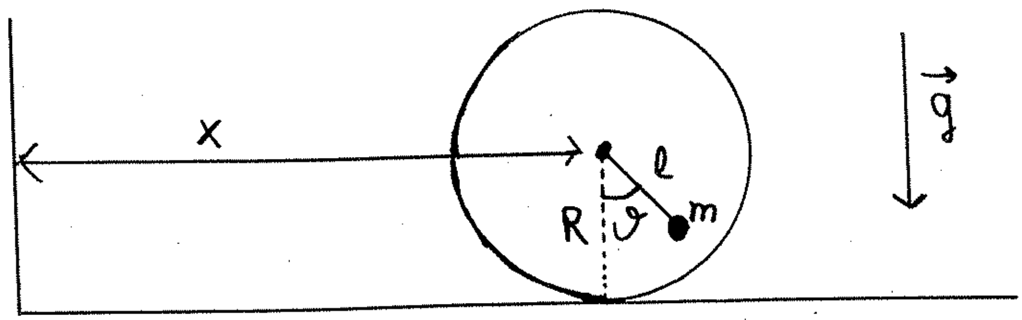
Physics Department Comprehensive Examination #64.

Part I. Tuesday 7 January 1992, 1-4pm.

Problem 1.

A uniform disk, having mass M and radius R, can roll without slipping on a horizontal surface. A frictionless pendulum, made from a mass m and a massless rod of length l (l<R), swings from the axis of the disk. The motion takes place in a vertical plane, with a downward gravitational field $\vec{g}$.



a. Find Lagrange's equations of motion. Do NOT assume $\theta \ll 1$.

b. Calculate the frequency ω_0 for small oscillations of the pendulum: $\theta \ll 1$.

Problem 2.

A one-dimensional quantum mechanical harmonic oscillator is described by the Schrödinger equation $H\psi(x)=(-\frac{d^2}{dx^2}+x^2)\psi(x)=E\psi(x)$. Consider the operator $A=x+\frac{d}{dx}$.

a. What is the Hermitian conjugate, $A^\dagger$, of A?

b. Calculate the commutator $[A^\dagger,A]$.

c. Express H in terms of A and $A^\dagger$.

d. Show that $E \geq 1$.

e. Find the ground-state energy and wave function of the Hamiltonian H.

f. Calculate the ground-state energy and wave function of the Hamiltonian $H'=H+i\alpha\frac{d}{dx}$, with α real.

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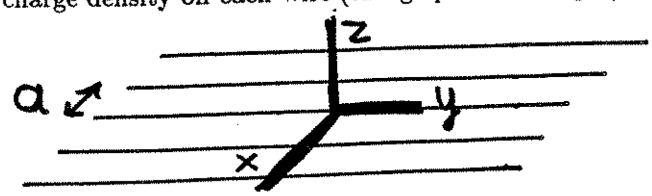
Problem 3.

Consider an infinite array of infinitely long, infinitely thin, parallel wires lying in the x-y plane (z=0). The wires run parallel to the y-axis with uniform spacing, a, between them. One wire coincides with the y-axis. The linear charge density on each wire (charge per unit length) has a constant value, λ.



Assume that the scalar potential for z>0 can be expanded in a Fourier series of the form:

$$\phi(x,y,z)=\sum_{n=0}^{\infty} F_n(z)\cos\left(\frac{2\pi nx}{a}\right)$$

a. Find the most general functional form for $F_n(z)$ for every n for this problem.

b. Argue that one of the two constants of integration in each $F_n(z)$ can or should be set equal to zero.

c. How far from the grid must you be for the sinusoidal variation in the potential to fall off by at least a factor of $\frac{1}{e}$?

d. Write an expression for the surface charge density $\sigma(x,y)$ on the x-y plane, z=0, due to the charged wires.

e. Calculate the unknown constants in part (a).

Problem 4.

Each member of a family of 4 opens the refrigerator 8 times a day. Every time it is opened, 75% of the 1 m^3 of air in the refrigerator is replaced by warm air from the room. In addition to the air, the refrigerator contains 200 liters of food and beverages, which are essentially all water. A thermostat keeps the inside of the refrigerator at 2°C, and the house is kept at 20°C. The refrigerator's insulation is ideal, so no heat is lost through the walls or door (when closed). Assume that the refrigerator is cooled by a heat pump that is ideally efficient, discharging the heat inside the house. The vapor pressure of water at 20°C is 0.1 atm; at 2°C it is negligible. The heat of vaporization of water is 2.26 MJ/kg. The ideal gas constant is R=8.31 J-mol^{-1}K^{-1}. A mole of gas occupies 22.4 liter at 0°C (273K). Give answers to 10% accuracy.

a. What are the molar heat capacities C_p and C_v for air?

b. What is the average power needed to operate the refrigerator, if the air in the house is dry?

c. Now suppose the air in the house is saturated with water vapor. How much additional power is needed?

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Part 2. Wednesday 8 January 1992, 1-4pm.

Problem 5.

Schrödinger's equation in one dimension has the form $\{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\}\ \psi(x) = E\psi(x)$. The potential is given by $V(x) = -g\{\delta(x-\frac{1}{2}a)+\delta(x+\frac{1}{2}a)\}$, with $g>0$, $a>0$.

a. Find the energy of all bound states ($E<0$) for this problem. If you need to solve a transcedental equation, you may indicate how to do it graphically.

b. For each bound state, make a sketch of the wave function.

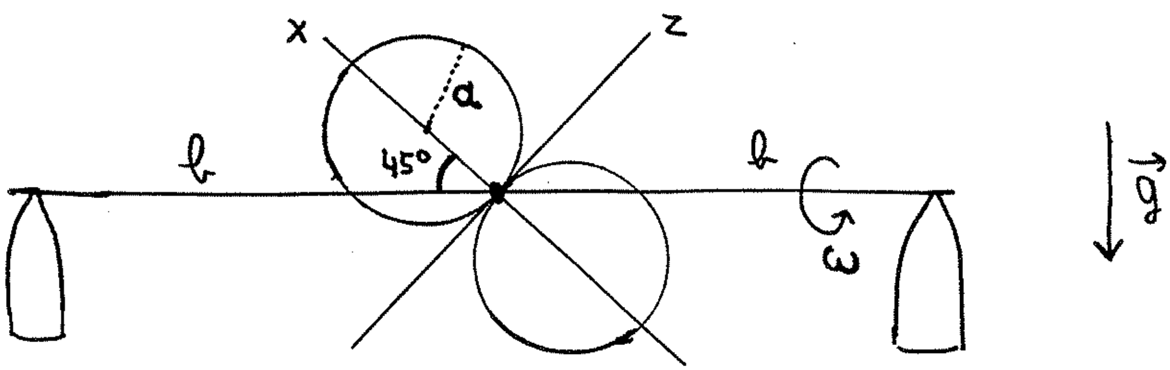
c. How is the symmetry of the potential reflected in the wave functions?

d. Calculate the energy of the ground state in the limit $a \to 0$.

e. Is the last result related to the results for the ground state of a single delta-function? If yes, how are they related? If no, why are they not related?

Problem 6.

Two identical homogeneous solid spheres, each having a mass m and radius a, are held rigidly in contact and rotate about a horizontal axle of negligible mass that passes through the point of contact and makes a 45° angle with the line through the centers. The point of contact for the spheres is located at the midpoint of the axle, located a distance b from each bearing. Gravity acts downward.



a. Find the inertia tensor with respect to the xyz coordinate system which rotates with the spheres.

b. Find the torque $\vec{N}$ on the system due to the bearings.

c. What is the largest angular velocity ω with which the system can rotate without tending to lift one of the bearings?

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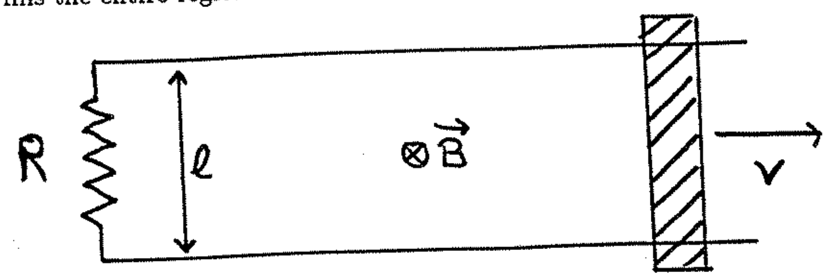
Problem 7.

A metal bar of mass m slides frictionlessly on two parallel conducting rails, which are a distance l apart. A resistor R is connected across the rails. A uniform magnetic induction B, perpendicular to the plane of the rails, fills the entire region.



In this picture, the direction of the magnetic induction is into the paper.

a. If the bar moves to the right at speed v, what is the current in the resistor? In which direction does it flow?

b. What is the magnetic force on the bar? In which direction?

c. If the bar starts out with speed v_0 at time t=0, and is left to slide, what is the speed at a later time t?

d. Prove that energy is conserved in this process by showing that the kinetic energy lost by the bar at time t is exactly equal to the energy gained elsewhere.

Problem 8.

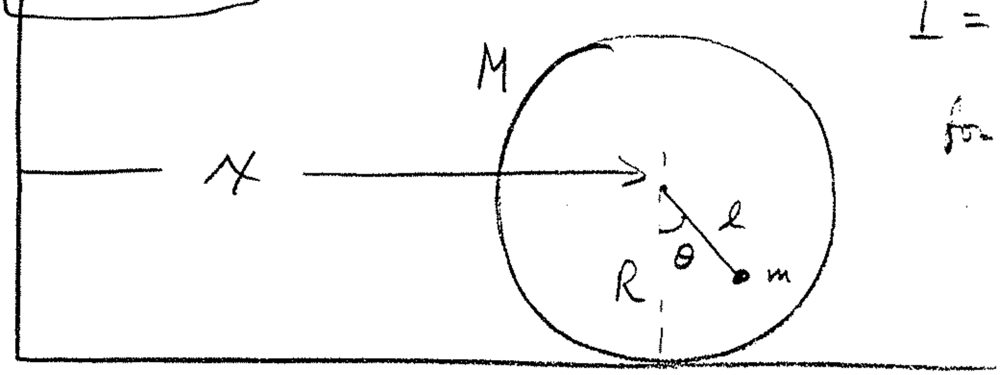
Consider a collection of droplets of liquid dispersed in a vapor of the same material (fog). Assume that the droplets and the vapor are in equilibrium with a reservoir of the liquid.

a. What is the appropriate thermodynamic potential or free energy X to describe this system?

b. The potential X of a droplet of N molecules is given by $X=\alpha N+\sigma S$, where S(N) is the surface area and σ is the surface tension. What is the physical meaning of α?

c. Show that the number of droplets containing N molecules is proportional to $\exp(-\sigma S)$. Why does it not depend on α?

(K) Problem 1



$I = \frac{1}{2} MR^2$ for disk.

(a) $L = T - U$.

$$T = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} \overset{\frac{1}{2}MR^2}{I}\, \underset{\omega = \frac{\dot{x}}{R}\ (\text{rolling})}{\omega^2} + \frac{1}{2} m\left[\left(\frac{d}{dt}(x + l\sin\theta)\right)^2 + \left(\frac{d}{dt}(l\cos\theta)\right)^2\right]$$

$$= \frac{1}{2} M\dot{x}^2 + \frac{1}{4} M\dot{x}^2 + \frac{1}{2} m\left[\left(\dot{x} + l\cos\theta\,\dot{\theta}\right)^2 + \left(l\sin\theta\,\dot{\theta}\right)^2\right]$$

$$U = -mgl\cos\theta$$

$$\therefore\ L = \frac{3}{4} M\dot{x}^2 + \frac{m}{2}\left(\dot{x}^2 + 2l\cos\theta\,\dot{x}\dot{\theta} + l^2\dot{\theta}^2\right) + mgl\cos\theta$$

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \implies 0 = \frac{d}{dt}\left(\frac{3}{2} M\dot{x} + m\dot{x} + ml\cos\theta\,\dot{\theta}\right)$$

or $\boxed{\frac{3}{2} M\dot{x} + m\dot{x} + ml\cos\theta\,\dot{\theta} = \text{constant.}}$

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} \implies -ml\sin\theta\,\dot{x}\dot{\theta} - mgl\sin\theta = \frac{d}{dt}\left[ml\cos\theta\,\dot{x} + ml^2\dot{\theta}\right]$$

so that $\cos\theta\,\ddot{x} - \cancel{\sin\theta\,\dot{x}\dot{\theta}} + l\ddot{\theta} + \cancel{\sin\theta\,\dot{x}\dot{\theta}} + g\sin\theta = 0$

or $\boxed{\cos\theta\,\ddot{x} + l\ddot{\theta} + g\sin\theta = 0}$

(b) If $\theta \ll 1$: $\cos\theta \cong 1$; $\sin\theta \cong \theta$

Then
$$\begin{cases} \frac{3}{2}M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = 0 \\ \ddot{x} + l\ddot{\theta} + g\theta = 0. \end{cases}$$
and

From first equation, $\ddot{x} = \dfrac{-ml\ddot{\theta}}{m + \frac{3}{2}M}$

$$\therefore \quad \frac{-ml\ddot{\theta}}{m + \frac{3}{2}M} + l\ddot{\theta} + g\theta = 0$$

$$l\underbrace{\left(1 - \frac{m}{m + \frac{3}{2}M}\right)}_{\frac{3}{2}M/(m + \frac{3}{2}M)}\ddot{\theta} + g\theta = 0$$

or $$\ddot{\theta} + \left(\frac{m + \frac{3}{2}M}{\frac{3}{2}M}\,\frac{g}{l}\right)\theta = 0$$

This is a harmonic oscillator differential equation with angular frequency given by $\omega_0^2 = (\ )$

or $$\boxed{\omega_0 = \sqrt{\frac{2m + 3M}{3M}\,\frac{g}{l}}}$$

$E_n = E_n^{(0)} + \langle n|W|n\rangle + \sum_k \dfrac{\langle k|W|n\rangle}{E_n^{(0)} - E_k^{(0)}}$

$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)}$

Problem 2

a) $$\int_{-\infty}^{\infty} \varphi^*(x)\left\{x+\frac{d}{dx}\right\}\psi(x)\,dx = \int_{-\infty}^{\infty}\left[\left(x-\frac{d}{dx}\right)\varphi(x)\right]^*\psi(x)\,dx$$

$$A^\dagger = x - \frac{d}{dx}$$

b) $$[A^\dagger, A]\varphi(x) = \left[\left(x-\frac{d}{dx}\right)\left(x+\frac{d}{dx}\right) - \left(x+\frac{d}{dx}\right)\left(x-\frac{d}{dx}\right)\right]\varphi(x) = -2\varphi(x)$$

$$[A^\dagger, A] = -2$$

c) $$H = A^\dagger A + 1$$

d) $$E = E\int|\psi|^2dx = \int dx\,\psi^\dagger(x)\{A^\dagger A+1\}\psi(x)$$

$$= \int dx\,|A\psi(x)|^2 + 1 \geq 1$$

e) Ground state needs $A\psi_0(x) = 0 \Rightarrow H\psi_0(x) = \psi_0(x)$

$E_g = 1$

$$\left(x+\frac{d}{dx}\right)\psi(x) = 0 \Rightarrow \psi(x) = \alpha e^{-\frac{1}{2}x^2}$$

$$\int|\psi|^2dx = 1 \Rightarrow \alpha.$$

f) $$H' = \left(A - \tfrac{1}{2}ia\right)^\dagger\left(A - \tfrac{1}{2}ia\right) + 1 - \tfrac{1}{4}a^2$$

since $$\frac{d}{dx} = \frac{A-A^\dagger}{2}$$

need $$\left(A - \tfrac{1}{2}ia\right)\psi_0(x) = 0 \Rightarrow E_0 = 1 - \frac{a^2}{4}$$

$$\psi_0(x) = \alpha e^{-\frac{1}{2}\left(x-\frac{1}{2}ia\right)^2}$$

Problem 3

a) $\phi(x,y,z) = \phi(x,z)$ must satisfy Laplace's eqn.

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}\left[-\frac{4\pi^2 n^2}{a^2}\cos\frac{2\pi n x}{a}F_n(z) + \frac{\partial^2 F_n(z)}{\partial z^2}\cos\frac{2\pi n x}{a}\right] = 0$$

$\cos\frac{2\pi n x}{a}$ are independent $\Rightarrow$ remove sum

$$\Rightarrow \frac{d^2 F_n}{dz^2} - \frac{4\pi^2 n^2}{a^2}F_n = 0$$

$$F_n = A_n e^{\frac{2\pi n z}{a}} + B_n e^{-\frac{2\pi n z}{a}} \qquad n \neq 0$$

$$F_0 = A_0 z + B_0$$

b) Let $B_0 = 0$ since an additive const in ϕ is physically irrelevant.
$A_n = 0$ since ϕ cannot increase as you move away from the wires

c) $F_n = B_n e^{-\frac{2\pi n z}{a}}$

F_1 falls off by a factor of $\frac{1}{e}$ when $z = \frac{a}{2\pi}$, less than $\frac{1}{4}$ of a grid spacing! Higher harmonics fall off even faster.

d) $\sigma = \lambda \sum_{l=-\infty}^{\infty} \delta(x - la)$

3 e) $\sigma = -\frac{1}{4\pi} \frac{\partial \Phi}{\partial z}\Big|_{z=0}$

$$= -\frac{1}{4\pi}\left[A_0 - \sum_{n=1}^{\infty} \frac{2\pi n}{a} B_n \cos\frac{2\pi n x}{a}\right] = \lambda \sum_{\ell=-\infty}^{\infty} \delta(x-\ell a)$$

$$\cancel{\int_{-\frac{a}{2}}^{\frac{a}{2}} \lambda\,\delta(x)\cos\frac{2\pi \ell x}{a}}\,dx$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} \lambda\,\delta(x)\cos\frac{2\pi m x}{a}\,dx = \lambda$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} A_0 \cos\frac{2\pi m x}{a}\,dx = A_0\, \frac{a}{2}\, \delta_{m,0} \Rightarrow A_0 = -\frac{4\pi\lambda}{a}$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{2\pi n}{a} B_n \cos\frac{2\pi n x}{a}\cos\frac{2\pi m x}{a}\,dx = \frac{2\pi n}{a} B_n \frac{a}{2}\delta_{n,m}$$

$$\Rightarrow B_n = -4\pi\,\frac{a}{2\pi n}\,\frac{2}{a}\,\lambda = -\frac{4\lambda}{n}$$

$$\boxed{A_0 = -\frac{4\pi\lambda}{a}, \quad B_n = -\frac{4\lambda}{n}}$$

Physics Department Comprehensive Exam # 63, January 7-8, 1992

SOLUTIONS

Problem #4

(a) Air is nearly an ideal non-interacting gas. $C_p = C_V + R$, $C_V = \frac{5}{2}R$ for diatomic molecules: count R/2 for each degree of freedom, 3 translation + 2 rotation since axially symmetric.

(b) let dQ_{out}/dt = rate at which heat is added to air. $\Delta Q_{out} = N\ C_p\ \Delta T$, because refrigerator is probably not pressure-sealed,otherwise it would be C_V. $dQ_{out}/dt = C_p \Delta T\ dN/dt$, ΔT = 18 K.

$$\frac{dN}{dt} = \frac{4 \text{ x } 8 \text{ x } 0.75 \text{ x } 1 \text{ m}^3}{24 \text{ hours}} = \frac{1 \text{ m}^3}{\text{hr}} = \frac{1000\ l}{3600 \text{ sec}} = \frac{1}{3.6}\ l/\text{sec} = \frac{1 \text{ mole}}{3.6 \text{ x } 22.4 \text{ sec}} = 0.0124 \frac{m}{s}$$

$$\frac{dQ_{out}}{dt} = C_p \text{ x } \frac{18}{3.6\text{x}22.4} \frac{\text{K moles}}{\text{sec}} = 0.23 \frac{\text{k moles}}{\text{sec}} \text{ x } C_p$$

rate at which energy is used = dQ_{in}/dt = (efficiency) x dQ_{out}/dt, efficiency = $\frac{T_2 - T_1}{T_1} = \frac{18 \text{ K}}{275 \text{ K}}$

$$\text{power used} = \frac{dQ_{in}}{dt} = \frac{18}{275} \text{ x } \frac{1}{4.48} \frac{\text{K moles}}{\text{sec}} \text{ x } C_p = \frac{18}{275} \text{ x } \frac{1}{4.48} \frac{\text{K moles}}{\text{sec}} \text{ x } \frac{7}{2} \text{ x } 8.31 \frac{\text{J}}{\text{mole K}}$$

$$= 0.425 \text{ J/sec}$$

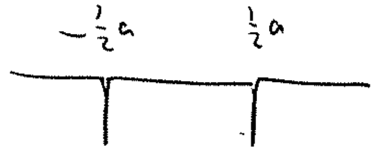
(c) When the warm moist air enters the refrigerator, the moisure it contains will be cooled and then will condense to the liquid phase. At the end the refrigerator will contain the same amount of cool dry air, which it takes the same power to supply as in part b. In addition the heat energy has to be supplied to cool and condense the water vapor:

$$\frac{dQ_{out}}{dt} = \frac{dN}{dt}(C_p \Delta T + \Delta Q_V)\text{ , } \Delta Q_V = \text{heat of vaporization per mole}$$

C_p = 4 R for water since its molecule is not axially symmetric so all 3 rotations exist

$dN/dt = 0.1\ dN/dt$ (above), ΔQ_V = 2.26 MJ/kg x 18 g/mole

Problem 5.

$V(x)$ 

$$\frac{\hbar^2 \kappa^2}{2m} = -E \qquad E<0$$

Bound state: $\quad x < -\frac{1}{2}a \qquad \psi(x) = A\, e^{\kappa x}$

$$-\tfrac{1}{2}a < x < \tfrac{1}{2}a \qquad \psi(x) = B\, e^{-\kappa x} + C\, e^{\kappa x}$$

$$x > \tfrac{1}{2}a \qquad \psi(x) = D\, e^{-\kappa x}$$

$$\lim_{\epsilon \to 0} \; -\frac{\hbar^2}{2m} \int_{\pm\frac{1}{2}a-\epsilon}^{\pm\frac{1}{2}a+\epsilon} dx \, \frac{d^2\psi}{dx^2} - g\, \psi(\pm\tfrac{1}{2}a) = 0$$

$$\Rightarrow \qquad \psi'(\pm\tfrac{1}{2}a+\epsilon) - \psi'(\pm\tfrac{1}{2}a-\epsilon) = -\frac{2mg}{\hbar^2}\,\psi(\pm\tfrac{1}{2}a)$$

ψ continuous.

at $-\frac{1}{2}a$:

$$A\, e^{-\frac{1}{2}\kappa a} = B\, e^{\frac{1}{2}\kappa a} + C\, e^{-\frac{1}{2}\kappa a}$$

$$-\kappa B\, e^{\frac{1}{2}\kappa a} + \kappa C\, e^{-\frac{1}{2}\kappa a} - \kappa A\, e^{-\frac{1}{2}\kappa a} = -\frac{2mg}{\hbar^2} A e^{-\frac{1}{2}\kappa a}$$

at $\frac{1}{2}a$

$$B\, e^{-\frac{1}{2}\kappa a} + C\, e^{\frac{1}{2}\kappa a} = D\, e^{-\frac{1}{2}\kappa a}$$

$$-\kappa D\, e^{-\frac{1}{2}\kappa a} - \left[-\kappa B\, e^{-\frac{1}{2}\kappa a} + \kappa C\, e^{\frac{1}{2}\kappa a}\right] = -\frac{2mg}{\hbar^2}\, D e^{-\frac{1}{2}\kappa a}$$

or:

$$
\begin{cases}
A = B e^{\kappa a} + C \\
-B e^{\kappa a} + C = A\left(1 - \dfrac{2mg}{\kappa \hbar^2}\right) \\
\\
B + C e^{\kappa a} = D \\
B - C e^{\kappa a} = D\left(1 - \dfrac{2mg}{\kappa \hbar^2}\right)
\end{cases}
$$

$$\cancel{-2B e^{\kappa a} + C \frac{2mg}{\kappa \hbar^2} = 0}$$

$$\cancel{B \frac{2mg}{\kappa \hbar^2}}$$

$$B e^{\kappa a} \left\{ 2 - \frac{2mg}{\kappa \hbar^2} \right\} - C \frac{2mg}{\kappa \hbar^2} = 0$$

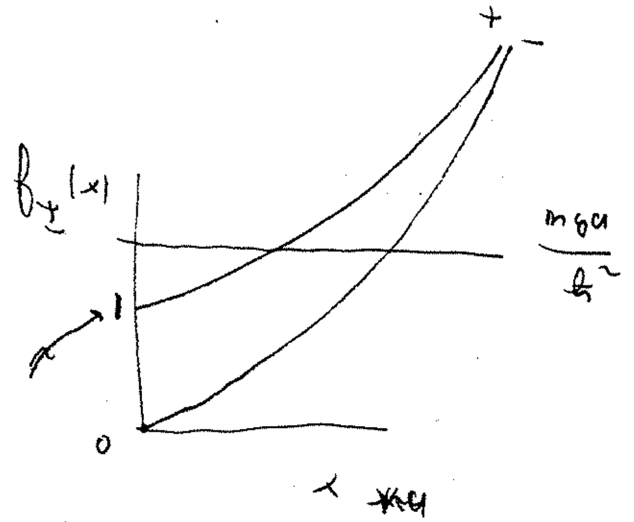
$$C e^{\kappa a} \left\{ 2 - \frac{2mg}{\kappa \hbar^2} \right\} - B \frac{2mg}{\kappa \hbar^2} = 0$$

$$\Rightarrow \left[e^{\kappa a} \left\{ 2 - \frac{2mg}{\kappa \hbar^2} \right\} \right]^2 = \left[\frac{2mg}{\kappa \hbar^2} \right]^2$$

$$e^{\kappa a} \left\{ 2 - \frac{2mg}{\kappa \hbar^2} \right\} = \pm \frac{2mg}{\kappa \hbar^2}$$

$$e^{\kappa a} = \frac{mg}{\kappa \hbar^2} \left\{ e^{\kappa a} \pm 1 \right\}$$

$$1 = \frac{\cancel{m}\,\hbar^2}{mga} \frac{\kappa a\, e^{\kappa a}}{e^{\kappa a} \pm 1} = \frac{\hbar^2}{mga} f_{\pm}(\kappa a)$$



two solutions $\frac{mga}{\hbar^2} \geq 1$ one solution $\frac{mga}{\hbar^2} < 1$

b) $\pm$ sign $\Rightarrow$ $B = \pm C$

$\Rightarrow$ Ψ_+  Ψ_- 

c) $V(+x) = V(-x)$ $\Rightarrow$ ψ has even / odd symmetry

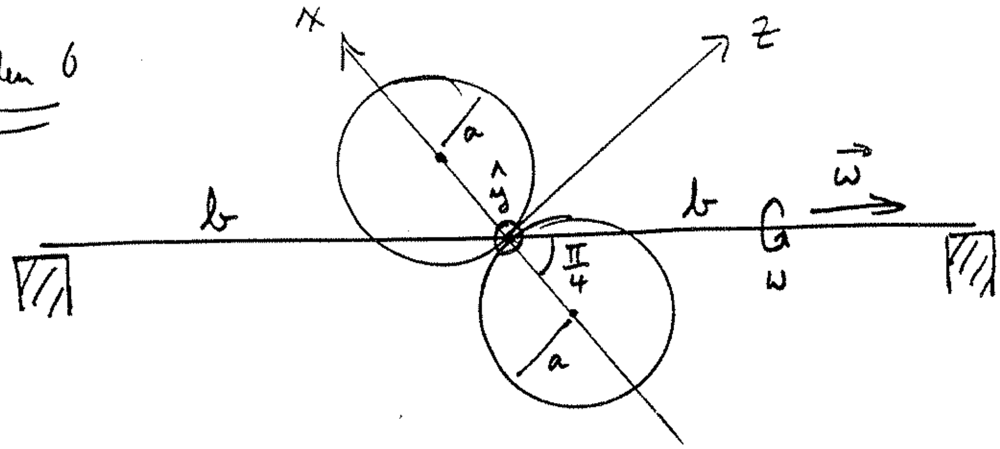
d) $a \to 0$ only even solution.

near 0 $f_+(x) \approx \frac{1}{2}x$

$$\Rightarrow \quad \frac{\hbar^2}{mga}\,\frac{1}{2}\kappa a = 1 \quad \text{or} \quad \kappa = \frac{2mg}{\hbar^2}$$

e) like single δ-function: $V = -2g\,\delta(x)$, double strength.

Problem 6



For one sphere:
$I_x = \frac{2}{5} m a^2$

$I_y = I_z = \frac{7}{5} m a^2$
(parallel axis theorem).

Torque on system: $\vec{N} = \left.\frac{d\vec{L}}{dt}\right|_{space} = \underbrace{\left.\frac{d\vec{L}}{dt}\right|_{rot}}_{zero} + \vec{\omega} \times \vec{L}$

$x\,y\,z$ axes rotate with spheres.
x-axis lies along line of centers; z-axis is in plane of x-axis and axle.

$\vec{L} = \overleftrightarrow{J} \cdot \vec{\omega}$

In $x\,y\,z$ system, $\overleftrightarrow{J} = \begin{pmatrix} \frac{4}{5} m a^2 & 0 & 0 \\ 0 & \frac{14}{5} m a^2 & 0 \\ 0 & 0 & \frac{14}{5} m a^2 \end{pmatrix}$
(diagonal)

and $\vec{\omega} = \frac{\omega}{\sqrt{2}}(\hat{z} - \hat{x})$.

$\therefore \vec{L} = \frac{2}{5} m a^2 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} \cdot \frac{\omega}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \frac{\sqrt{2}\, m a^2 \omega}{5} \begin{pmatrix} -2 \\ 0 \\ 7 \end{pmatrix}$

$\vec{N} = \vec{\omega} \times \vec{L} = \frac{\sqrt{2}\, m a^2 \omega}{5} \cdot \frac{\omega}{\sqrt{2}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1 & 0 & 1 \\ -2 & 0 & 7 \end{vmatrix} = \frac{m a^2 \omega^2}{5} (-\hat{y})(-7+2)$

$\therefore \vec{N} = m a^2 \omega^2 \hat{y}$; Torque about one bearing, due to weight of spheres, is $N = 2 m g b$.

$m a^2 \omega^2 = 2 m g b \implies \boxed{\omega = \sqrt{\frac{2 g b}{a^2}}}$

Problem 7.

a) $IR = \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_{\text{area of loop}} \overset{\text{into page}}{B\cdot da} = -\frac{d}{dt}(B\,\ell\,\overset{\text{length of loop at time } t}{x}) = -B\ell v$

$I = -\frac{B\ell v}{R}$ counterclockwise

b) $F_{\text{mag on bar}} = Q\vec{v}\times\vec{B} = I\ell\times B = -\frac{B^2\ell^2 v}{R}$ to left (slows the bar down)

Q: total charge in bar

$\vec{v}$: (component in direction perpendicular to bar, the other component produces current, not force on bar)

c) $F = \frac{dp}{dt} = m\frac{dv}{dt} = -\frac{B^2\ell^2 v}{R}$

$\int\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int dt \Rightarrow \ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}t$

$v = v_0 e^{-\lambda t}$ where $\lambda = \frac{B^2\ell^2}{mR}$

d) Power dissipated in resistor $= P = I^2R = \frac{B^2\ell^2v^2}{R}$

$E = \int_0^t P\,dt = \int_0^t \frac{B^2\ell^2v^2}{R}dt = \int_0^t \frac{B^2\ell^2v_0^2}{R}e^{-2\lambda t}dt$

$= \frac{B^2\ell^2v_0^2}{R}\left(-\frac{1}{2\lambda}\right)\left(e^{-2\lambda t}-1\right)$

$= \frac{B^2\ell^2}{R}\left(-\frac{1}{2\lambda}\right)(v^2-v_0^2) = +\frac{1}{2}m(v_0^2-v^2) =$ kinetic energy lost by bar

Problem #8

(a) vapor and liquid are in equilibrium $\Rightarrow$ P, T equal in both phases, fixed $\Rightarrow$ use Gibbs free energy $X = G(P, T, N) = U + PV - TS = \mu N$; may also use thermodynamic potential Ω

(b) α is the chemical potential of the liquid

(c) $N \sim \exp - (X_{drop} - X_{gas})/T = \exp - (\mu_{liq}N + \sigma S - \mu_{gas}N)$, $\mu_{liq} = \mu_{gas}$ by phase equilibrium