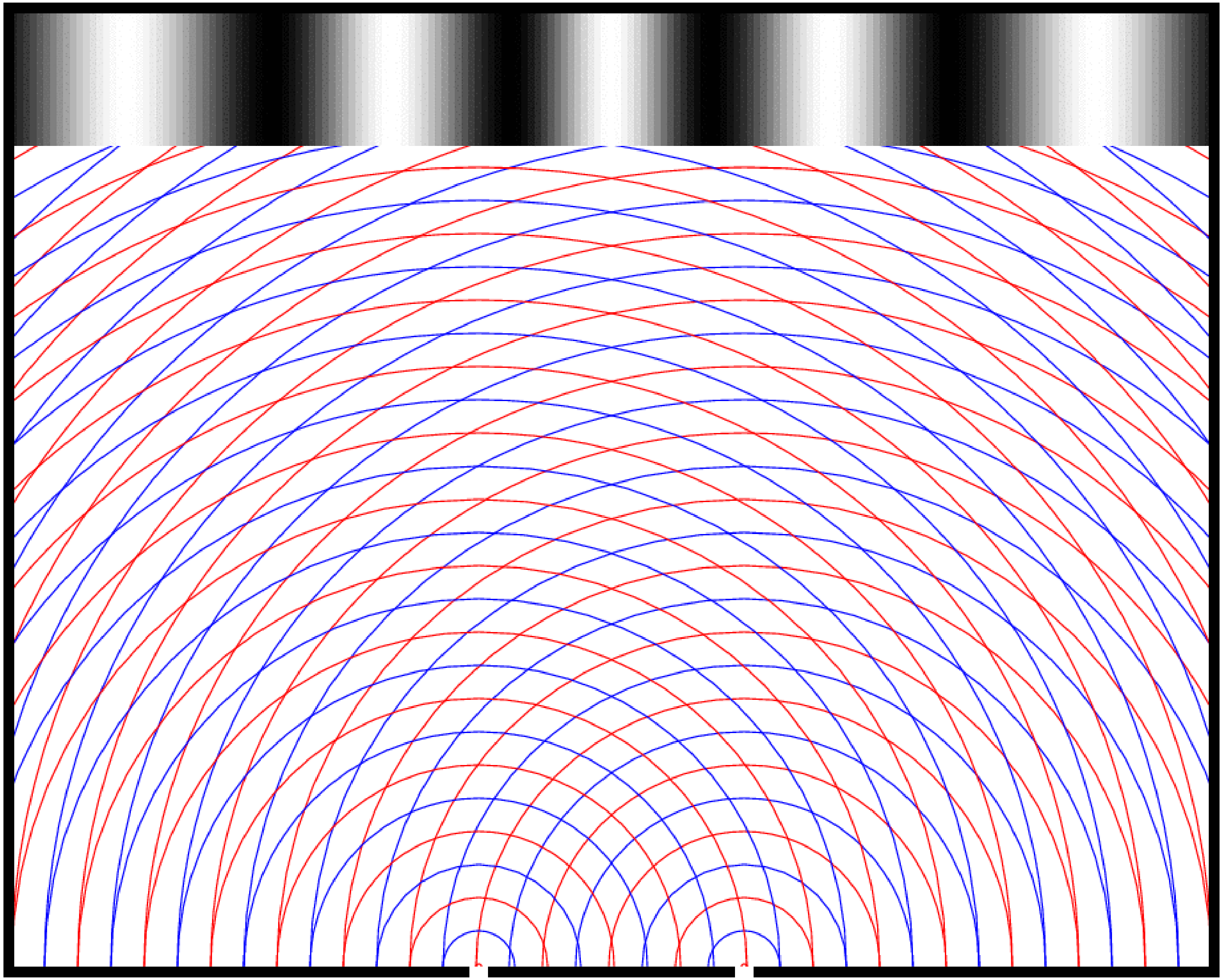
Original figure from Young's 1805 report.

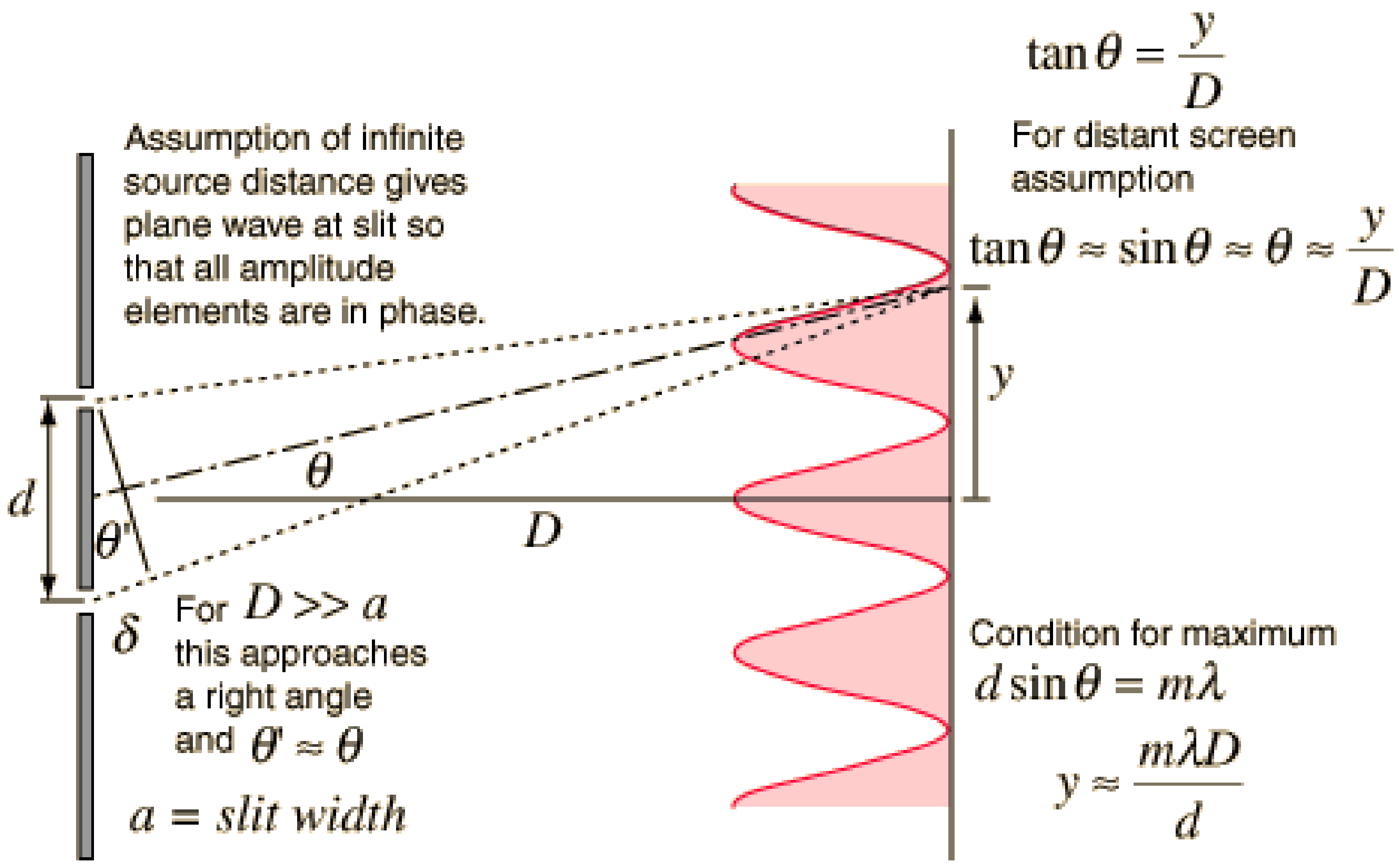
The next two slides show a simulation of the Young's experiment. In the original Power Point presentation, which has been converted to the present PDF vesion, the two slides were animated (they were .GIF graphic files. However, when a PPT presentation is converted to a PDF file, the animations no longer work!

So, if you want to watch the animated version, please open a small Power Point file, which contains only those two animations – by clicking
on this link!

The red circles emerging from the slits are wave crests, and the blue circles are the wave troughs.

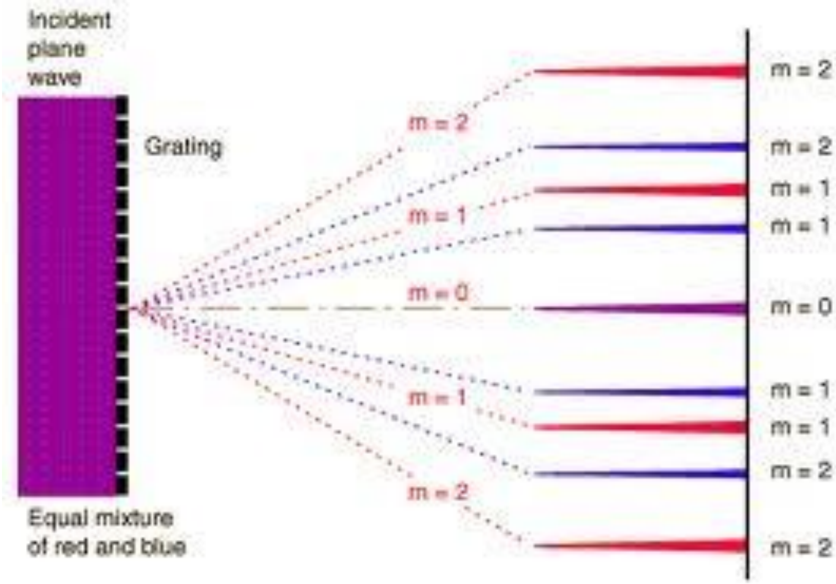


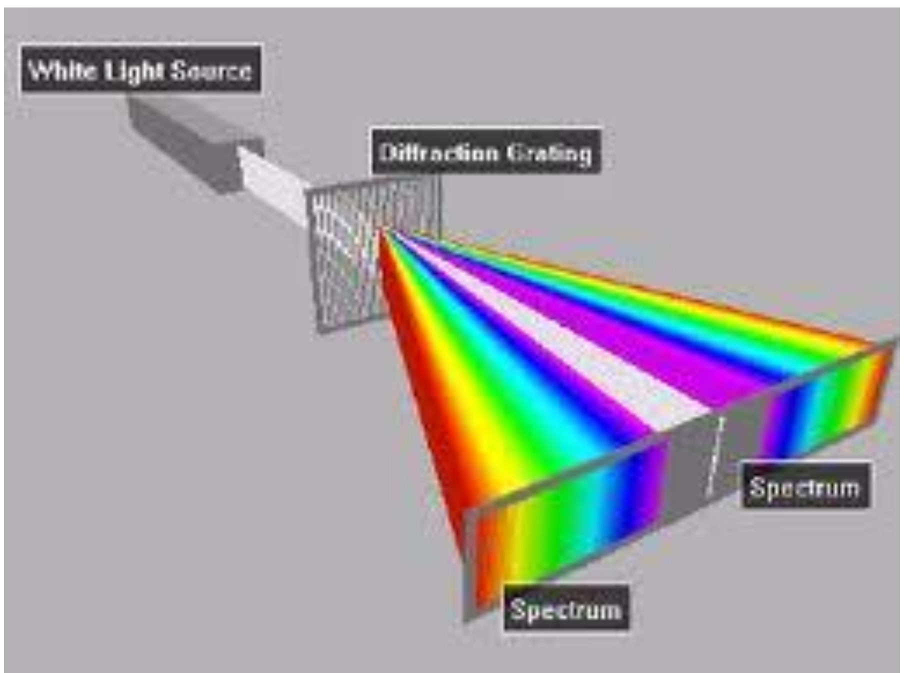




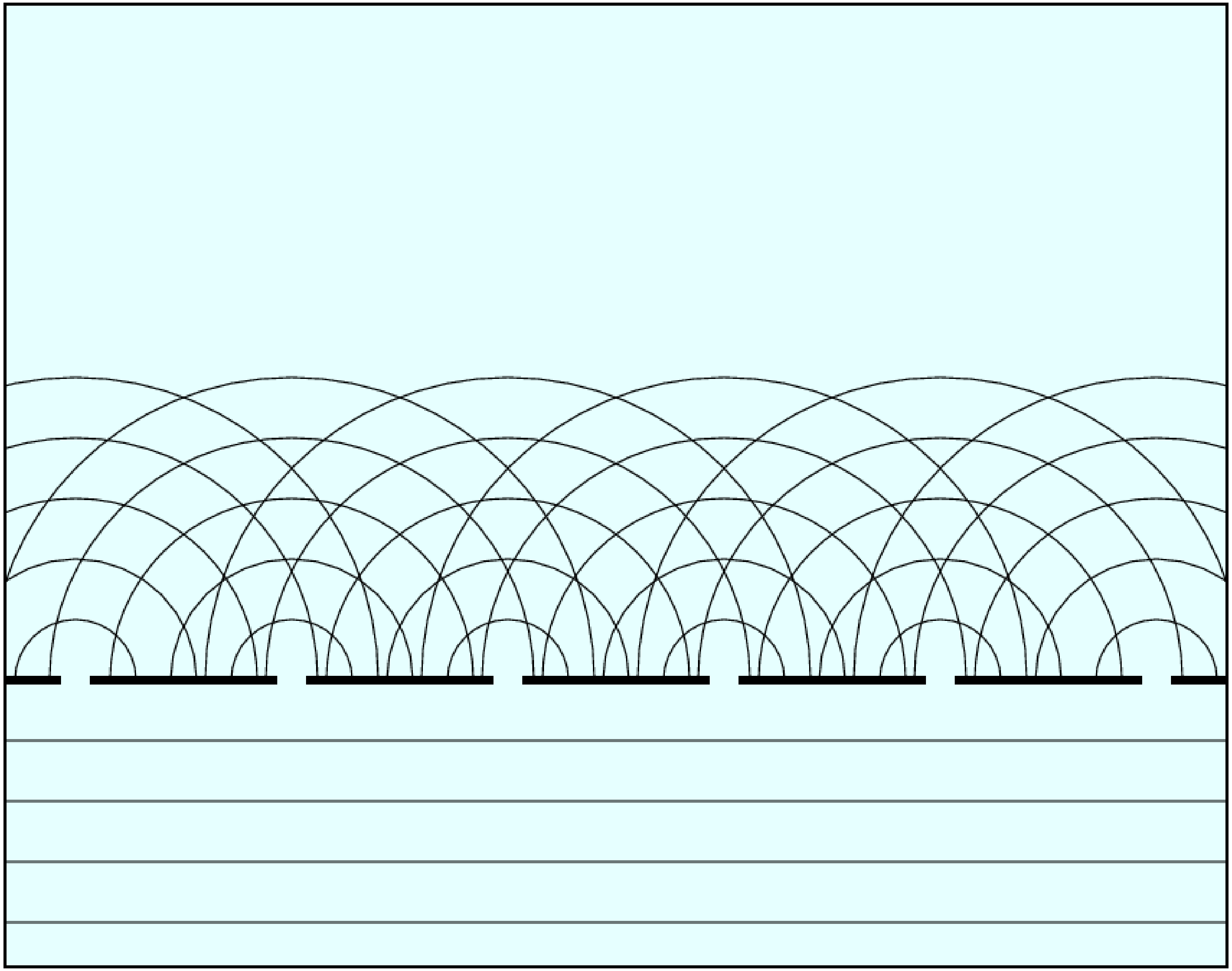
$\tan\theta = \frac{y}{D}$
For distant screen assumption
$\tan\theta \approx \sin\theta \approx \theta \approx \frac{y}{D}$
Assumption of infinite source distance gives plane wave at slit so that all amplitude elements are in phase.
d
θ
θ'
D
y
δ
For $D >> a$ this approaches a right angle and $\theta' \approx \theta$
$a = slit\ width$
Condition for maximum
$d\sin\theta = m\lambda$
$y \approx \frac{m\lambda D}{d}$

Grating (or "diffraction grating"). A plate with a large number of equally spaced narrow slits.

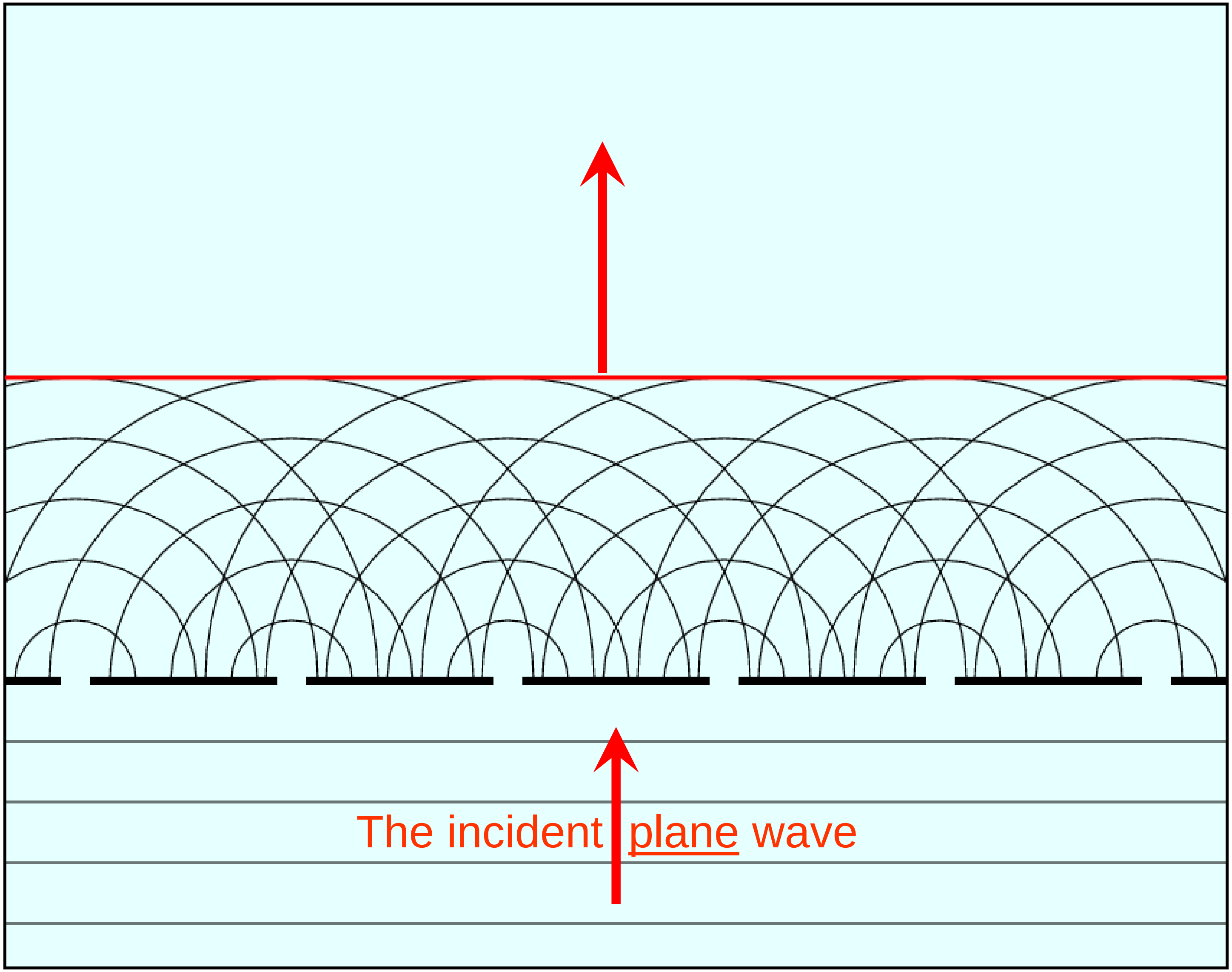




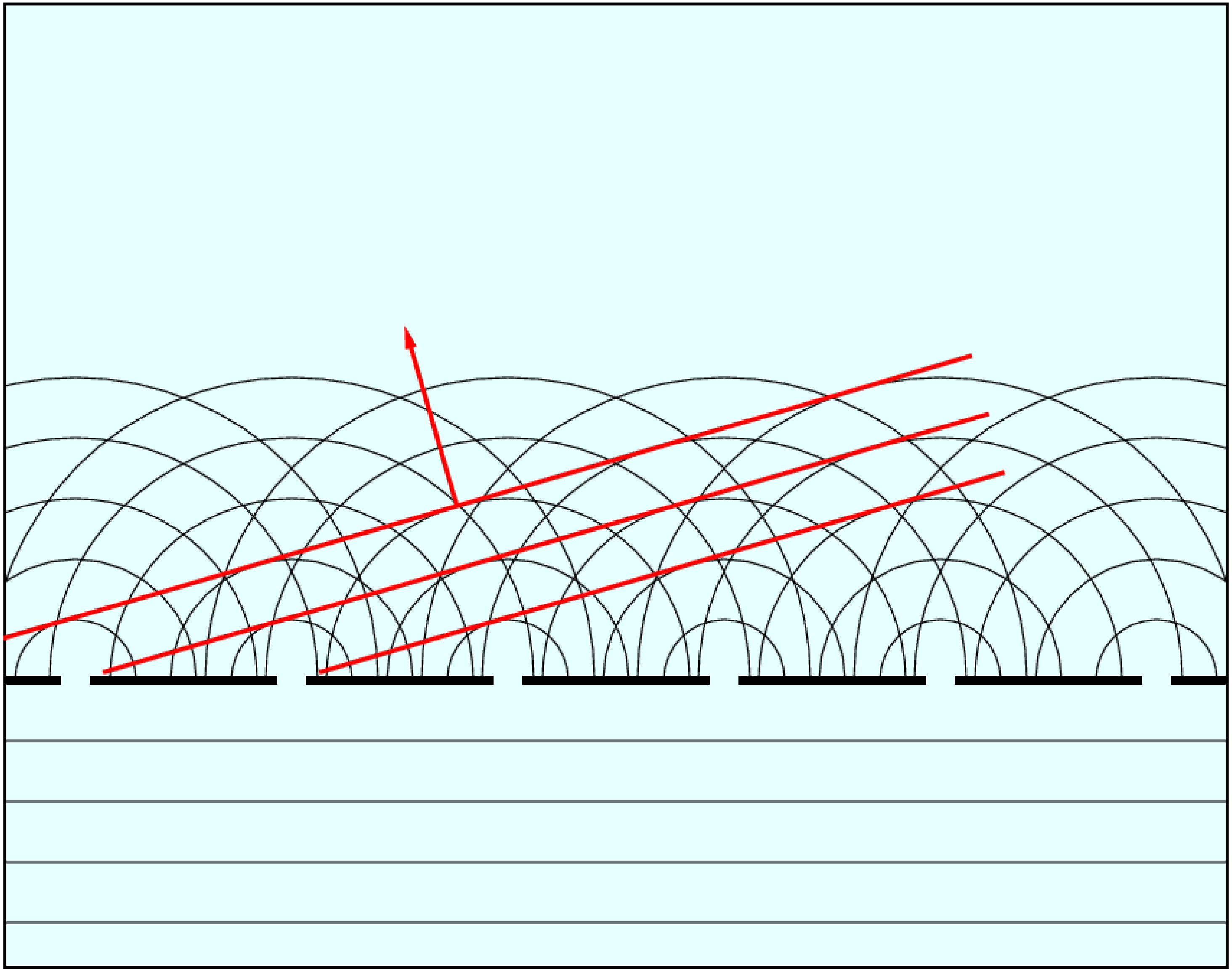
Grating: each of the many slits is the source of an elementary "Huygens wave":



Plot explaining how the "zero-order" wave is produced:



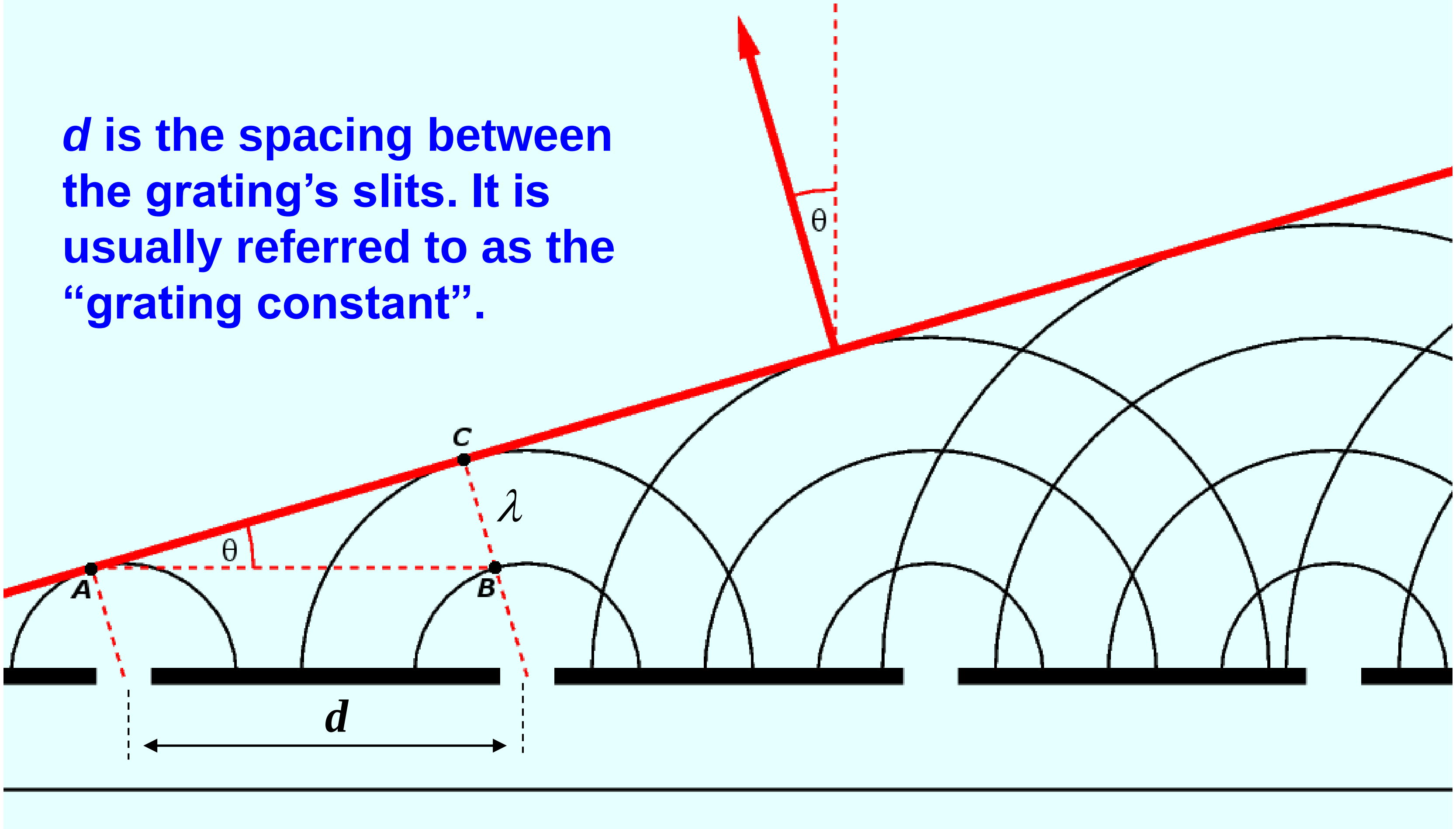
Here, the graph shows how the wave crests emerging from one slit interfere with the crests from the successive slits, phase-shifted by one full wavelength – thus forming the deflected wave "of the first order".



Of course, there is an analogous process for-ming a wave deflected to the right – but those wavefronts are not shown (the plot would become too messy, I'm afraid).

The first-order wave – a blown-up plot:

d **is the spacing between the grating's slits. It is usually referred to as the "grating constant".**

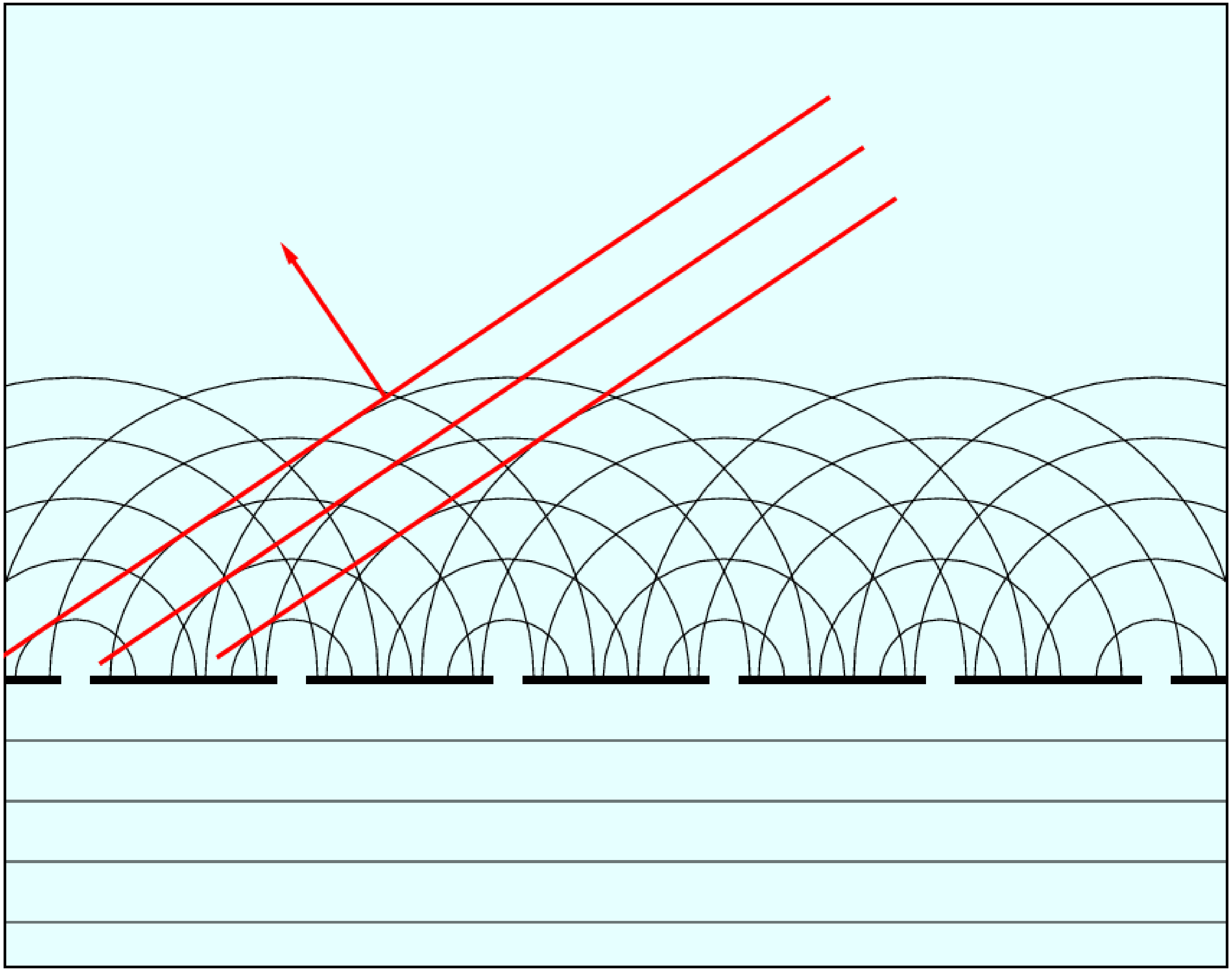


The angle θ its propagation direction makes with the propagation direction of the incident beam can be readily obtained from basic trigonometry:

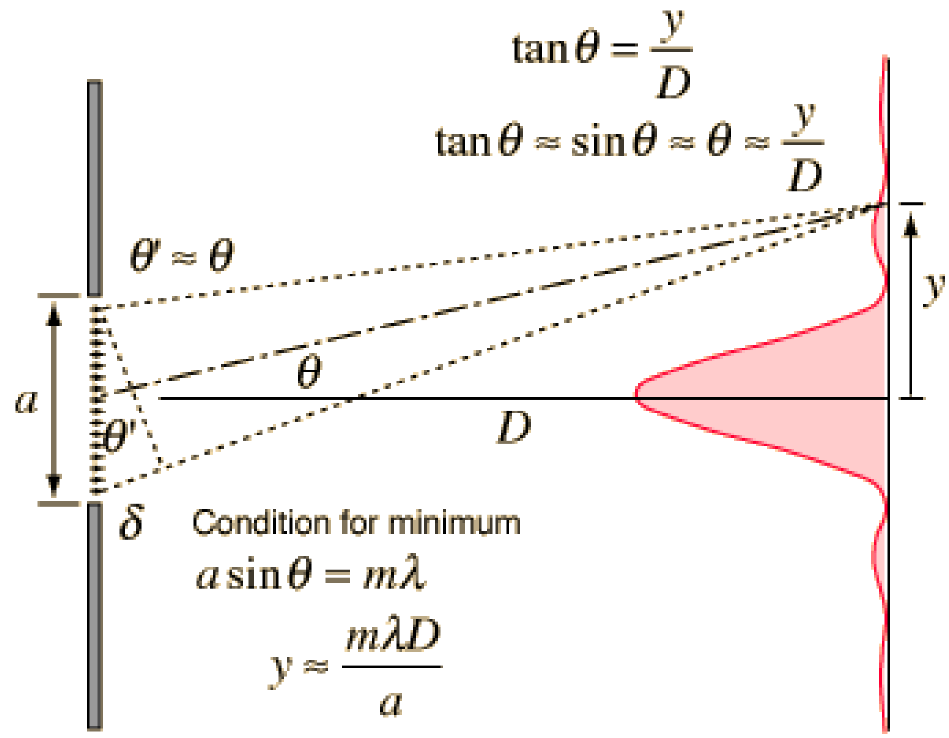
$$\sin\theta = \frac{BC}{AB} = \frac{\lambda}{d}$$

A graph explaining how the “second-order” deflected wave is created (a more appropriate term than “deflected” would be “diffracted”, but “deflection” is more intuitive).

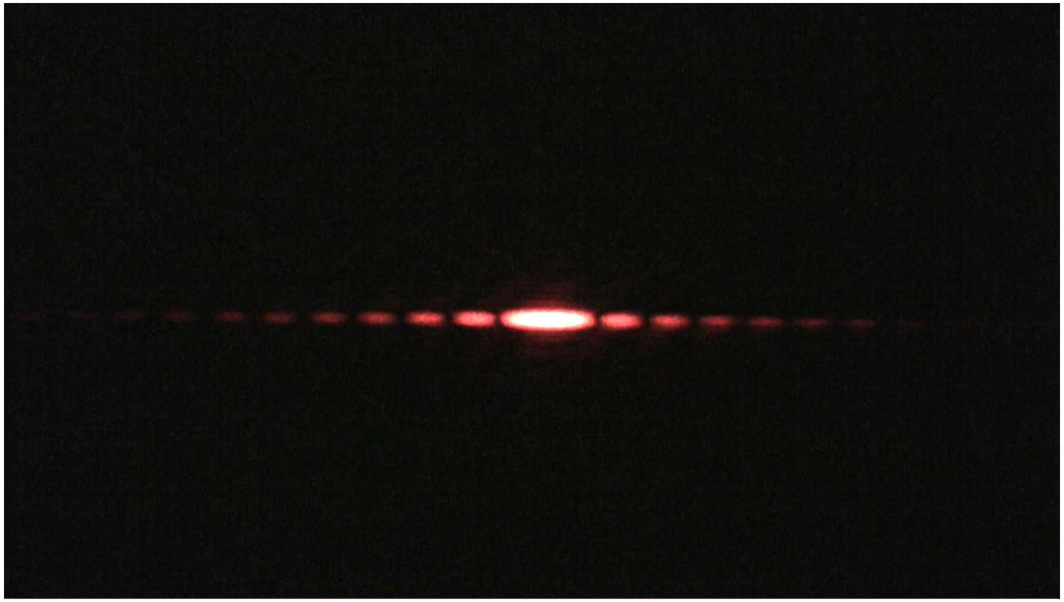
Now the wave crest emerging from one slit interferes with with the crest from the next slit that is phase-shifted by **two wavelengths.**



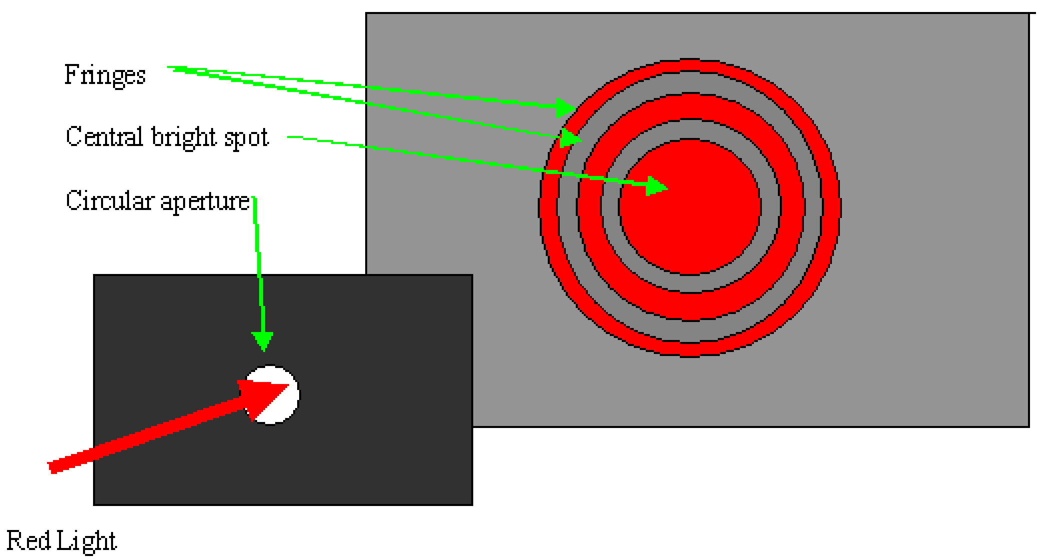
Diffraction by a single slit: there is a central maximum, and a num-Ber of weaker maxima (called “side-bands”) on both sides. The top plot explains where the minima between the side maxima are located (*m* in the for-mulae can take the value of ±1, ±2, ±3…).



Lower picture – single slit diffraction of red light from a laser pointer (an experiment one can easily make at home – a good slit can be made, e.g., from two disposable blades for standard box cutters).



Light diffraction on a single circular aperture (“aperture” is an elegant word meaning “hole”, or “opening”).

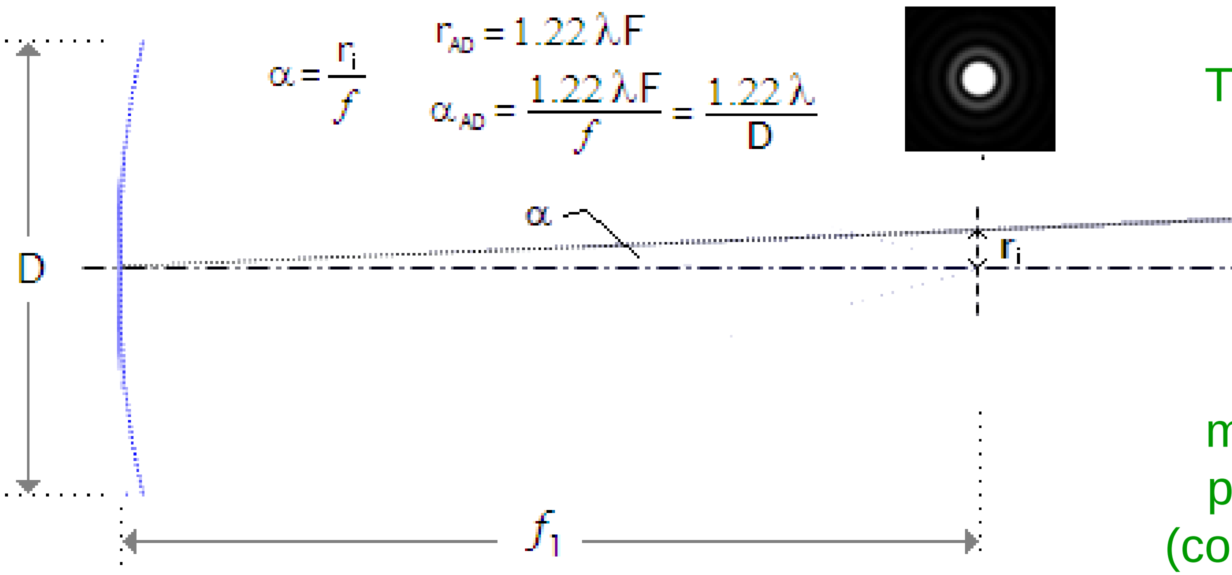


A YouTube video clip from MIT about diffraction on a circular aperture: Please click!

NEXT TWO SLIDES: diffraction on a circular opening is a serious problem in observational astronomy!

The objective lens of a telescope can be thought of as a "circular opening". Therefore, images of point objects (and distant stars are, in practice, "nearly point objects") are not ideal points, but "discs" of the size of the central maximum in the diffraction pattern from such opening (continued in the next slide).

SIZE OF DIFFRACTION PATTERN IN A TELESCOPE



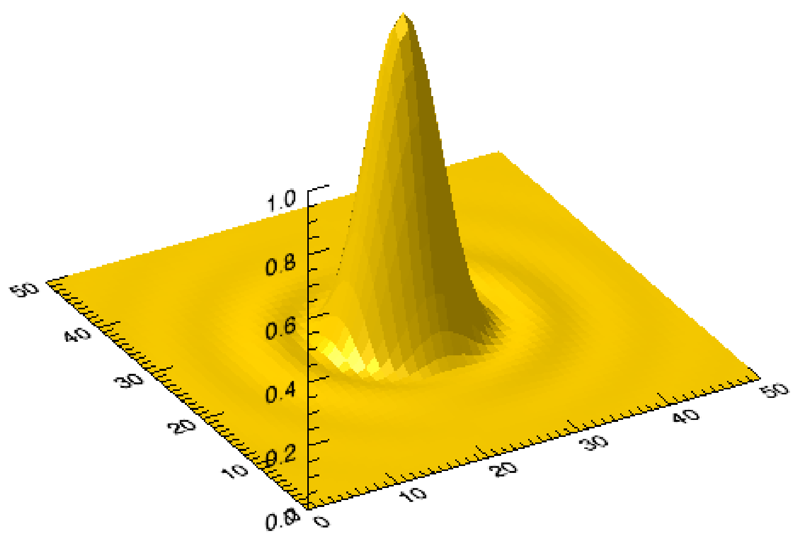
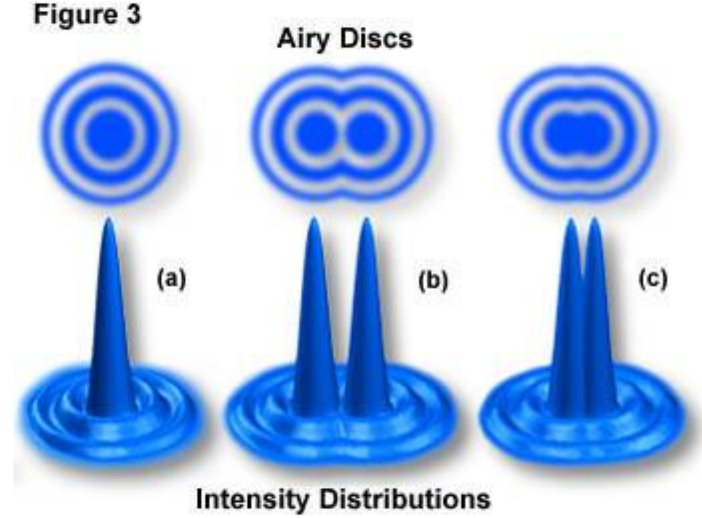


Figure 3



By astronomers, such diffraction-broadened images are called "Airy discs".

The Airy discs limit the ability of the telescope to resolve stars that are too close to each other in the sky. Let's show this by considering a simple example. The opening of the largest OSU telescope has a 12 inch diameter (0.30 m). We can take the average wavelength in starlight as 500 nm. From the formula in the preceding slide we can calculate the angular size of the Airy disc in this telescope:

$$\alpha_{\mathrm{AD}} = \frac{1.22\times 500\,\mathrm{nm}}{0.3\,\mathrm{m}} = \frac{1.22\times 500\times 10^{-9}\ \mathrm{m}}{0.3\,\mathrm{m}} = 2.03\times 10^{-6}\,\mathrm{radians} = 0.42\,\mathrm{sec.\ of\ arc}$$

This result means that any two objects in the skies, the "angular distance" between which is smaller than 0.4 second of arc, will merge in this telescope into a single "spot" – as illustrated by the rightmost blue shape in the preceding slide. This is a sufficient resolving power to see the Mizar A and Mizar B stars (we talked about them some tiime ago) as two separate objects, because the angular distance between them is 14 sec. of arc – but not two other stars in the five-star Mizar cluster.

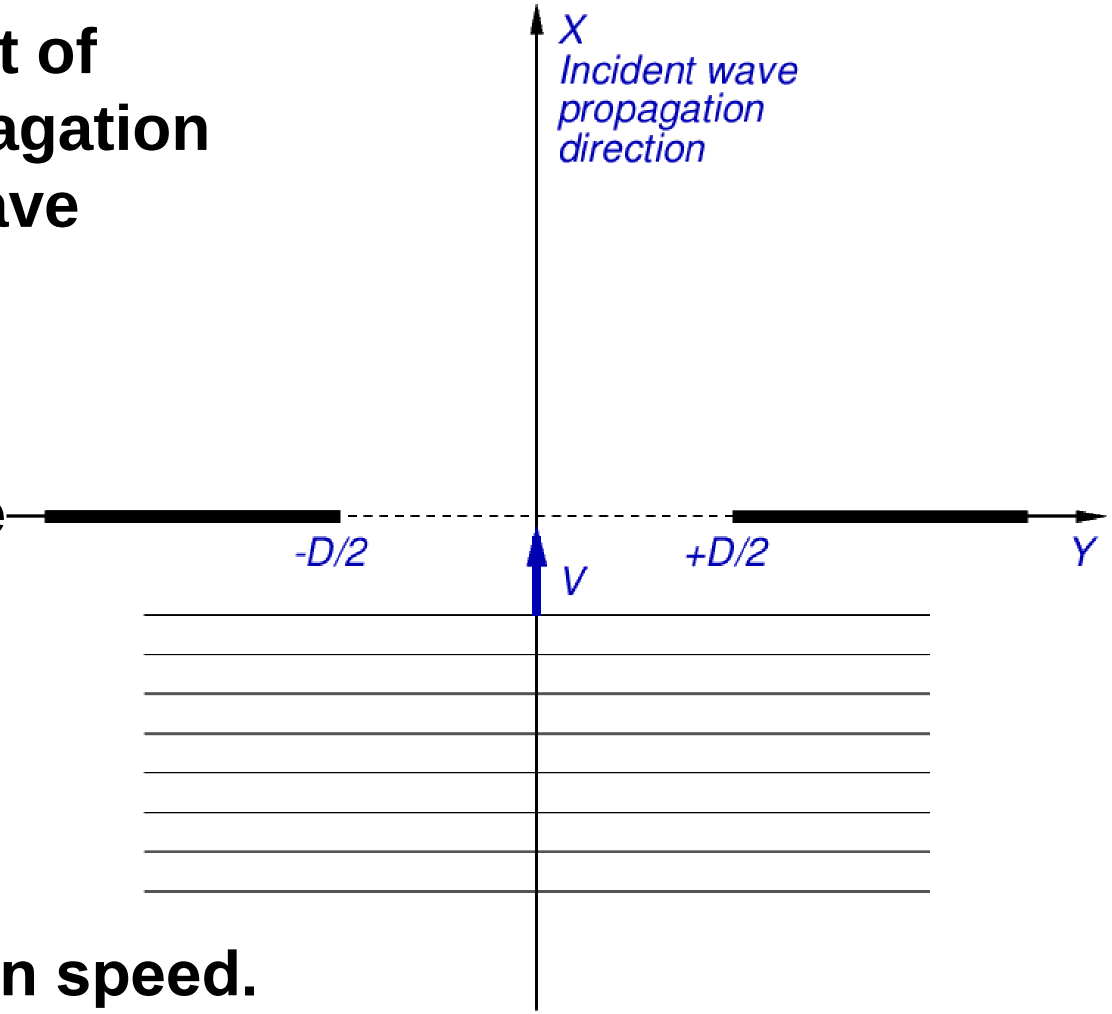
The effect of "Airy disc" also explains why astronomers want telescopes with objective diameters as large as possible!

Let’s return now to a single slit diffraction:

Single slit diffraction: derivation of the equation describing the image forming on a distant screen.

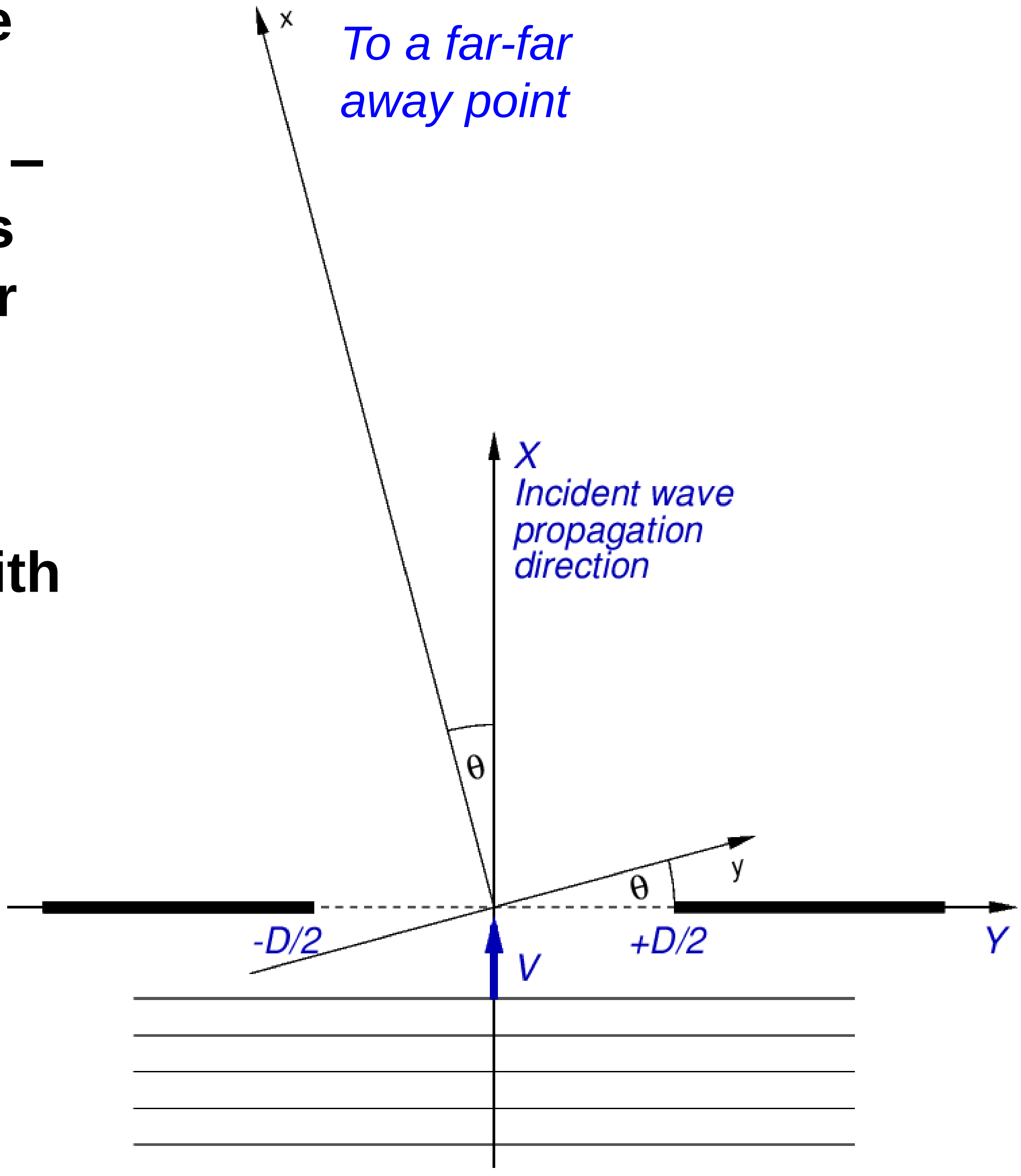
Suppose that a light wave is incident on a slit of width *D*. The propagation direction of the wave is perpendicular to the slit's plane.

Let's consider one wavefront of the impinging wave. We mark it with a blue vector, symbolizing the wave's propagation speed.



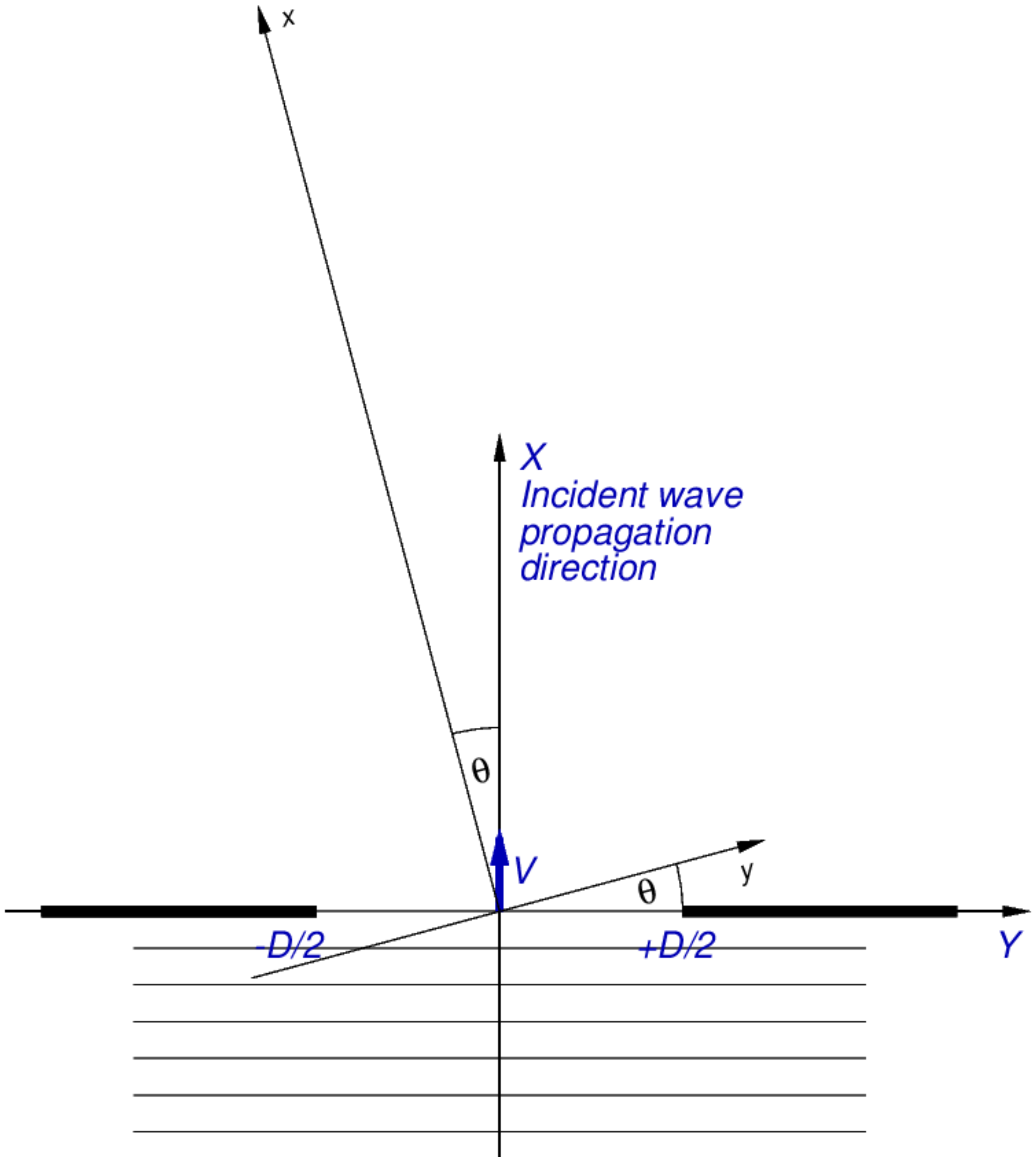
We want to calculate the "deflected", or – more professionally – the diffracted wave's amplitude at a far-far away point lying on the line we call *x* (small), joining the point (not shown) with the slits center, and making a Θ angle with the Incident wave direction.

We also introduce a coordinate axis *y*, orthogonal to i.



We will call the time instant when the wave-front with the blue *V* vector "attached" reaches the slit center as t_0.

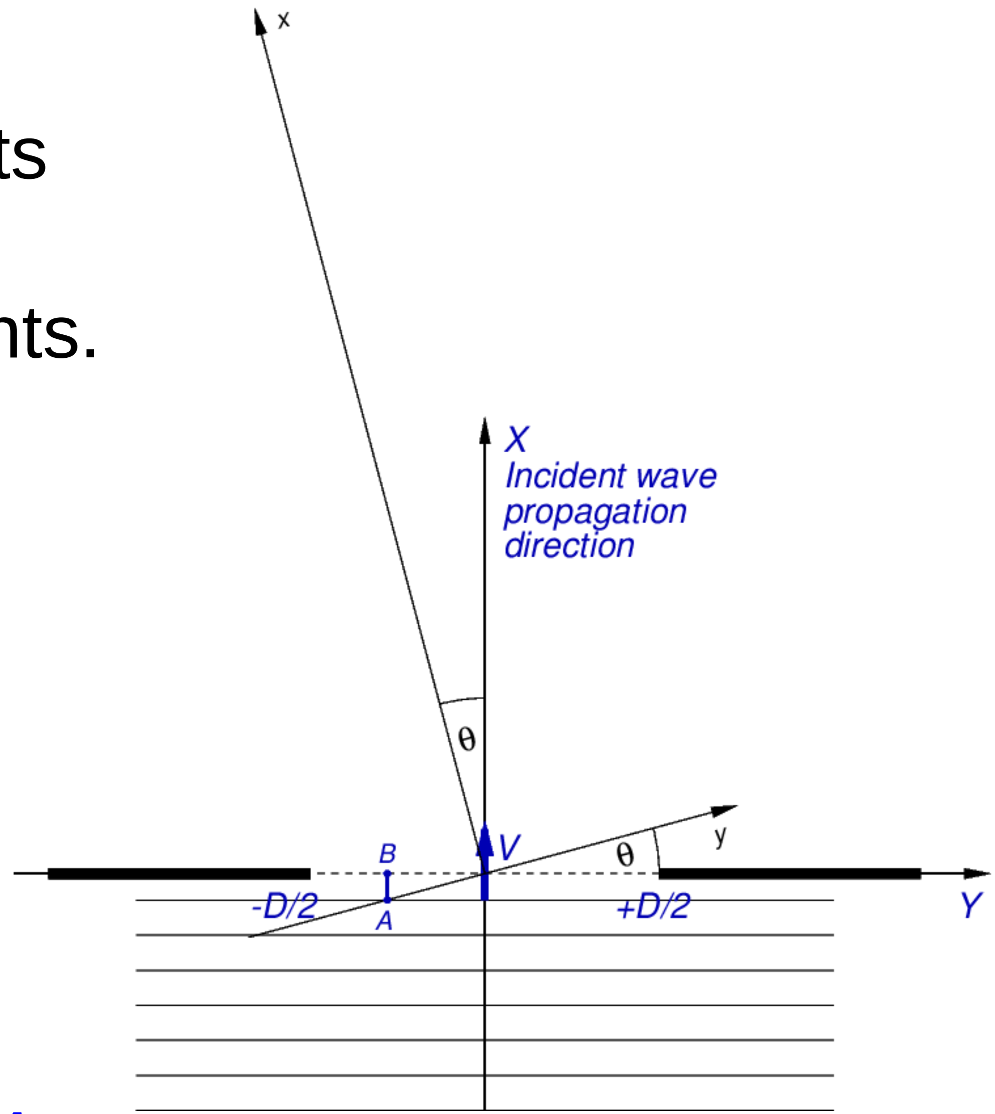
The wavefront creates a "Huyghens wavelet" at the slit center.



The wavefront “with the *V* attached” creates many wavelets along the *y* axis, but at different time instants.

The graph shows a moment <u>earlier</u> than t_0.

Here, for instance, the wavefront intersects the y axis at a point marked as *A*.



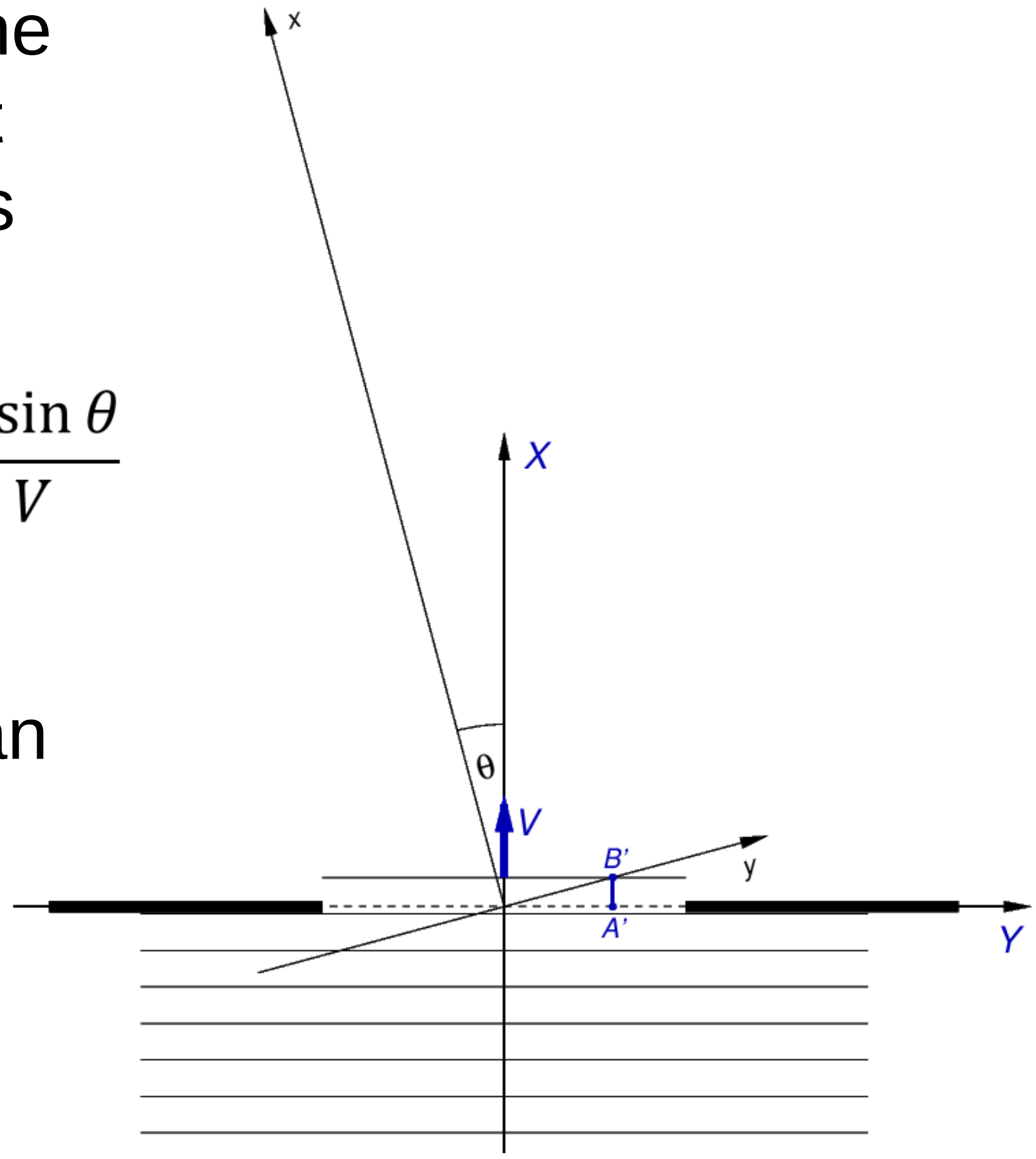
$$t' = t_0 + \frac{AB}{V} = t_0 + \frac{y \sin \theta}{V}$$

(note that t’ is lower than t_0 because *y* is negative)

Here, in contrast, the "marked" wavefront intersects the y axis <u>later</u> than at t_0.

$$t'' = t_0 + \frac{A'B'}{V} = t_0 + \frac{y\sin\theta}{V}$$

Now, t'' is <u>larger</u> than t_0 because y is positive.



What needs to be done now is math only – much pretty tedious math. Namely, we want to find what happens in the distant point x when all Huygens wavelets created along the abstract line y converge at this point at a time instant t.

We will use the wave equation which was presented in Chapter 2 as the Eq. 2.15. Only we have to use not y, but another symbol for the displacement, as y now has another meaning. Let it be Δz:

$$\Delta z = A \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \tag{4.1}$$

Here T is the time period of the wave, but we want to use instead $T = \lambda/V$ (remember the "Dr. Tom's triangles"?):

$$\Delta z = A \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi V}{\lambda}t\right) \tag{4.2}$$

We assumed that at the origin of the xy coordinate system the wavelet was created at time t_0. So tat the contribution of this very wavelet at x and t is:

$$\Delta z = A \sin\left[\frac{2\pi}{\lambda}x - \frac{2\pi V}{\lambda}(t - t_0)\right] \tag{4.3}$$

But along the "abstract line y" different wavelets were created at different times, as was shown in the slides above – at $t_0 + y \sin\theta / V$. So, the contribution of any wavelet reaching the distant point x at time t is:

$$\Delta z = A \sin\left[\frac{2\pi}{\lambda}x - \frac{2\pi V}{\lambda}(t - t_0 - \frac{y \sin\theta}{V})\right]$$

We can take the time t_0 as zero (it's always permitted), and rewrite the above as:

$$\Delta z = A \sin\left[\frac{2\pi \sin\theta}{\lambda} y + \frac{2\pi}{\lambda}(x - Vt)\right] \tag{4.4}$$

Well, in order to obtain the contribution of ***all the wavelets***, we have to integrate the individual ones given by the Eq. 4.4, for all relevant y values, i.e., those between $y_{\text{min}} = -D/(2\cos\theta)$ and $y_{\text{max}} = +D/(2\cos\theta)$:

$$\Delta z_{\text{tot}} = \int_{y_{\text{min}}}^{y_{\text{max}}} A \sin\left[\frac{2\pi \sin\theta}{\lambda} y + \frac{2\pi}{\lambda}(x - Vt)\right] dy \tag{4.5}$$

Looks scary? Not really, all parameters in the integral are constant, with the exception of y. We can use the known integral, written in a general form:

$$\int_{y'}^{y''} \sin(ay+b)dy = \frac{1}{a}\left[\cos(ay''+b) - \cos(ay'+b)\right]$$

And then, the trigonometric identity:

$$\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

After processing the integral, we end up with a "monster expression":

$$\Delta z_{\text{tot}} = A \frac{\sin\left(\frac{2\pi D \sin\theta}{\lambda \cos\theta}\right)}{\frac{2\pi D \sin\theta}{\lambda}} \times \sin\left[\frac{2\pi}{\lambda}(x - Vt)\right]$$

But this "monster" can be tamed, if we take into account that we are interested only in the spectrum for low angles θ, for which we can take with a vary good approximation: $\sin\theta \approx \theta$, and $\cos\theta \approx 1$. Then the expression simplifies to:

$$\Delta z_{\text{tot}} = A \frac{\sin\left(\frac{2\pi D\theta}{\lambda}\right)}{\frac{2\pi D\theta}{\lambda}} \times \sin\left[\frac{2\pi}{\lambda}(x - Vt)\right]$$

The rightmost sine function represent a wave equation for x and t, while the rest can be thought of as the **amplitude** of this wave. We can write the final result in the form:

$$\Delta z_{\text{tot}} = A\left(\frac{\sin\xi}{\xi}\right) \times \sin\left[\frac{2\pi}{\lambda}(x - Vt)\right]$$

where $\xi = 2\pi D\theta/\lambda$.

The $\sin\xi/\xi$ function describes the shape of the single-slit diffraction spectrum. In fact, this is the **amplitude** of the diffracted wave – and what we observe, is the **intensity** of this wave, proportional to the **square of the amplitude**. Therefore, we can write:

$$I(\theta)_{\text{obs.}} \propto \left(\frac{\sin\xi}{\xi}\right)^2 \tag{4.6}$$