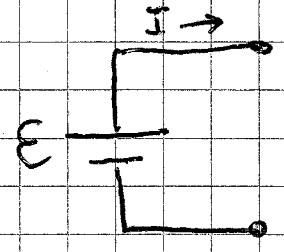
Tuesday, 15:00 data (next to Weniger)

Volts across R (Volts)	Current in R (Amps)	Power (Watts)
33.0	0.37	
31.0	0.62	
30.0	0.72	
29.0	0.80	
28.0	0.85	
27.0	0.89	
26.0	0.91	
25.0	0.91	
23.0	0.94	
21.0	0.94	
19.0	0.95	
17.0	0.97	
15.4	0.97	

Basic theory of real batteries (cells).

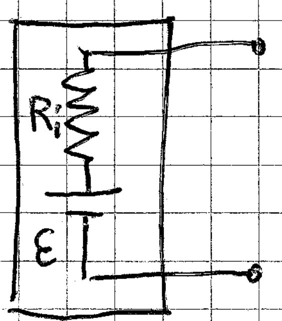
1. An ideal cell:
(ideal voltage source)



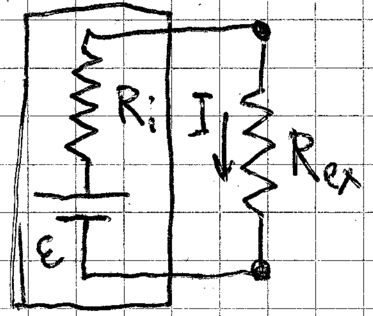
The voltage between the terminals is always the same, no matter how much current is taken from the battery: $\mathcal{E} \neq \mathcal{E}(I)$

The symbol "ε" stands for "Electromotive Force", this term was coined ~200 years ago, when people knew very little about electric currents – and it "stuck", even though it's simply voltage.

2. A real cell:
(real voltage source): it can be thougt of as a "black box" containing an ideal voltage source, an a resistor representing the so-called "internal resistance".



So, if you connect an external resistor R_{ex} to a real voltage source:



the current I in the resistor is $I = \dfrac{\mathcal{E}}{R_{TOT}} = \dfrac{\mathcal{E}}{R_i + R_{ex}}$

The ~~vo~~ votage across the external resistor is:

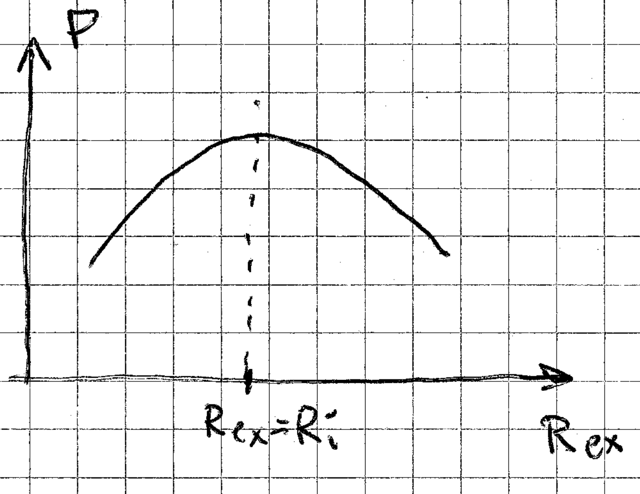
$$R_{ex} = \frac{V}{I} \Rightarrow V = R_{ex} \cdot I = \frac{\mathcal{E} \cdot R_{ex}}{R_i + R_{ex}}$$

The power dissipated in the external resistor is:

$$P = V \cdot I = \frac{\mathcal{E} \cdot R_{ex}}{R_i + R_{ex}} \cdot \frac{\mathcal{E}}{R_i + R_{ex}} = \mathcal{E}^2 \frac{R_{ex}}{(R_i + R_{ex})^2}$$

<u>Question</u>: If we change R_{ex}, then, as follows from , the <u>power P</u> also changes. How does it change? Does it have a maximum? For which value of R_{ex}?

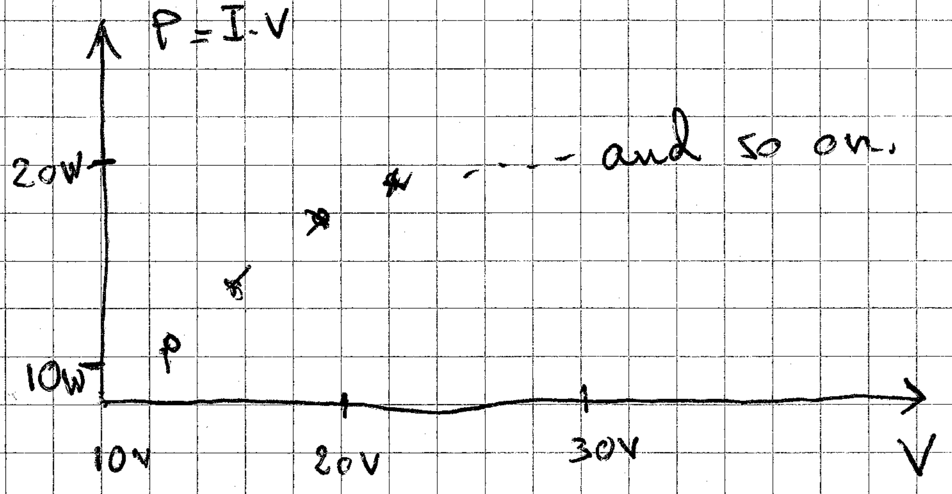
<u>Answer</u>: Yes, it does have a maximum, when the two resistances are equal, $R_{ex} = R_i$



But, in our experiment, we are interested primarily in determining the maximum power attainable from our PV cell. The question of what is the value of R_i of the cell is not so important today.

So, if we <u>measure</u> <u>both</u> I and V across R_{ex}, we can obtain P simply by multiplying the two values: $P = I \cdot V$

Next, plot $P = I \cdot V$ as a function of V:



Plot a continuous smooth curve going through the data points, and find the maximum power.

If you want to know why the maximum power occurs when $R_{ex} = R_i$?

$$P = \mathcal{E}^2 \frac{R_{ex}}{(R_i + R_{ex})^2}$$: $\mathcal{E}$ and R_i are constants, so P is a function of a single variable R_{ex}.

If $P(R_{ex})$ has an extremum – minimum or maximum – then at the R_{ex} value where the extremum occurs, the derivative $\frac{dP}{dR_{ex}} = 0$

$$\frac{dP}{dR_{ex}} = \mathcal{E}^2 \frac{d}{dR_{ex}} \left[\frac{R_{ex}}{(R_i + R_{ex})^2} \right] =$$

$$= \mathcal{E}^2 \left[\frac{1}{(R_i + R_{ex})^2} - \frac{2R_{ex}}{(R_i + R_{ex})^3} \right] = \mathcal{E}^2 \frac{R_i - R_{ex}}{(R_i + R_{ex})^3}$$

This expression is equal zero only if $R_i = R_{ex}$.

But is it a maximum, or a minimum?
To find the answer, we have to calculate the <u>second derivative</u>:

$$\frac{d^2P}{dR_{ex}^2} = \frac{d}{dR_{ex}} \left(\frac{dP}{dR_{ex}} \right) = \frac{d}{dR_{ex}} \left[\mathcal{E}^2 \frac{R_i - R_{ex}}{(R_i + R_{ex})^3} \right]$$

$$= \mathcal{E}^2 \left[- \frac{1}{(R_i + R_{ex})^3} - 3 \frac{(R_i - R_{ex})}{(R_i + R_{ex})^4} \right]$$

$$= \mathcal{E}^2 \left[\frac{-R_i - R_{ex}}{(R_i + R_{ex})^4} - \frac{3R_i - 3R_{ex}}{(R_i + R_{ex})^4} \right]$$

$$= \mathcal{E}^2 \left[\frac{-R_i - R_{ex} - 3R_i + 3R_{ex}}{(R_i + R_{ex})^4} \right] = \mathcal{E}^2 \frac{2R_{ex} - 4R_i}{(R_i + R_{ex})^4}$$

At the extremum, where $R_{ex} = R_i$, the value of the second derivative is then:

$$\left(\frac{d^2P}{dR_{ex}^2} \right)_{R_{ex} = R_i} = \mathcal{E}^2 \cdot \frac{-2R_i}{(2R_i)^4} = -\mathcal{E}^2 \frac{1}{8R_i^3}$$

which is a <u>negative</u> value, so that at the extremum there is a <u>maximum</u>

Q.E.D.