# OSU PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #72

Sept. 26 and 27, 1994

Comprehensive examination for Fall 1994

## PART I

### General Instructions

This Comprehensive Examination for Fall 1994 consists of eight problems of equal weight (20 points each). It has four parts. The first part (Problems 1-2) is handed out at 9:00 am on Monday, Sept. 26, and lasts three hours. The second part (Problems 3-4) will be handed out at 1:30 pm on the same day and will also last three hours. The third and fourth parts will be administered on Tuesday, Sept. 27.

Work carefully, indicate your reasoning, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit – especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, work each problem in its own numbered bluebook, and be certain that your chosen student letter (but not your name) is inside the back cover of every booklet. Be sure to make note of your student letter for use in the remaining parts of the examination.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers on the floor, except the exam, bluebooks and the collection of formulas and data distributed with the exam. Calculators are not allowed. Please return all bluebooks and formula sheets at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solutions. "Scratch" work will not be graded.

**Problem 1.** An observer moving with velocity $\boldsymbol{v}$ through a static electric field $\boldsymbol{E}$ experiences a magnetic field

$$\boldsymbol{B} = -\gamma\boldsymbol{\beta} \times \boldsymbol{E}.$$

As usual, $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$.

(a) Using this formula, derive an expression for the spin-orbit interaction energy of an electron with magnetic moment $\boldsymbol{\mu}$ moving in a classical orbit around a particle whose charge is $+e$. Note: We will get to quantum mechanics in part (b). Do this part of the problem in a purely classical and non-relativistic way, *i.e.* work to first order in $\beta$. This is not intended as an exercise in deriving the Thomas precession factor. If you know how to get the Thomas factor we will be terribly impressed; if not, just multiply the non-relativistic answer by 1/2.

(b) Use your results from (a) to derive the quantum mechanical interaction Hamiltonian for spin-orbit coupling.

(c) Derive a formula for the spin-orbit splitting in hydrogen. There will be a difficult radial integral that you do not have to work out. Indicate how your result depends on the quantum numbers.

**Problem 2.** A plane polarized monochromatic electromagnetic wave, frequency $\omega$, propagating in a dielectric medium with real index of refraction $n$ is normally incident upon a non-magnetic conductor where it undergoes reflection. If the conductor has complex index of refraction $n_c = n + in'$, show that the reflected electric field vector undergoes a phase shift $\phi$ where

$$\phi = \tan^{-1}\left(\frac{2n}{n'}\right).$$

Problem #1

1 (a)
6

$$\vec{B} \cong -\frac{\vec{v}}{c} \times \vec{E} = -\frac{\vec{p}}{mc} \times \vec{E}$$

$$H' = -\underset{\substack{\uparrow \\ \text{Thomas} \\ \text{factor}}}{\frac{1}{2}} \vec{\mu} \cdot \vec{B} = -\frac{1}{2} \frac{\vec{\mu}}{mc} \cdot (\vec{E} \times \vec{p})$$

$$\vec{E} = \frac{e\vec{r}}{r^3}$$

so $$H' = -\frac{\vec{\mu}\, e}{2mcr^3} \cdot (\vec{r} \times \vec{p}) = -\left(\frac{e}{2mcr^3}\right) \vec{\mu} \cdot \vec{L}$$

(b) For an electron $\vec{\mu} = \left(\frac{e}{mc}\right) \vec{S}$
6

(ignoring QED corrections). Now replace $\vec{L}$ & $\vec{S}$ by the operators $\hat{L}$, $\hat{S}$. Note that

$$\vec{J}^2 = (\vec{L}+\vec{S})^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L}\cdot\vec{S}$$

$$\hat{H}' = -\left(\frac{e^2}{2m^2c^2r^3}\right)\frac{1}{2}\left[\hat{J}^2 - \hat{L}^2 - \hat{S}^2\right]$$

(c) The first order corrections $E' = \langle \psi | H' | \psi \rangle$

8 let $f(r) = \frac{-e^2}{2m^2c^2r^3}$

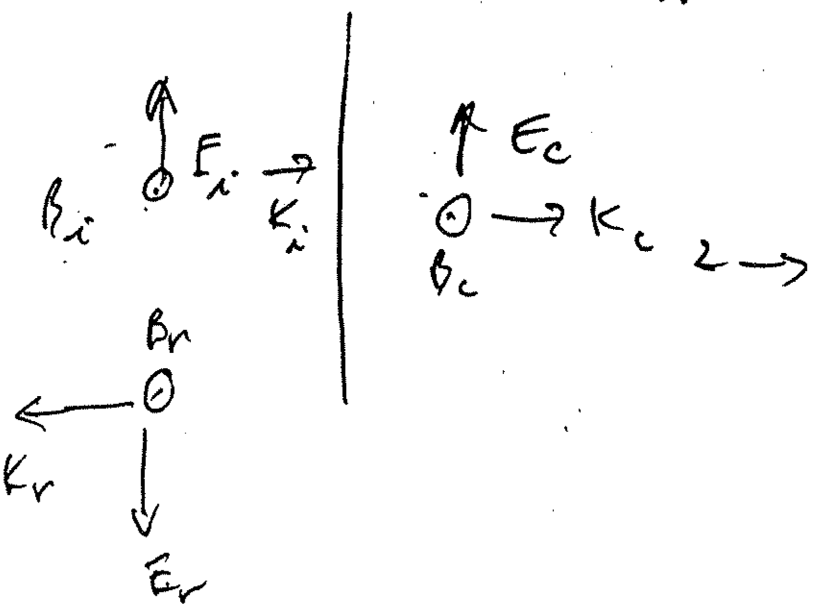
Then $$E' = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - \frac{3}{4}\right] \langle f(r) \rangle_{n,l}$$

Since $j = l \pm 1/2$

$$E'^{(+)} = \frac{\hbar^2}{2} l \langle f \rangle_{n,l} \quad \text{and} \quad E'^{(-)} = -\frac{\hbar^2}{2}(l+1)\langle f$$

$N = n$ $\qquad\qquad$ $N = n + in'$

$\vec{B} = \dfrac{\vec{k} \times \vec{E}}{\omega}$




(2)

Incident
$$\vec{E}_i = \vec{E}_{i0}\, e^{i(k_i z - \omega t)} \qquad k_i = \frac{\omega}{c} n$$
$$\vec{B}_i = \vec{B}_{i0}\, e^{i(k_i z - \omega t)} \quad (2)$$

$$|\vec{B}_{i0}| = \frac{n}{c} |E_{i0}| \quad (2)$$

reflect propagate $-z$ dir
$$\vec{E}_r = \vec{E}_{r0}\, e^{-i(k_r z + \omega t)} \qquad k_r = \frac{\omega}{c} n$$
$$\vec{B}_r = B_{r0}\, e^{-i(k_r z + \omega t)}$$

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In conductor
$$\vec{E}_{c} = \vec{E}_{c0}\, e^{i(k_c z - \omega t)}$$
$$\vec{B}_c = \vec{B}_{c0}\, e^{i(k_c z - \omega t)}$$

$$k_c = \frac{\omega}{c}(n + in')$$

$$|B_{c0}| = \frac{(n + in')}{c} |E_{c0}| \quad (2)$$

**Problem 3.** An electron in a one-dimensional potential well

$$V = \frac{1}{2}Kx^2$$

is placed in a constant, uniform electric field $E$ which points in the $x$ direction. The corresponding perturbation to the Hamiltonian is

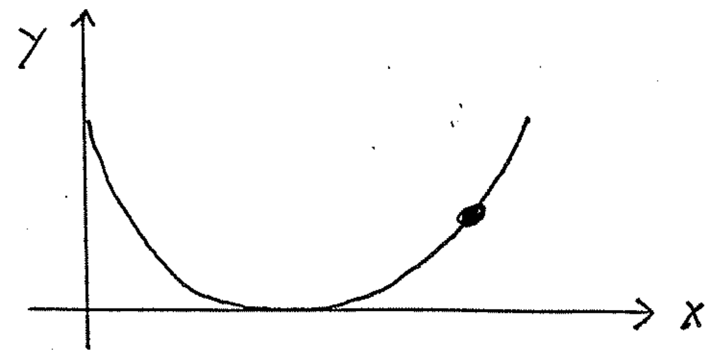
$$H' = eEx$$

(a) Find the exact eigenenergies of the full Hamiltonian.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}Kx^2 + eEx.$$

(b) Now solve the problem using stationary-state perturbation theory. Regard $H'$ as a perturbation to the simple harmonic oscillator Hamiltonian. Show that the first-order corrections to the energy vanish and that the second-order corrections give the exact result from part (a).

**Problem 4.** A bead of unit mass is confined to slide without friction along a wire in the vertical plane as shown in the diagram. The wire has




the shape of a cycloid with parametric equations

$$x = \theta - \sin\theta$$

$$y = 1 + \cos\theta,$$

where $0 \leq \theta \leq 2\pi$.

(a) Find the Lagrangian describing the bead's motion along the cycloid shaped wire in terms of the generalized coordinate $\theta$.

(b) Derive the equation of motion of the bead.

(c) Show that the bead executes small oscillations about the bottom of the trajectory with frequency

$$\omega = \frac{1}{2}\sqrt{g}$$

Problem # 2

2.(a) $V = \frac{1}{2}K x^2 + eEx$

$$= \frac{1}{2}K\left(x^2 + \frac{2eE}{K}x + \frac{e^2E^2}{K^2}\right) - \frac{e^2E^2}{2K}$$

$$= \frac{1}{2}K\xi^2 - \frac{e^2E^2}{2K}$$

where $\xi = x + \frac{eE}{K}$,

change variables $x \rightarrow \xi$ and we get the usual harmonic oscillator with a constant energy offset.

$$E = (n+1/2)\hbar\omega - \frac{e^2E^2}{2K}$$

$$\omega = \sqrt{K/m}$$

(b) $\hat{a} = \frac{\beta}{\sqrt{2}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$

$$\hat{a}^+ = \frac{\beta}{\sqrt{2}}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)$$

so $\hat{x} = \frac{\hat{a}+\hat{a}^+}{\sqrt{2}\,\beta}$ where $\beta^2 = m\omega_0/\hbar$

so we can write $H' = \frac{eE}{\sqrt{2}\,\beta}\left(\hat{a}+\hat{a}^+\right)$

Clearly $\langle n|\hat{H}'|n\rangle = 0$, so the first-order corrections vanish. To get the second-order correction we need the off-diagonal terms

$$H'_{ni} = \frac{eE}{\sqrt{2}\,\beta}\left\{\underbrace{\sqrt{i+1}\;\delta_{n,i+1}}_{i=n-1} + \underbrace{\sqrt{i}\;\delta_{n,i-1}}_{i=n+1}\right\}$$

Boundry cond.

$E_t$ cont

$$E_{i0} - E_{r0} = E_{c0} \quad ③$$

$H_t$ continuous $\quad \vec{B} = \mu \vec{H}$

Assume Non magnetic $\mu = \mu_0$

$$B_{i0} + B_{r0} = B_{c0} \quad ③$$

$$\Rightarrow E_{i0} + E_{r0} = \frac{(n + i n')}{n} E_{c0}$$

$$E_{r0} = \frac{(n + i n') - n}{(n + i n') + n} E_{i0} \quad ②$$

$$= \frac{i n'}{2n + i n'} E_{i0}$$

$$= \frac{i n' (2n - i n')}{4n^2 + n'^2} E_{i0} \quad ②$$

$$= \rho e^{i\phi} E_{i0}$$

($\rho$ is damping) not of interest here

Then $\varphi = + \frac{2nn'}{n'^2} = 2\frac{n}{n'}$ ②

Problem II 5 — continued

$$\text{Second order correction} = \sum_{i \neq n} \frac{|H'_{ni}|^2}{E_n^{(0)} - E_i^{(0)}}$$

$$= \frac{|H'_{n,n-1}|^2}{E_n^{(0)} - E_{n-1}^{(0)}} + \frac{|H'_{n,n+1}|^2}{E_n^{(0)} - E_{n+1}^{(0)}}$$

$$E_n^{(0)} = (n+1/2)\hbar\omega$$

$$\text{correction} = \left(eE/\sqrt{2}\,\beta\right)^2 \left\{ \frac{n}{\hbar\omega} - \frac{n+1}{\hbar\omega} \right\}$$

$$= \frac{-e^2E^2}{2\beta^2\hbar\omega} = \frac{-e^2E^2}{2k}$$

(1) Problem #4

$$T = \frac{1}{2}(\dot{x}^2 + \dot{y}^2)$$

$\left\{ \begin{array}{l} \text{constraint} \\ x = \theta - \sin\theta \\ y = 1 + \cos\theta \end{array} \right\}$

$$= \frac{1}{2}\left\{ (1-\cos\theta)^2 \dot\theta^2 + [-\sin\theta\,\dot\theta]^2 \right\}$$

$$= (1-\cos\theta)\,\dot\theta^2$$

$$V = gy$$

$$= g(1+\cos\theta)$$

(a) $L = (1-\cos\theta)\dot\theta^2 - g(1+\cos\theta)$

Equation of Motion

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\theta} - \frac{\partial L}{\partial\theta} = 0$$

(b) $\Rightarrow ((1-\cos\theta)\ddot\theta + \frac{1}{2}\sin\theta\,\dot\theta^2 - \frac{g}{2}\sin\theta = 0$

(c) Minimum potential energy is at

$\theta = \pi$

Expanding about $\theta = \pi$ for small parametric displacements (generalized coord)

$$2\ddot\theta + \frac{\pi}{2}\dot\theta^2 + \frac{g}{2}\theta - \frac{g\pi}{2} = 0$$

$$\Rightarrow \omega^2 = \frac{g}{4}$$

**Problem 5.** White dwarf stars may be regarded as highly degenerate, non-relativistic electron systems in a non-degenerate positive background of protons. Moreover, if the white dwarf is regarded as a sphere with mass $M$ and radius $R$, then its gravitational selfenergy $U$ may be taken as

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

where $G$ is the gravitational constant.

(a) Show that the electronic contribution to the Grand Thermodynamic Potential, $\Omega$, for the white dwarf star is

$$\Omega_{\text{elec}} = -\frac{V}{15\pi^2}\left(\frac{\hbar^2}{m}\right)\left(\frac{2m\mu_\circ}{\hbar^2}\right)^{\frac{2}{5}}$$

where $m$ is the electron mass, $V$ is the star's volume, and $\mu_\circ$ is the electron chemical potential at $T = 0° \ K$. (For this highly degenerate system assume the limit $T = 0° \ K$.)

In a mean field approximation, the Helmholtz free energy $F$, for the white dwarf may be constructed as the sum of the Helmholtz free energy of the degenerate electrons, $F_{elec}$, and the gravitational potential energy, $U$, of the star $F_{MF} = F_{\text{elec}} + U$.

(b) Show that the electronic contribution to the Helmholtz free energy is

$$F_{\text{elec}} = \frac{V}{10\pi^2}\left(\frac{\hbar^2}{m}\right)\left(\frac{2m\mu_\circ}{\hbar^2}\right)^{\frac{2}{5}}$$

(c) Assuming the star has a mass $M$, where $M = N_{\text{protons}}\, m_{\text{proton}}$, with $N_{\text{protons}}$ the number of protons and $m_{\text{proton}}$ is the mass of the proton, minimize the mean field Helmholtz free energy $F_{MF}$ with respect to the white dwarf's radius and show that a white dwarf satisfies the universal relation

$$M^{1/3}R = \text{constant}$$

**Problem 6.** A flux of uncharged, spherical dust grains approaches the solar system from interstellar space. For simplicity, assume that all the grains are moving directly toward the sun, *i.e.* they are moving along a radius vector drawn from the center of the sun. They are acted upon by the gravitational attraction of the sun and by radiation pressure due to the electromagnetic radiation from the sun.

(a) Show that the net force acting on the grains can be written as

$$F_{\text{net}} = \frac{GM_{\odot}m}{r^2}(1-\beta).$$

The term $\beta$ represents the effect of radiation pressure. Derive an expression for $\beta$ in terms of the solar mass $M_{\odot}$, radius $R_{\odot}$, and surface temperature $T_{\odot}$, as well as the emissivity $\epsilon$, mass $m$, and radius $r_{o}$ of the dust grain.

(b) Find an expression for the radius of a grain that will pass straight through into the sun with its motion unaffected.

Problem #5

$$Z_{gr} = \prod_i \left(1 + e^{-\beta(\epsilon_i - \mu)}\right)$$

$$\Omega = -\frac{1}{\beta} \log Z$$

$$= -\frac{1}{\beta}\, 2 \sum_i \log\left(1 + e^{-\beta(\epsilon_i - \mu)}\right)$$

$$= -\frac{1}{\beta} \times 2 \times \frac{V}{(2\pi)^3}\, 4\pi \int dk\, k^2 \log\left(1 + e^{-\beta\left(\frac{\hbar^2 k^2}{2m} - \mu\right)}\right)$$

Integrate by parts

$$\Omega = -\frac{1}{\beta} \times 2 \times \frac{V}{(2\pi)^3}\, 4\pi\, \beta \int dk\, \frac{k^4}{3}\left(\frac{\hbar^2}{m}\right) \frac{1}{e^{\beta\left(\frac{\hbar^2 k^2}{2m} - \mu\right)} + 1}$$

AT $T = 0$

(a)
$$\Omega = -\frac{1}{3}\left(\frac{\hbar^2}{m}\right)\frac{V}{\pi^2} \int_0^{K_F} dk\, K^4$$

$$= -\frac{1}{3} \times \frac{1}{5}\left(\frac{\hbar^2}{m}\right)\frac{V}{\pi^2} K_F^5 = -\frac{1}{15}\left(\frac{\hbar^2}{m}\right)\frac{V}{\pi^2}\left(\frac{2m}{\hbar^2}\mu_0\right)^{5/2}$$

$$\mu_0 = \frac{\hbar^2 K_F^2}{2m}$$

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(b)
$$\Omega = E - TS - \mu N$$

$$= F - \mu N$$

$$F_{elec.} = \Omega + \mu N$$

$$\mu = \frac{\hbar^2 K_F^2}{2m}$$

Problem 3.

$$N = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{T,V} \qquad \mu = \frac{\hbar^2 k_F^2}{2m}$$

$$\frac{\partial \mu}{\partial k_F} = \frac{2\hbar^2}{2m} k_F$$

$$= - \frac{\partial \Omega}{\partial k_F} \frac{\partial k_F}{\partial \mu}$$

$$= - \frac{\partial \Omega}{\partial k_F} \left(\frac{\partial \mu}{\partial k_F}\right)^{-1}$$

$$= \frac{\frac{1}{15} \times 5 \left(\frac{\hbar^2}{m}\right) \frac{V}{\pi^2} k_F^4}{\left(\frac{\hbar^2}{m}\right) k_F}$$

$$= \frac{1}{3} \cancel{\left(\frac{\hbar^2}{m}\right)} \frac{V}{\pi^2} k_F^3$$

$$F_{elec.} = -\frac{1}{15} \left(\frac{\hbar^2}{m}\right) \frac{V}{\pi^2} k_F^5 + \frac{\hbar^2}{2m} k_F^2 \left(\frac{1}{3}\right) \frac{V}{\pi^2} k_F^3$$

$$= \left(\frac{1}{6} - \frac{1}{15}\right) \left(\frac{\hbar^2}{m}\right)\left(\frac{V}{\pi^2}\right) k_F^5$$

$$= \frac{1}{10} \left(\frac{\hbar^2}{m}\right)\left(\frac{V}{\pi^2}\right) k_F^5$$

But $k_F^5 = \left(\frac{N}{V}\right)^{5/3} (3\pi^2)^{5/3}$

and $M = N_{proton} M_{proton}$ $\qquad N_{proton} = N_{electron}$

$= N \, \mu_{proton}$

So $N = \frac{M}{\mu_{proton}}$

#5 - cont.

$$F_{elect} = \frac{1}{10}\left(\frac{\hbar^2}{m}\right)\frac{1}{\pi^2}\,\frac{V}{V^{5/3}}\,\frac{M^{5/3}}{(\mu_{proton})^{5/3}}\,(3\pi^2)^{5/3}$$

$$= \frac{1}{10}\,\frac{\hbar^2}{\pi^2 m}\,\frac{1}{V^{2/3}}\,M^{5/3}\left(\frac{3\pi^2}{\mu_{proton}}\right)^{5/3}$$

$$F_{white\ Dwarf} = \frac{1}{10}\,\frac{\hbar^2}{\pi^2 m}\,\frac{1}{\left(\frac{4}{3}\pi\right)^{2/3}R^2}\,M^{5/3}\left(\frac{3\pi^2}{\mu_{proton}}\right)^{5/3}$$

$$- \frac{3}{5}\,\frac{M^2 G}{R}$$

$$\frac{\partial F}{\partial R} = 0 = \frac{1}{10}\,\frac{\hbar^2}{\pi^2 m}\left(\frac{4}{3}\pi\right)^{-2/3}(-2)\frac{1}{R^3}\,M^{5/3}\left(\frac{3\pi^2}{\mu_{proton}}\right)^{5/3}$$

$$+ \frac{3}{5}\,\frac{M^2 G}{R^2}$$

$$M^{1/3}R = \frac{1}{6}\,\frac{1}{G}\,\frac{\hbar^2}{m\pi^2}\left(\frac{4}{3}\pi\right)^{-2/3}\left(\frac{3\pi^2}{\mu_{proton}}\right)^{5/3}$$

$$= \frac{1}{6}\,\frac{\pi^{2/3}}{G}\,\frac{\hbar^2}{m}\left(\frac{4}{3}\right)^{-2/3}\left(\frac{3}{\mu_{proton}}\right)^{5/3}$$

#6 $F_{rad} = \frac{L}{c} A$ ; $L = \sigma T_\odot^4 \left(\frac{R_\odot}{r}\right)^2$

(a) $\therefore$ $F_{rad} = \frac{\pi R^2}{c} \sigma T_\odot^4 \frac{R_\odot^2}{r^2}$

$F_{gr} = \frac{4}{3}\pi G M_\odot \rho \frac{R^3}{r^2}$

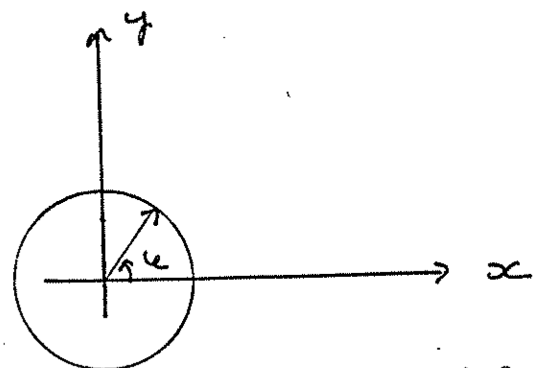
same $\frac{1}{r^2}$ dep.
also central forces approx.

$\therefore$ $F_{res} = \frac{G M_\odot m}{r^2}(1-\beta)$, where $\beta = \frac{3\sigma T_\odot^4 R_\odot^2}{4\rho c G M_\odot R}$.

(b) unaffected motion means $\beta = 1$

$\therefore$ $R = \frac{3\sigma T_\odot^4 R_\odot^2}{4\rho c G M_\odot} \approx 0.6\,\mu m.$

**Problem 7.** A very long cylindrical surface carries the surface current density $\boldsymbol{k} = \boldsymbol{z} k_o \cos\varphi$. The radius of the cylinder is $R$, and the interior and exterior regions are vacuum with $\mu = 1$.




(a) In which direction does the vector potential $\boldsymbol{A}$ point?
(b) What are the relevant boundary conditions?
(c) Find the vector potential $\boldsymbol{A}$ and the field $\boldsymbol{B}$ for $\rho > R$ and $\rho < R$.

**Problem 8.** A magnetic field can exist in some superconductors as a result of the formation of a lattice of singularities called vortices. The microscopic magnetic field vector $\boldsymbol{b}$ in such a superconductor is described by the famous London equation,

$$\boldsymbol{b}(\boldsymbol{r}) + \lambda^2 \nabla \times (\nabla \times \boldsymbol{b}(\boldsymbol{r})) = \boldsymbol{\phi}_\circ \delta^{(2)}(\boldsymbol{r})$$

Here $\delta^{(2)}(\boldsymbol{r})$ is the two-dimensional Dirac delta function placing a single vortex along the line $x = 0$, $y = 0$, $\boldsymbol{\phi}_\circ$ is a vortex vector with magnitude $|\boldsymbol{\phi}_\circ| = h/2e$ (where $\boldsymbol{\phi}_\circ$ is a unit of magnetic flux) taken to point in the $z$-direction and $\lambda$ is a constant called the penetration depth.

Consider a beam of neutrons polarized with spins parallel to $\boldsymbol{b}$ and normally incident (propagation vector $\boldsymbol{q}$ perpendicular to $\boldsymbol{\phi}_\circ$) upon a superconductor supporting only a single vortex line.

(a) Find the Fourier components of the magnetic field $\boldsymbol{b}$ and write an expression for the Fourier expansion of the Hamiltonian of interaction between these neutrons and a vortex line. (Hint: $\nabla \cdot \boldsymbol{b} = 0$)

(b) Derive an expression in the Born approximation for the scattering amplitude of elastically scattered neutrons in terms of $\boldsymbol{q}$ and $\boldsymbol{q}'$ (the incident and scattered wave vectors.)

(c) Find the differential scattering cross-section $\frac{d\Gamma_{\boldsymbol{q}-\boldsymbol{q}'}}{d\theta_{\boldsymbol{q}'}}$ for these neutrons in terms of $|\boldsymbol{q}|$ and $\theta$ where $d\theta_{\boldsymbol{q}'}$ is the angle subtended by the scattered neutrons, and $\theta$ is the angle between the incident and scattered neutron wave vectors.

[redacted]

Problem 7

a) $\vec{A} = A\hat{z}$ (2)

b) $B \to 0$ as $\rho \to \infty$, $B$ = finite at $\rho = 0$; $A$ and normal $B$ are continuous and tangential $H$ is discontinuous due to the surface current. (4)

c) $\vec{B} = \vec{\nabla} \times \vec{A}$ and $\vec{\nabla} \times \vec{B} = 4\pi \vec{J}/c \Rightarrow \nabla^2 \vec{A} = -4\pi \vec{J}/c$ if $\vec{\nabla} \cdot \vec{A} = 0$.
So, need solutions to $\nabla^2 \vec{A} = 0$ for $\rho < R$ and $\rho > R$. (2)

$$A(\rho, \varphi) = C_0 + \sum_{n=1}^{\infty} [S_n \rho^n \sin n\varphi + C_n \rho^n \cos n\varphi] \text{ for } \rho < R \quad (2)$$

$$= C_0' + D \ln \rho + \sum_n [S_n' \rho^{-n} \sin n\varphi + C_n' \rho^{-n} \cos n\varphi] \text{ for } \rho \geq R \quad (2)$$

Apply continuity of $A$ at $\rho = R$ (2) and discontinuity of tangential $B$. (2)

$$\left.\frac{\partial A}{\partial \rho}\right|_{\rho=R} - \left.\frac{\partial A}{\partial \rho}\right|_{\rho=R} = \frac{4\pi k_0 \cos\varphi}{c}$$

So $A = \frac{4\pi k_0}{c} \frac{\rho}{2} \cos\varphi$ for $\rho < R$ and $A = \frac{4\pi}{c} \frac{k_0}{2} \frac{R^2}{\rho} \cos\varphi$ for $\rho > R$ (2)

$$\vec{B} = \vec{\nabla} \times \vec{A} = \hat{\rho} \frac{1}{\rho} \frac{\partial A_z}{\partial \varphi} - \hat{\varphi} \frac{\partial A_z}{\partial \rho}$$

for $\rho < R$, $\vec{B} = -\frac{4\pi k_0}{2c} [\hat{\rho} \sin\varphi + \hat{\varphi} \cos\varphi]$ (2)

for $\rho > R$, $\vec{B} = \frac{4\pi k_0 R^2}{2c\rho^2} [-\hat{\rho} \sin\varphi + \hat{\varphi} \cos\varphi]$

The problem can also be solved by integrating $\vec{A}(\vec{\rho}) = \int \frac{\vec{J}(\vec{\rho}\,')}{|\vec{\rho} - \vec{\rho}\,'|} da'$ using the two-dimensional expansion for $|\vec{\rho} - \vec{\rho}\,'|^{-1}$!

#8

$$f(\theta) = \frac{m}{2\pi\hbar^2} \langle k'|V|k\rangle$$

$$FT[\vec{b}] + FT[\lambda^2 \vec{\nabla}\times\vec{\nabla}\times\vec{b}] = FT[\phi_0\, \delta^{(2)}(\vec{r})]$$

using $\vec{\nabla}\times\vec{\nabla}\times = \vec{\nabla}(\vec{\nabla}\cdot - \nabla^2$, yields

$$b_{\underline{k}}(1+\lambda^2 k^2) = \phi_0$$

In our case $\underline{k} \to \underline{q}$, where $\underline{q} = \underline{k}' - \underline{k}$.

and since $V = \mu_N b(\vec{r})$, one gets

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \left[\frac{0.955}{1+\lambda^2 4k^2 \sin^2\theta/2}\right]^2,$$

since Elastic scattering means $|q| = 2k\sin\theta/2$