Department of Physics Comprehensive Examination # 90

Part I

2 April 2001

This Comprehensive Examination for Spring 2001 consists of eight problems each worth 20 points. The problems are grouped into four sessions:

Session 1	problems 1, 2	8-11 AM	Mon 2 April
Session 2	problems 3, 4	12-3 PM	Mon 2 April
Session 3	problems 5, 6	8-11 AM	Tues 3 April
Session 4	problems 7, 8	12-3 PM	Tues 3 April

Work carefully, indicate your reasoning, and display your work clearly. Even if you do not complete a problem, it is possible to obtain partial credit, especially if you demonstrate conceptual understanding. Do all work in the bluebooks, work each problem in its own numbered bluebook, and be certain that your chosen student letter, but not your name, is on the inside of the back cover of every bluebook. Be sure to remember your student letter for use in the remaining sessions of the examination. If something is omitted from the statement of the problem or you feel there is an ambiguity, please ask your question quietly and privately, so as not to disturb the others. Only your bluebooks and the examination should be on the table before you. Any other items should be stored on the floor. Calculators are not allowed. Please return all bluebooks and formula sheets at the end of the exam.

Use the last pages of your bluebooks for scratch work separated by at least one page from your solutions. Scratch work will not be graded.

1. A spring with a linear spring constant k is attached to a mass m and is free to oscillate in one dimension. There is a viscous frictional force $-bv$, where b is a constant and v is the velocity. The mass has an initial velocity of v_0 and an initial position of x_0.

   (a) Derive the solution for the position as a function of time $x(t)$ in the absence of friction.

   (b) Deduce the frequency ω_0 that the system oscillates with in the absence of friction.

   (c) Derive the time-dependent solution $x(t)$ in the presence of friction.

   (d) Deduce the frequency ω that the system oscillates with in the presence of friction, and the condition(s) on b for there to be oscillations.

   (e) A sinusoidal force $F \cos \Omega t$ now drives the mass (which remains attached to the spring). Deduce what must be the most general form of the new solution $x(t)$ for this system. If your solution contains parameters, make sure to indicate how they are to be determined. While you must give the complete form of all equations that must be solved, you do *not* actually have to solve these equations.

2. A magnetic material is in the shape of a right circular cylinder of length L and radius a. The cylinder has a uniform constant permanent magnetization $\mathbf{M}_o$ parallel to its axis.

   (a) Determine the magnetic field $\mathbf{H}$ and magnetic induction $\mathbf{B}$ at all points on the axis of the cylinder, both inside and outside.

   (b) Sketch the field lines of both $\mathbf{H}$ and $\mathbf{B}$.

   (c) Sketch the ratios $\dfrac{\mathbf{B}}{\mu_o M_o}$ and $\dfrac{\mathbf{H}}{M_o}$ on the axis as functions of z. Describe the physical reasoning you used to arrive at the shapes of these curves.

3. A one-dimensional quantum oscillator of frequency ω and mass m is perturbed by the addition of a potential $V(x) = \frac{1}{2}m\lambda x^2$.

(a) Using non-degenerate perturbation theory, find the energy of the ground state up to second order in λ. (*Hint:* You may find it useful to employ the completeness relation $\hat{1} = \sum_i |i\rangle\langle i|$.)

(b) The exact solution is easy to find for this potential. Expand the exact ground state energy in powers of λ and compare with the perturbation result of part (a).

4. Consider a degenerate Fermi gas of free electrons which has a chemical potential of 20 MeV (million electron volts), and which is at a temperature of 10^8 K.

Derive an expression for the pressure P in such a gas.

5. In the so-called "porous plug" experiment utilizing the Joule-Kelvin (Joule-Thomson) effect, a gas is made to pass through a constriction from a region of higher to lower pressure. This process is often referred to as "throttling", and can be well characterized as *isenthalpic.* Under some conditions a gas will cool down throttled, under other conditions it will heat up, and under certain conditions it will do neither.

   (a) Find an expression for the partial derivative $\left(\frac{\partial T}{\partial P}\right)_H$ describing this process.

   (b) Consider a gas obeying the Dieterici equation of state,

   $$P \exp\left(\frac{a}{RTV}\right)(V-b) = RT,$$

   where a and b are constants characteristic of the gas. Show that the temperature T_0 at which no change of temperature will occur in such a gas being throttled is given by

   $$T_0 = \frac{2a}{bR}\left(\frac{V-b}{V}\right)$$

6. A long straight wire carries a constant current I. It is made of a material with resistivity η and cross-sectional area a.

   (a) Find the magnetic induction $\mathbf{B}$ and electric field $\mathbf{E}$ outside the wire. (The electric field $\mathbf{E}$ is ambiguous unless some boundary conditions are specified. You may assume that the wire is stretched between two infinite parallel conducting plates perpendicular to the wire.)

   (b) Sketch the lines of force for these two fields.

   (c) Find the radiated power per unit area.

   (d) In what direction is it radiated?

   (e) What is the total power radiated?

   (f) What is the physical meaning of your result?

7. Consider the tent map

$$x_{n+1} = \mu\left(1 - 2\left|x_n - \frac{1}{2}\right|\right) = \begin{cases} 2\mu x_n, & \text{for } 0 \le x_n \le \frac{1}{2} \\ 2\mu(1 - x_n), & \text{for } \frac{1}{2} \le x_n \le 1. \end{cases}$$

(a) Determine algebraically the *fixed points* for $x \le \frac{1}{2}$ (state conditions on μ).

(b) Determine algebraically the *fixed points* for $x \ge \frac{1}{2}$ (state conditions on μ).

(c) Deduce (preferably algebraically) whether any of the fixed points are stable.

(d) Use a graphical technique (ask for graph paper) to show the properties of *fixed attractors* for $\mu = 0.25, 0.65, 0.75$.

8. Consider the potential

$$V(x) = \begin{cases} 0 & \text{for } x > \frac{L}{2}, \\ -|V_0| & \text{for } \frac{L}{2} > x > 0, \\ +\infty & \text{for } 0 > x. \end{cases}$$

Find the allowed energies and the corresponding (unnormalized) wavefunctions for this potential. If you deduce an equation that can be solved only numerically, you may give that equation as your answer; however, in that case you should explain the number of solutions expected.

1) a) friction

(x_0, v_0)

a) $x(t)$ (no friction)

b) $F = ma$

$-kx = m\, d^2x/dt^2$

$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega_0^2 x \quad , \quad \underline{\omega_0 = \sqrt{k/m}}$

Soln $\quad x = A\cos\omega_0 t + B\sin\omega_0 t \qquad$ (substitution)

$x(0) = A \Rightarrow A = x_0$

$v(0) = \left(-\omega_0 A \sin\omega_0 t + \omega_0 B \cos\omega t\right)\Big|_{t=0} = \omega_0 B \Rightarrow B = v_0/\omega_0$

$$\underline{x(t) = x_0 \cos\omega_0 t + (v_0/\omega)\sin\omega_0 t}$$

c) with friction

$-kx - b\frac{dx}{dt} = m\, d^2x/dt^2$

$$\underline{\frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}dx/dt = 0}$$

let $\omega_0 = \sqrt{k/m}$, $\beta = \frac{1}{2}\frac{b}{m} \Rightarrow \qquad \underline{\ddot{x} + \omega_0^2 x + 2\beta\dot{x} = 0}$

(not freq. for this case)

guess $\quad x = Ae^{\omega t}$

$Ae^{\omega t}\left[\omega^2 + \omega_0^2 + 2\beta\omega\right] = 0$

$\omega = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$

$$\underline{x(t) = Ae^{-\beta t}e^{+\sqrt{\beta^2-\omega_0^2}\,t} + Be^{-\beta t}e^{-\sqrt{\beta^2-\omega_0^2}\,t}}$$

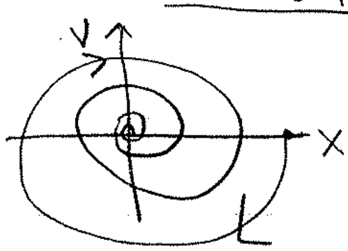
d) Oscillations require complex exponent $\Rightarrow$ $\omega_0 > \beta$

then

$x(t) = e^{-\beta t}\left[Ae^{i\omega t} + Be^{-i\omega t}\right]$

$\underline{\omega = \sqrt{\omega_0^2 - \beta^2}} \quad < \omega_0 \qquad$ (slows down)

e)



(f) $\ddot{x} + \omega_0^2 x + 2\beta\dot{x} = \frac{F}{m}\cos\lambda t \equiv f\cos\lambda t$ $(f = F/m)$

This part not asked to work out

The general solution is the sum of a particular solution (one which reproduces the RHS) and a solution of the homogeneous equation ($f=$ i.e. the solution of part (c).

Guess particular solution

$$x_p(t) = P\cos(\lambda t - \delta)$$

$$\dot{x}_p(t) = -\lambda P\sin(\lambda t - \delta), \quad \ddot{x}_p = -\lambda^2 P\cos(\lambda t - \delta)$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta, \quad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

substitute: $-\lambda^2 P\cos\lambda t\cos\delta - \lambda^2 P\sin\lambda t\sin\delta + \omega_0^2 P\cos\lambda t\cos\delta + \omega_0^2 P\sin\lambda t\sin\delta$

$$+ 2\beta(-\lambda P)(\sin\lambda t\cos\delta - \cos\lambda t\sin\delta) = f\cos\lambda t$$

$$\begin{cases} \cos\lambda t\,[-\lambda^2 P\cos\delta + \omega_0^2 P\cos\delta + 2\beta\lambda P\sin\delta - f] = 0 \\ +\sin\lambda t\,[-\lambda^2 P\sin\delta + \omega_0^2 P\sin\delta - 2\beta\lambda P\cos\delta] \end{cases}$$

to be true for all times, each [ ] must vanish

$\sin\lambda t$: $(\omega_0^2 - \lambda^2)\cancel{P}\sin\delta = 2\beta\lambda\cos\delta$

$$\tan\delta = \frac{2\beta\lambda}{\omega_0^2 - \lambda^2} \Rightarrow \delta = \tan^{-1}\left(\frac{2\beta\lambda}{\omega_0^2 - \lambda^2}\right)$$

$\cos\lambda t$: $P\left[(\omega_0^2 - \lambda^2)\cos\delta + 2\beta\lambda\sin\delta\right] = f$

solve $\cos\delta$, $\sin\delta$: $\cos^2 + \sin^2 = 1 \Rightarrow 1 + \tan^2\delta = \frac{1}{\cos^2}$, $1 + \frac{1}{\tan^2} = \frac{1}{\sin^2}$

$$\Rightarrow \frac{1}{\cos^2\delta} = 1 + \tan^2\delta = \frac{4\beta^2\lambda^2 + (\omega_0^2 - \lambda^2)^2}{(\omega_0^2 - \lambda^2)^2}, \quad \frac{1}{\sin^2\delta} = \frac{4\beta^2\lambda^2 + (\omega_0^2 - \lambda^2)^2}{4\beta^2\lambda^2}$$

$$\sin\delta = \frac{2\beta\lambda}{\sqrt{4\beta^2\lambda^2 + (\omega_0^2 - \lambda^2)^2}}$$

$$\cos\delta = \frac{\omega_0^2 - \lambda^2}{\sqrt{4\beta^2\lambda^2 + (\omega_0^2 - \lambda^2)^2}}$$

$$\Rightarrow \underline{\rho} = \frac{f}{(\omega_0^2-\Omega^2)\cos\delta + 2\beta\Omega\sin\delta} = \frac{f\sqrt{4\beta^2\Omega^2+(\omega_0^2-\Omega^2)^2}}{(\omega_0^2-\Omega^2)^2+4\beta^2\Omega^2}$$

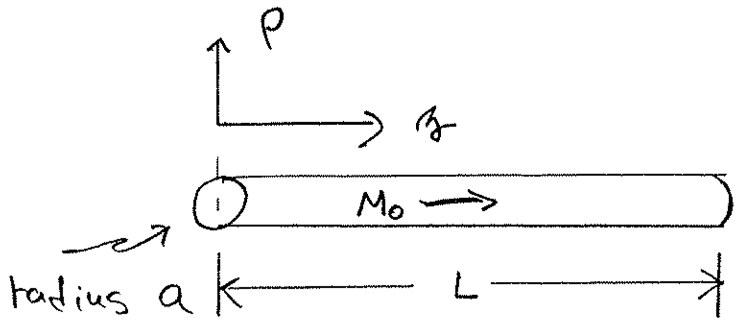
$$\rho = \frac{f}{\sqrt{(\omega_0^2-\Omega^2)^2+4\beta^2\Omega^2}}$$

$$\boxed{x(t) = Ae^{-\beta t}e^{+\sqrt{\beta^2-\omega_0^2}\,t} + Be^{-\beta t}e^{-\sqrt{\beta^2-\omega_0^2}\,t} + \frac{f}{\sqrt{(\omega_0^2-\Omega^2)^2+4\beta^2\Omega^2}}\cos(\Omega t-\delta)}$$

g) $\delta = \tan^{-1}\left(\frac{2\beta\Omega}{\omega_0^2-\Omega^2}\right)$ = phase Δ;

positive below, negative above resonance

2.(a)



The magnetism produces surface "charges" at the ends of the cylinder $\sigma_M = \hat{n}\cdot\vec{M}$

$$\Phi_M = \frac{1}{4\pi}\int_S \frac{\sigma_M(\vec{\rho}')\,da'}{|\vec{\rho}-\vec{\rho}'|}$$

Take the origin $z=0$ at left end of the cylinder.

$$\Phi_M = \frac{1}{4\pi}\int_0^{2\pi} d\varphi' \int_0^a \frac{(-M_0)\,\rho'\,d\rho'}{(\rho'^2+z^2)^{1/2}}$$

$$+\ \frac{1}{4\pi}\int_0^{2\pi} d\varphi' \int_0^a \frac{(+M_0)\,\rho'\,d\rho'}{(\rho'^2+(L-z)^2)^{1/2}}$$

use the fact that

$$\int_0^a \frac{\rho\,d\rho}{\sqrt{\rho^2+z^2}} = \sqrt{\rho^2+z^2}\,\Big|_0^a = +\sqrt{a^2+z^2} - |z|$$

be careful with signs!

$$\vec{H} = -\vec{\nabla}\Phi_m$$

$$H_z = -\frac{d\Phi_m}{dz} \qquad \text{(only one component)}$$

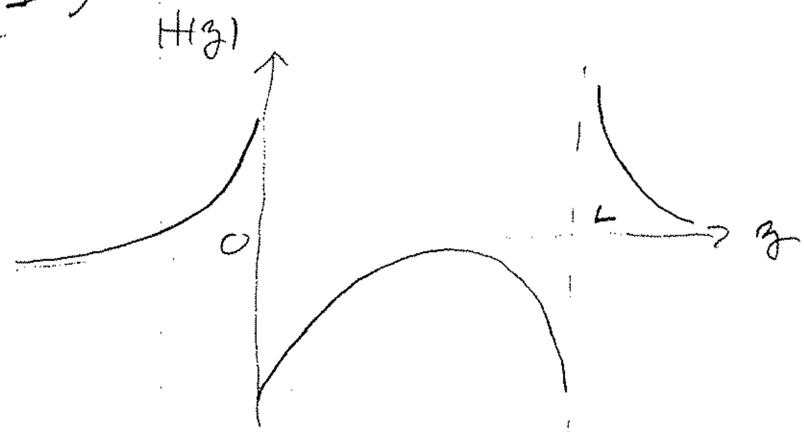
$$\Phi = \frac{M_0}{2}\left\{ -\sqrt{a^2+z^2} + |z| + \sqrt{a^2+(L-z)^2} - |L-z| \right\}$$

$$H = \frac{M_0}{2}\left\{ \frac{z}{\sqrt{a^2+z^2}} - sg(z) - \frac{(z-L)}{\sqrt{a^2+(z-L)^2}} + sg(z-L) \right\}$$

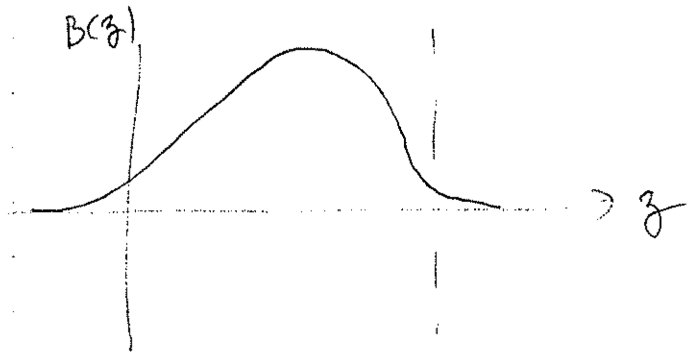
$$sg(z) = \begin{matrix} +1 & z>0 \\ -1 & z<0 \end{matrix}$$

Finally $B = \mu_0 (H + M)$

(1b)



H is analogous to E in the presence of two charged plates



B is continuous since $\nabla \cdot B = 0$

(3) $H = \frac{p^2}{2m} + V(x) = \frac{p^2}{2m} + \frac{1}{2}kx^2 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$

$\omega = \sqrt{k/m} \quad \omega^2 = k/m \quad k = m\omega^2$

$E_{gs} = \hbar\omega/2 = E_0$

a) $V(x) \to \frac{1}{2}kx^2 + \frac{1}{2}m\lambda x^2$, $E_0' = ?$ (order λ^2)

$$E_0' = E_0 + \langle 0|\tfrac{1}{2}m\lambda x^2|0\rangle + \sum_{m \neq n=0} \frac{|\langle 0|\frac{1}{2}m\lambda x^2|m\rangle|^2}{\frac{\hbar\omega}{2} - E_n}$$

$E_n = \hbar\omega(n+1/2)$

$$E_0' = \hbar\omega/2 + \tfrac{1}{2}m\lambda\langle 0|x^2|0\rangle + \frac{m^2\lambda^2}{4}\sum_{m\neq 0}\frac{|\langle 0|x^2|m\rangle|^2}{\frac{\hbar\omega}{2} - \hbar\omega(m+1/2)} + \cdots$$

The 0 state has even parity and x^2 is even, so only m = even contribute

$|\langle 0|x^2|m\rangle|^2 = ?$ yet we only need first term for expansion

$\langle x^2\rangle_{0m} = x_{0m}\,x_{m0}$

(4) $\langle n|x|m\rangle = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{m\omega}}\begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} \cdots \end{pmatrix}$ $\quad \sum_i |i\rangle\langle i| = 1$

(Hermite Polynomials)

$$\langle n|x^2|m\rangle = \sum_i^{\infty} \langle n|x|i\rangle\langle i|x|m\rangle \text{ — completeness! } \boxed{\text{HINT}}$$

$$\langle x^2\rangle = \frac{\hbar}{2m\omega}\begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \\ \sqrt{1} & 0 & \sqrt{2} & 0 & \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 \; 0 \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} \end{pmatrix}\begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 \; - \\ \sqrt{1} & 0 & \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 \; 0 \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} \; 0 \; 0 \end{pmatrix}$$

$$= \frac{\hbar}{2m\omega}\begin{pmatrix} 1 & 0 & \sqrt{2} & 0 & 0 & -- \\ 0 & 3 & 0 & \sqrt{6} & & -- \end{pmatrix}$$

only $m=2$

$$E = \frac{\hbar\omega}{2} + \frac{1}{4}\frac{\hbar\lambda}{\omega} - \frac{1}{16}\frac{\lambda^2}{\omega^4} + \cdots$$

(4) given H.O.: $\langle n|x|n'\rangle = \sqrt{\frac{\hbar}{m\omega}}\sqrt{\frac{n+1}{2}}\,\delta_{n+1,n'} + \sqrt{\frac{\hbar}{m\omega}}\sqrt{\frac{n}{2}}\,\delta_{n-1,n'}$

$\langle n|p|n'\rangle = -i\sqrt{m\hbar\omega}\sqrt{\frac{n+1}{2}}\,\delta_{n+1,n'} + i\sqrt{m\hbar\omega}\sqrt{\frac{n}{2}}\,\delta_{n-1,n'}$

b) $H_0 = p^2/2m + \frac{1}{2} m\omega^2 x$

$H_0 + H' = \frac{p^2}{2m} + \frac{1}{2} m(\omega^2 + \lambda) x^2$

$\omega^2 \to \omega^2 + \lambda$

$E = \frac{1}{2}\hbar\omega \to \frac{1}{2}\hbar\sqrt{\omega^2+\lambda}$

$= \frac{1}{2}\hbar\omega\sqrt{1+\lambda/\omega^2}$

$= \frac{1}{2}\hbar\omega[1 + \frac{1}{2}\lambda/\omega^2 - \frac{1}{8}\frac{\lambda^2}{\omega^4} + \cdots$

Same as part a!

GRADUATE THERMAL PHYSICS PROBLEM - Spring Comp Exam: April 2 & 3, 2001

4) Consider a degenerate Fermi gas of electrons which has a chemical potential of 20 MeV, and which is at a temperature of 10^8 K.

Derive an expression for the pressure, P, in such a gas.

(1) A chemical potential of this value means that the average energy of the electrons is $\varepsilon_{avg} = 0.6\,\mu_0$, which, in turn, corresponds to a Fermi temperature of about $T_{fermi} \approx 0.6\,\mu_0 / k \approx 2.3 \times 10^{11}\,K$. Since this is so much greater than the temperature of the gas, we are entitled to apply the zero-temperature approximation for Fermi statistics.

(2) As well, since these electrons have energies >> m_0c^2, the relativistic relation $E = \sqrt{(pc)^2 + (m_0c^2)^2}$ must be used to express this quantity.

$$E = 2\sum_{|\vec{p}|<p_o} \sqrt{(pc)^2 + (m_oc^2)^2} = \frac{2V}{h^3}\int_0^{p_o} 4\pi p^2\sqrt{(pc)^2 + (m_oc^2)^2}\,dp$$

The value of p_o can be determined from $N = \frac{8\pi V}{h^3}\int_0^{p_o} p^2\,dp$. Thus, $p_o = \hbar\left(\frac{3\pi^2 N}{V}\right)^{1/3}$.

Putting $y = \frac{p}{m_oc}$, allows one to write the integral for E as

$$E = \frac{m_o^4 c^5 V}{\pi^2 \hbar^3}\int_0^{y_o} y^2\left(1 + y^2\right)^{1/2} dy = \frac{m_o^4 c^5 V}{\pi^2 \hbar^3} G(y_o)$$

Now the pressure is given by $P = -\left(\frac{\partial E}{\partial V}\right)_T$. Realizing that $G(y_o)$ has a dependence on V, we get the following expression $P = -\left(\frac{\partial E}{\partial V}\right)_T = \frac{m_o^4 c^5}{\pi^2 \hbar^3}\left\{-G(y_o) - V\frac{\partial G(y_o)}{\partial V}\right\}$,

where $y_o = \frac{p_o}{m_oc} = \frac{\hbar}{m_oc}\left(\frac{3\pi^2 N}{V}\right)^{1/3}$.

Now, $\frac{\partial G}{\partial V} = \left(\frac{\partial G}{\partial y_o}\right)\left(\frac{\partial y_o}{\partial V}\right) = -\frac{1}{3}\frac{y_o}{V}\left(\frac{\partial G}{\partial y_o}\right)$, where $\frac{\partial G}{\partial y_o} = y_o^2\left(1 + y_o^2\right)^{1/2}$.

So, finally, $P = \frac{m_o^4 c^5}{\pi^2 \hbar^3}\left\{-\int_o^{y_o} y^2\left(1 + y^2\right)^{1/2} dy + \frac{y_o^3}{3}\left(1 + y_o^2\right)^{1/2}\right\}$.

For the relativistic situation that we are examining here, $y_o \gg 1$, and so the expression for P becomes

$$P = \frac{m_o^4 c^5 y_o^2}{12\pi^2 \hbar^3}\left(y_o^2 - 1\right) \approx \frac{m_o^4 c^5 y_o^4}{12\pi^2 \hbar^3}.$$

Thus, finally,
$$P \approx \frac{m_o^2 c^3}{12\pi^2 \hbar}\left(\frac{3\pi^2 N}{V}\right)^{4/3}.$$

5.) Tent map:

$$x_{n+1} = 2\mu \begin{cases} x, & 0 < x < 1/2 \\ 1-x, & 1/2 < x < 1 \end{cases}$$

a) Fixed point $\Rightarrow x_{n+1} = x_n$

for $x < 1/2$ $\quad x_{n+1} = 2\mu x_n$
so fixed when $2\mu x_n = x_n \Rightarrow \underline{x = 0}$

if $x \neq 0$, then $\div x \Rightarrow 2\mu = 1 \Rightarrow \mu = 1/2$ fixed for all x

b) $\underline{x > 1/2}$: $x_{n+1} = 2\mu - 2\mu x_n$

Fixed: $x_n = 2\mu - 2\mu x_n$

$\Rightarrow x^* = \dfrac{2\mu}{1+2\mu}$, $\quad x^* = \dfrac{1}{1+\frac{1}{2\mu}} > \frac{1}{2} \Rightarrow \frac{1}{2\mu} > 1$

yet

so $\mu > \frac{1}{2}$, all $x^* > 1/2$ $\quad \Rightarrow \mu > 1/2$

c) Stability?

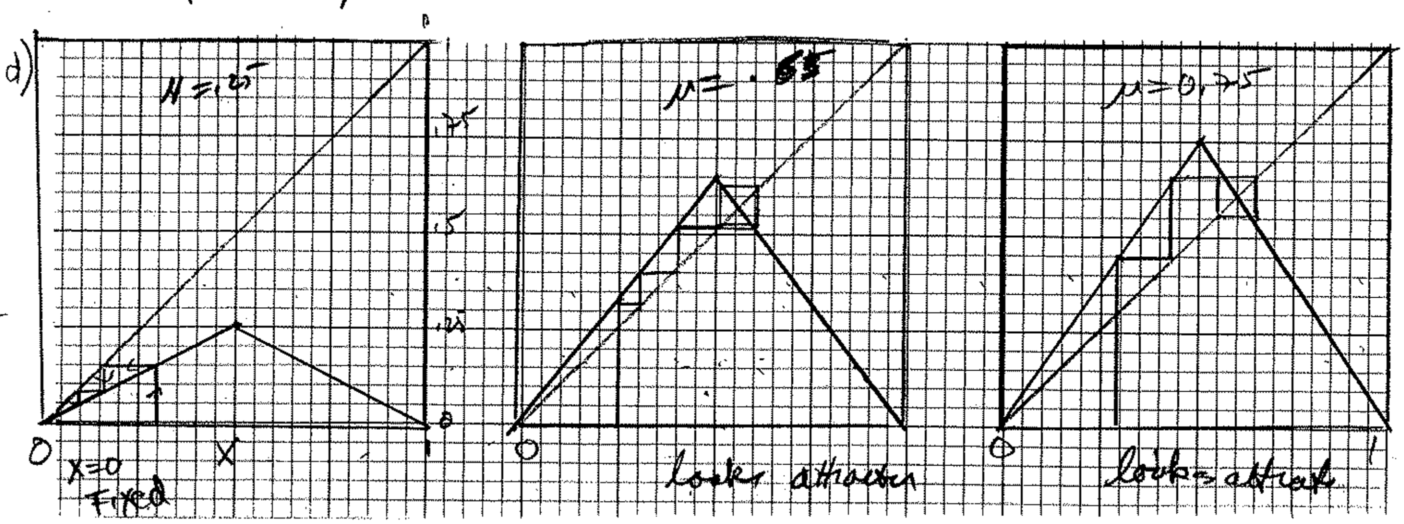
for $x_{n+1} = f(x_n)$, stability $\Rightarrow \left|\frac{df}{dx}\right| < 1$

if $f(x) = 2\mu x$, $\frac{df}{dx} = 2\mu$, stable $\Rightarrow \underline{\mu < 1/2}$

if $f(x) = 2\mu - 2\mu x$, $\frac{df}{dx} = -2\mu$, " " $\underline{\mu < 1/2}$

only $x = 0$ stable
plus $\mu = 1/2$ all $x < 1/2$, special case

d)



6. Long straight wire

(a)

Use gauss's law for $\vec{E}$

$$\int_S \vec{E}\cdot\hat{n}\, da = Q/\epsilon_0 \quad \text{so} \quad E = \sigma/\epsilon_0$$

σ = charge/area on plates.

ampere's law for $\vec{B}$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$2\pi r B = \mu_0 I$$

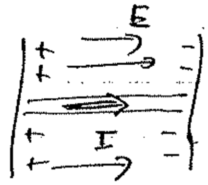
$$B = \frac{\mu_0 I}{2\pi r}$$

We don't know σ however, so use the resistivity:

$$\eta \vec{J} = \vec{E} = \frac{\eta I}{a} = \frac{\sigma}{\epsilon_0}$$

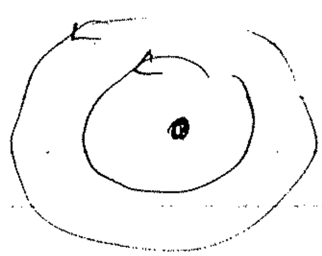
$$\text{So } \vec{E} = \frac{\eta I}{a}$$

Directions: E)



(assume conventional current)

B)



I up from paper

2(B) $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Note $\vec{S}$ is radial & pointed <u>in</u> to wire.

Total power $= \vec{S} \times$ area

$$= \frac{1}{\mu_0} \left(\frac{\kappa I}{a} \right) \left(\frac{\mu_0 I}{2\pi r} \right) \times 2\pi r$$

$$= I^2 R \qquad (!)$$

Power is being drawn <u>in</u> from the field & ~~turned~~ into heat.

7

UNDERGRADUATE THERMAL PHYSICS PROBLEM FOR SPRING 2001 COMP EXAM

In the so-called "porous plug" experiment, utilizing the Joule-Kelvin (Joule-Thomson) effect, a gas is made to pass through a constriction from a region of higher to lower pressure. This process is often referred to as throttling, and can be well characterized as *isenthalpic*. Under some conditions, a gas will cool down upon being throttled, under other conditions, it will heat up, and under certain conditions it will do neither.

(a) Find an expression for the partial derivative $\left(\frac{\partial T}{\partial P}\right)_H$ describing this process.

(b) The Dieterici equation of state is given by $P\exp\left(\frac{a}{RTV}\right)(V-b) = RT$.

Consider a gas obeying this equation of state. Show that the temperature, T_0, at which no change of temperature will occur in such a gas being throttled is given by

$$T_0 = \frac{2a}{bR}\left(\frac{V-b}{V}\right)$$

(a) The enthalpy is given by $H = U + PV$, and so its differential by

$dH = TdS + VdP$. Expanding the entropy differential as $dS = \left(\frac{\partial S}{\partial P}\right)_T dP + \left(\frac{\partial S}{\partial T}\right)_P dT$

allows $dH = T\left(\frac{\partial S}{\partial P}\right)_T dP + T\left(\frac{\partial S}{\partial T}\right)_P dT + VdP$. Now, a process occurring

isenthalpically is one for which $dH = 0$. Equating our expression to zero yields

$$0 = C_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP,$$

where use has been made of the following Maxell relation, namely that

$$-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P.$$

Thus, we see that $\left(\frac{\partial T}{\partial P}\right)_H = \dfrac{T\left(\frac{\partial V}{\partial T}\right)_P - V}{C_P}$, an expression that is always zero for the

case of an ideal gas.

(b) The Dieterici EOS is given by $P \exp\left(\frac{a}{RTV}\right)(V - b) = RT$. Taking partials with respect to temperature of both sides of the equation, and gathering terms together yields

$$T\left(\frac{\partial V}{\partial T}\right)_P - V = \frac{(RTV + a)V(V - b)}{RTV^2 - a(V - b)} - V = 0$$

solving for T produces a form for T_o of $T_o = \frac{2a}{Rb}\left(\frac{V - b}{V}\right)$

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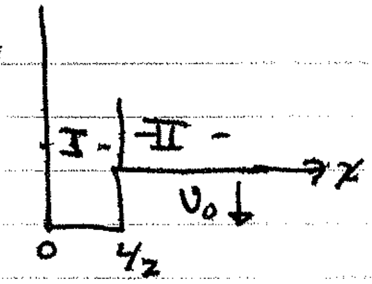
QM



$$V(x) = \begin{cases} 0 & x > L/2 \\ -|V_0| & \frac{L}{2} > x > 0 \\ +\infty & 0 > x \end{cases}$$

1D Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi \Rightarrow \underline{\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}(E-V)\psi(x) = -q^2\psi(x)}$$

<u>Case A $E > 0$</u> (Scattering) $\qquad \frac{d^2\psi}{dx^2} + q^2\psi(x) = 0$

region I: $V = -V_0$, let $q^2 = \frac{2m}{\hbar^2}(E+V_0) \Rightarrow$

$$\psi_I = A \sin qx + B \cos qx$$

B.C. $\psi_I(0) = 0 \Rightarrow B = 0 \Rightarrow \psi_I = \sin qx$

<u>II:</u> $q^2 = p^2 = 2mE/\hbar^2$

$$\psi_{II} = C \sin px + D \cos px$$

ψ, ψ' continuous at $x = L/2$: (1) $\psi_I(L/2) = \psi_{II}(L/2) \Rightarrow \sin q\frac{L}{2} = C \sin p\frac{L}{2} + D \cos p\frac{L}{2}$

(2) $\psi_I'(L/2) = \psi_{II}'(L/2) \Rightarrow q \cos q\frac{L}{2} = p\left[C \cos p\frac{L}{2} - D \sin p\frac{L}{2}\right]$

(1) $\cos pL/2 \pm$ (2) $\sin pL/2 \Rightarrow$

$$C = \sin p\tfrac{L}{2} \sin q\tfrac{L}{2} + \frac{q}{p} \cos p\tfrac{L}{2} \cos q\tfrac{L}{2}$$

$$D = \cos p\tfrac{L}{2} \sin q\tfrac{L}{2} - \frac{q}{p} \sin p\tfrac{L}{2} \cos q\tfrac{L}{2}$$

this is continuum and there are <u>no</u> restrictions on E!!
all E's are soltns.

<u>Case B: $-V_0 < E < 0$</u> (Bound)

I: (Same) $\psi_I = A \sin qx + B \cos qx$ (q smaller)

$\psi_{II} = F e^{-gx} + G e^{+gx} \Rightarrow F e^{-gx}$ to be normalizable

$\psi_I(L/2) = \psi_{II}(L/2) \Rightarrow \sin qL/2 = F e^{-gL/2}$

$\psi_I'(L/2) = \psi_{II}'(L/2) \Rightarrow q \cos qL/2 = -g F e^{-gL/2}$

Two equations, 2 unknowns

1) Squaring & adding : $q^2 \underbrace{(\sin^2 + \cos^2)}_{1} = F^2 (q^2 e^{-gL} + g^2$

$\Rightarrow$ $F = \dfrac{q}{\sqrt{q^2+g^2}} e^{gL/2}$

So if know q, know g & F.

<u>dividing:</u> $\tan qL/2 = -q/g$

must solve numerically. but will only have finite number of solutions

<u>Case C</u> $E < -V_0$