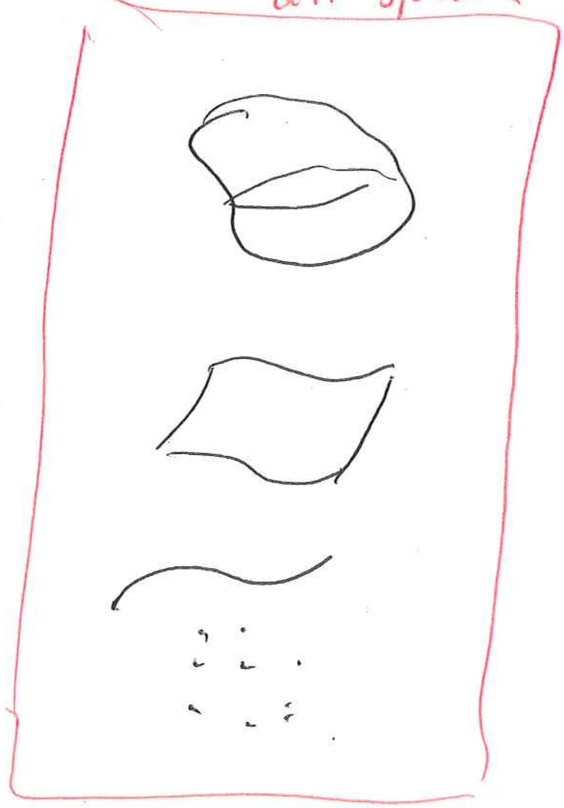
Charge

Integrate over all space

$$Q_{total} = \begin{cases} \int \rho \, d\tau \\ \int \sigma \, dA \\ \int \lambda \, |d\vec{r}| \\ \sum_i q_i \end{cases}$$



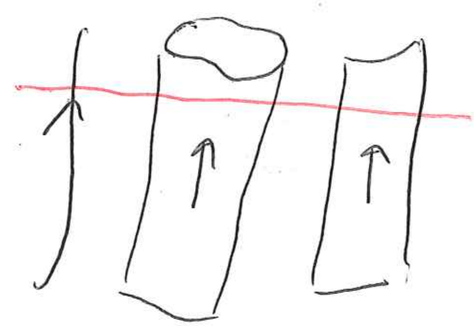
$\vec{J} = \rho \vec{v}$

$\vec{K} = \sigma \vec{v}$

$\vec{I} = \lambda \vec{v}$

Current

$$I_{total} = \begin{cases} \int \vec{J} \cdot \hat{n} \, dA \\ \int \vec{K} \cdot \hat{n} \, |d\vec{r}| \\ \vec{I} \cdot \hat{n} \end{cases}$$



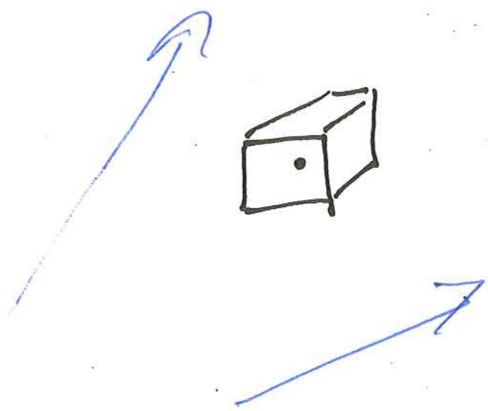
Flux through gate

$\perp$ to gate
tangent to current

Divergence

Flux through small cube/unit volume

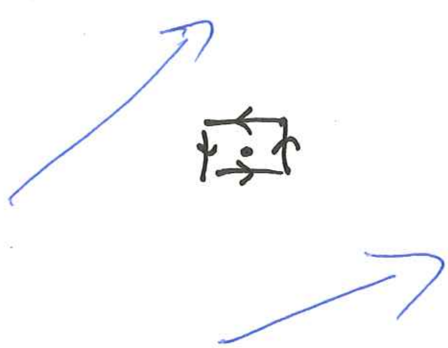
$$\sum_{\text{sides}} \vec{F}\cdot\hat{n}\,dA$$



(Component of) Curl

Circulation around small loop / unit area

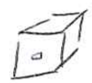
$$\oint \vec{F}\cdot d\vec{r}$$



For ~~any~~ vector field $\vec{F}$

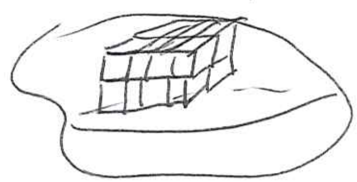
Electrostatics

Def of Divergence



$$\frac{\oint \vec{F}\cdot\hat{n}\,dA}{d\tau} = \vec{\nabla}\cdot\vec{F}$$

Divergence Theorem



$$\int_{\text{Boundary}} \vec{F}\cdot\hat{n}\,dA = \int_{\text{inside}} \vec{\nabla}\cdot\vec{F}\,d\tau$$

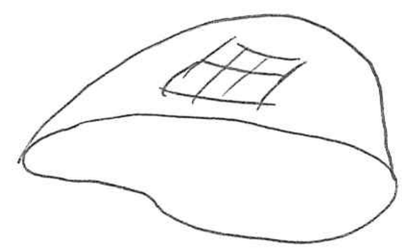
Magnetostatics

Mathematics

Def of Curl

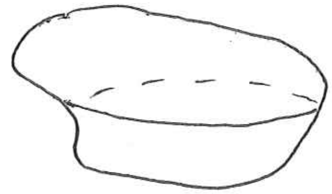
$$\frac{\oint \vec{F}\cdot d\vec{r}}{dA} = \vec{\nabla}\times\vec{F}\cdot\hat{n}$$

Stokes' Theorem



$$\int_{\text{boundary}} \vec{F}\cdot d\vec{r} = \int_{\text{inside (butterfly net)}} (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,dA$$

Gauss + Ampère



$$\oint_{\text{skin}} \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$\int_{\text{potato}} \vec{\nabla} \cdot \vec{E}\, d\tau \qquad\qquad \frac{1}{\varepsilon_0} \int \rho\, d\tau$$

All potatoes $\Rightarrow$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_{\text{loop}} \vec{B} \cdot d\vec{r} = \mu_0 \vec{J}$$

$$\int_{\text{net}} (\vec{\nabla} \times \vec{B}) \cdot \hat{n}\, dA \qquad\qquad \mu_0 \int \vec{J} \cdot \hat{n}\, dA$$

All butterfly nets $\Rightarrow$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Maxwell's Eqns (Static Cases)

Divergence Thm

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0} \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Gauss's Law

$$\oint \vec{B} \cdot \hat{n}\, dA = 0 \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{B} = 0$$

Stokes' Thm

$$\oint \vec{E} \cdot d\vec{r} = 0 \quad \Rightarrow \quad \vec{\nabla} \times \vec{E} = 0$$

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \quad \Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Ampère's Law

Conservative Fields

All of the following statements are equivalent:

1) $\vec{F}$ is conservative

2) $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of path

3) $\oint \vec{F} \cdot d\vec{r} = 0$

4) $\vec{F} = \vec{\nabla} U$ for a single-valued scalar field U

5) $\vec{\nabla} \times \vec{F} = 0$

6) $\vec{F} \cdot d\vec{r}$ is an exact differential

$$\left(\text{i.e. } dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz = \vec{\nabla} U \cdot d\vec{r} = \vec{F} \cdot d\vec{r} \right)$$

7) Candidate level curves and gradient vectors must agree on magnitude of change

Magnetostatic Field Relationships

$\vec{A}$

$\vec{B}$

$\vec{J}$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{J(\vec{r}\,')\, d\tau'}{|\vec{r}-\vec{r}\,'|}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r}-\vec{r}\,')\, d\tau'}{|\vec{r}-\vec{r}\,'|^3}$$

$$\mu_0 \vec{J} = \vec{\nabla} \times \vec{B}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$$

if $\vec{\nabla} \cdot \vec{A} = 0$ $\quad -\nabla^2 \vec{A} =$

Electrostatic Field Relationships

$\vec{E} \to \vec{g}$

$\frac{1}{4\pi\epsilon_0} \to -G$

$$
\begin{tikzcd}[row sep=huge]
V \arrow[d, bend left, "\vec{E} = -\vec{\nabla} V"] \arrow[dd, bend left=70, "\rho/\epsilon_0 = -\vec{\nabla}\cdot\vec{\nabla} V"] \\
\vec{E} \arrow[u, bend left, "V(B) - V(A) = -\int_A^B \vec{E}\cdot d\vec{r}"] \arrow[d, bend left, "\rho/\epsilon_0 = \vec{\nabla}\cdot\vec{E}"] \\
\rho \arrow[u, bend left, "\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|^3}"] \arrow[uu, bend left=70, "V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}"]
\end{tikzcd}
$$

Lorentz Force Law

$$\vec{F} = q_{test}\,(\vec{E} + \vec{v}_{test} \times \vec{B})$$

VEUF

$$\begin{tikzcd}[column sep=huge, row sep=huge]
V \arrow[r, bend left, "U = q_{test} V"] \arrow[d, bend left, "\vec{E} = -\vec{\nabla} V"] & U \arrow[d, bend left, "\vec{F} = -\vec{\nabla} U"] \\
\vec{E} \arrow[u, bend left, "V(B) - V(A) = -\int_A^B \vec{E} \cdot d\vec{r}"] \arrow[r, bend right, "\vec{F} = q_{test} \vec{E}"'] & \vec{F} \arrow[u, bend left, "U(B) - U(A) = -\int_A^B \vec{F} \cdot d\vec{r}"]
\end{tikzcd}$$

Conductors

← Many charges are free to move fast.

1) $\vec{E} = 0$ inside solid part

2) $\rho = 0$ inside solid part

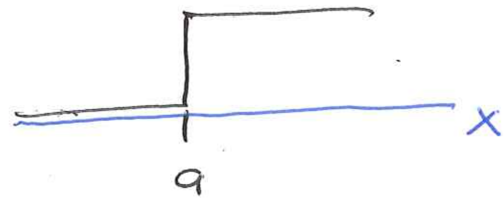
3) Any net charge resides on surface

4) Solid part is an equipotential

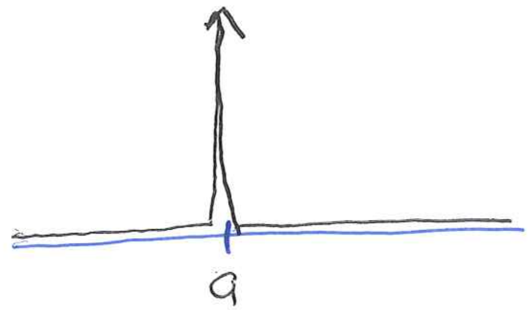
5) $\vec{E}$ is $\perp$ to surface (just outside)

Theta and Delta "Functions"

$$\Theta(x-a) = \begin{cases} 0 & x < a \\ 1 & x > a \end{cases}$$



$$\delta(x-a) = \begin{cases} 0 & x < a \\ \infty & x = a \\ 0 & x > a \end{cases}$$



$$\int_{-\infty}^{\infty} \delta(x-a)\, dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\, \delta(x-a)\, dx = f(a)$$

$$\frac{d}{dx}\, \theta(x-a) = \delta(x-a)$$

$$\int_{0}^{x} \delta(x-a)\, dx = \Theta(x-a)$$

Power Series

$$f(x) = \sum_{n=0}^{\infty} c_n (x - x_0)^n$$

$$c_n = \frac{1}{n!} \frac{d^n f}{dx^n} \Big|_{x = x_0} = \frac{1}{n!} f^{(n)}(x_0)$$

Memorize series for

$\sin x$

$\cos x$

e^x

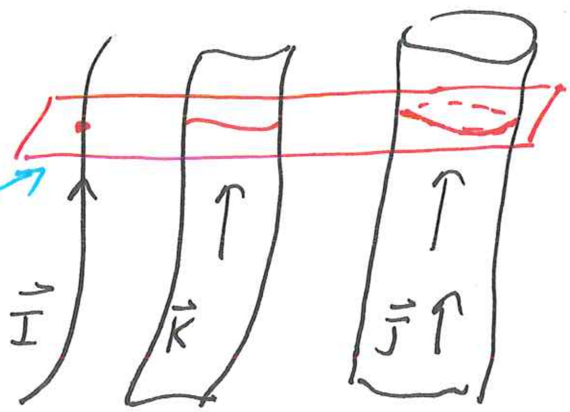
$\ln(1+x)$

$(1+x)^p$

Factor out the large thing

Two Different Types of Current Integrals

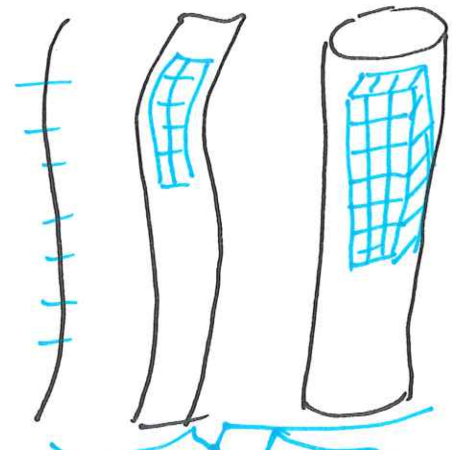
$$I_{total} = \begin{cases} \int \vec{J} \cdot \hat{n}\, dA \\ \int \vec{K} \cdot \hat{n}\, |d\vec{r}| \\ \vec{I} \cdot \hat{n} \end{cases}$$



These integrals chop up the gate

$$\vec{A}(\vec{r}) = \begin{cases} \frac{\mu_0}{4\pi} \int \frac{J(\vec{r}\,')\, d\tau'}{|\vec{r} - \vec{r}\,'|} \\ \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}\,')\, dA'}{|\vec{r} - \vec{r}\,'|} \\ \frac{\mu_0}{4\pi} \int \frac{I(\vec{r}\,')\, d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|} \end{cases}$$

(Similarly for $\vec{B}(\vec{r})$)



These integrals chop up the entire current

$$\vec{I}(\vec{r}\,')\, |d\vec{r}\,'| = I(\vec{r}\,')\, d\vec{r}\,'$$

Put the direction in one place