OSU PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #76

January 8 and 9, 1996

Comprehensive Examination for Winter 1996

PART I

General Instructions

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An iron meteorite of radius 2 cm is charged by its passage through the atmosphere until its electrostatic potential is 30 kV. It then strikes the ocean, where it is immediately immersed in the sea water. The dielectric constant of the water is $\kappa = 80$, and its resistivity is $\rho = 9\ \Omega$ cm.

(a) (5 points) What is the charge on the meteorite just before it enters the water? $Q_0 = ?$

(b) (15 points) Estimate the electrostatic potential of the meteorite 1 ns after it enters the water.

Note $\dfrac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9\, Nm^2 / C^2$

A surface has N sites, each of which can bind one gas molecule. The surface is in contact with an ideal gas at pressure, P, and temperature, T. An adsorbed molecule has energy, $-\varepsilon$, relative to the gaseous state. Assume that the molecules are monatomic, and find the adsorption isotherm, $\mathfrak{R}(P)$, which is the ratio of the average number of adsorbed molecules to the number of sites at equilibrium.

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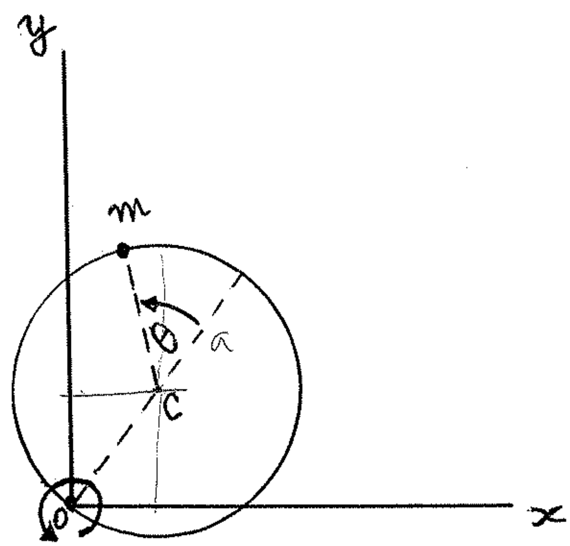
A bead, of mass m, slides without friction on a hoop of radius a, that, itself, rotates with **constant** angular velocity ω about an axis perpendicular to the plane of the hoop and passing through the edge of the hoop. The hoop lies in the x-y plane and the z-axis (rotation axis) is perpendicular out of the page at O. Consider this system to be in interstellar space and ignore any gravity. Measuring the angle, θ, as shown in the figure below, determine the equation of motion satisfied by it.



OSU Physics Dept. Comp. Exam #76. January 8-9, 1996 **PROB 4**

1. The canonical form of the density operator is given by

$$\rho = A \exp\left(\frac{-H}{k_B T}\right)$$

where k_B is Boltzmann's constant and T denotes temperature. It describes a system with a Hamiltonian H that is maintained in thermal equilibrium with a heat reservoir at the temperature T. Apply this to a simple system consisting of a one-dimensional harmonic oscillator with a fundamental frequency ω_o.

   (a) Find the diagonal elements of ρ.

   (b) Determine the normalization A.

   (c) Calculate the expectation $< E >$ of the oscillator.

Hint:

$$\sum_{n=0}^{\infty}(a+bn)q^n = \frac{1}{1-q} + \frac{bq}{(1-q)^2}$$

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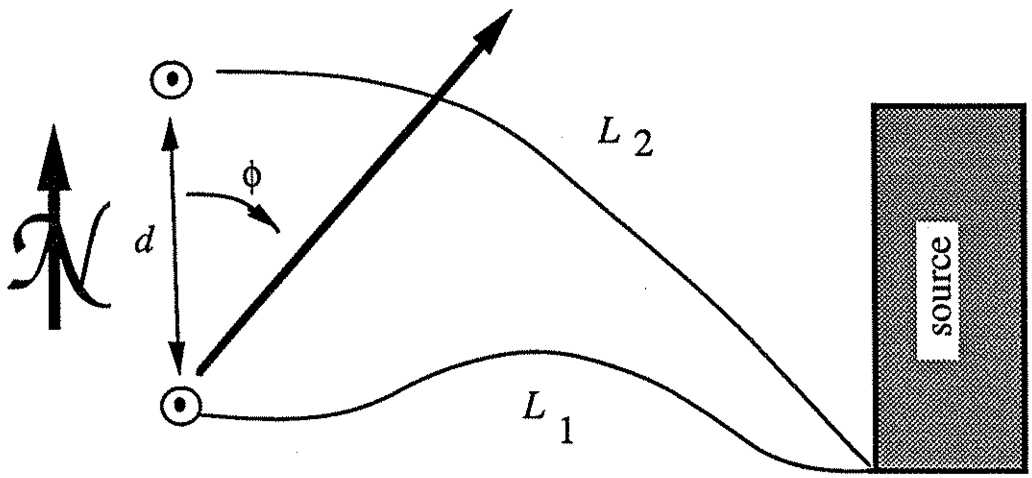
Two vertical center-fed linear dipole antennas, each of length $\lambda/2$, are placed at the same height so that the horizontal distance between antennas is d.



They are excited by a single ideal current source of frequency $f = c/\lambda$ via two transmission lines of unequal lengths $L_1 \neq L_2$. The phase velocity of the transmission lines is v.

(a) Show that, when the separation d is small enough, the radiated intensity is greatest along the line of centers, $\phi = 0$ or π .

In this case, what determines whether the maximum is toward the north or south?

(b) Show that, when the separation d is large enough, the radiated intensity will have maxima in directions other than north or south. In which directions are the maxima found? How large does d have to be for this to be the case?

(c) Find an expression for the angular dependence of the radiation intensity at distance R and angle θ in the horixontal plane, $I = I_0(R)\ F(\phi)$. You need not evaluate $I_0(R)$.

1) Consider a pool (or billiard) ball of diameter, D, sitting on a pool (or billiard) table, to receive a horizontal impulsive strike from a pool (or billiard) cue, delivered such that the line of the strike passes through the center of the ball.

(a) How far does the ball travel before the friction can be said to be pure rolling friction?

(b) What is the speed of the ball at this moment?

2) Now, analyze and describe the motion for the case where the ball is struck horizontally at a height greater than 0.7 D above the point of contact between the ball and the table.

OSU Physics Dept. Comp. Exam #76. January 8-9, 1996 **PROB 7**

This problem deals with two hypothetical, uncharged, spin-zero particles called "A" and "B." The laws of nature are such that one "A" particle can decay into two "B" particles, *i.e.* $A \to B + B$. The Hamiltonian responsible for this decay is

$$H_I = \int_L d\boldsymbol{r}\ g\phi_B^{\dagger 2}(\boldsymbol{r},t)\phi_A(\boldsymbol{r},t)$$

where g is a constant.

1. Assuming that the initial A particle is at rest, calculate the momentum and energy of the decay particles. Do not make any non-relativistic approximations.

2. Use first-order perturbation theory to derive a formula for the amplitude of this decay in terms of g and the two masses m_A and m_B. Strictly speaking this is a problem in field theory, but you can get get the same answer by simply using plane wave states

$$\phi(\boldsymbol{r},t) = \frac{1}{\sqrt{2E_p L^3}} e^{-i(E_p t - \boldsymbol{r}\cdot\boldsymbol{p})}$$

   for ϕ_A and ϕ_B.

3. Calculate the rate for this decay.

Liquid helium boils at a temperature of T_0 when its vapor pressure is equal to p_0. Even though the liquid is shielded from the outside world, an amount of heat per second, $\dot{Q}$, flows into the liquid and evaporates some of it. One way to reduce the temperature of the liquid is to pump on it. Consider a mechanical pump that removes a constant volume of gas every second, $\dot{V}$, irrespective of the pressure of the gas. The latent heat per mole of the helium gas can be taken to be L, independent of temperature.

Hint: You can make use of the ideal gas equation of state in treating this problem.

(a) Give an expression for the minimum vapor pressure, p_m, which this pump can maintain over the surface of the liquid. You can assume that the helium vapor has reached room temperature, T_r, by the time it gets to the pump.

(b) If the liquid is thus maintained in equilibrium with its vapor at this pressure p_m, show that its approximate temperature can be given by

$$T_m = T_o\left(1 - \frac{T_o R}{L}\ln\frac{p_m}{p_o}\right)^{-1}$$

where R is the gas constant.

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