PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #58

JAN. 14, 1989

Comprehensive Examination for Winter 1989

General Instructions

This Comprehensive Examination for Winter 1989 (#58) consists of eight problems of equal weight (20 points each). It has two parts. The first part (Problems 1-4) is handed out at 9:00 am (duration: 3 hours) and the second part (Problems 5-8) at 1:00 pm (duration 3 hours).

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

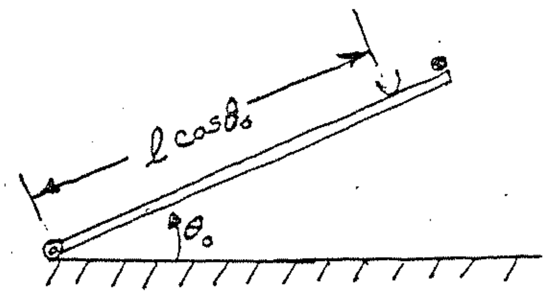
If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solutions. "Scratch" work will not be graded.

Physics Department Comprehensive Exam #58, Jan. 14, 1989

PART I

Problem # 1



A uniform rod of length ℓ is hinged at one end to a horizontal table; initially the rod is held at an angle θ_0 above the horizontal, as shown. A small particle is placed on the upper end of the rod and a massless cup is fixed to the rod a distance $\ell \cos \theta_0$ from the hinge. The system is released from rest. Show that if θ_0 be greater than a certain value θ_c the particle will be found in the cup at the end of the motion, and find θ_c.

Problem #2

The heat capacity at constant pressure for a certain solid is γT, with $\gamma = 5$ m cal/degree^2 mole. At T = 300 K, the solid changes into a classical ideal gas which is kept at constant pressure. The heat of the transition is 150 cal/mole. The entropy of this gas at T = 600 K is 5.5 cal/degree mole. Is this gas mono-atomic or di-atomic?
(For the value of the gas constant R use 2 cal/degree.)

Problem #3

A grounded solid spherical conductor of radius R is placed with its center at the origin of coordinates in a uniform applied electric field of magnitude E_o directed along the positive z-axis (Note: The total E-field of this problem is not uniform.). The sphere is grounded in its x-y plane and the entirety of the x-y plane is at zero potential.

(a)(5) Determine the distribution of charge on the sphere in terms of E_o and R. [Hint: Use a series expansion expression for the potential in spherical coordinates.]

(b)(8) The sphere of part (a) is now divided into two hemispheres by a cut through its center in the x-y plane; and the upper hemisphere (z>0) is given a constant angular velocity $\vec{\omega} = \omega_0 \hat{k}$, where $\hat{k}$ is a unit vector in the z-direction. Assuming the sphere to be magnetically inert, ($\mu = \mu_0$) calculate the magnetic field at its center.

(c)(3) The induced charge of part (a) is now "frozen" in place (with the upper half spinning, as in part (b)) and both the applied E-field and the conducting material are removed. Determine the value of ω for which the E- and B-fields at the origin of coordinates will be equal.

(d)(4) Illustrate in a sketch the net force on a particle of charge q (positive) as it moves through the origin of coordinates of part (c) with a velocity $\vec{v} = -\hat{i}$ m/s, where $\hat{i}$ is a unit vector in the x-direction. Determine the direction of the net force.

Problem #4

A flexible rope of mass m and length ℓ is hung over a smooth pin. Determine the motion, evaluate the arbitrary constants in the solution by requiring that at t = 0, the rope is at rest with 1/3 of its length on one side of the peg.

(a) How long does it take for the rope to slip off the peg?

(b) What is the tension in the rope?

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PART II

Problem #5

A particle is bound in a one-dimensional potential well of width 2a and depth $V = -V_o$. There are a finite number of bound states which are given by the expressions below which are obtained from the boundary conditions.

For even parity states: $k\tan(ka) = \kappa$ or $|\cos(ka)| = k/k_o$ & $\tan(ka) > 0$

For odd parity states: $k\cot(ka) = -\kappa$ or $|\sin(ka)| = k/k_o$ & $\tan(ka) < 0$

where: $\hbar k = [2m(|V| - |E|]^{1/2}$; $\hbar\kappa = [2m\,|E|]^{1/2}$; $k_o^2 = k^2 + \kappa^2 = 2m|V|/\hbar^2$.

For a potential of depth -80 MeV and width 2a = 4 fm with a nucleon in the well, how many energy levels exist and what is the parity of each. You do <u>not</u> need to calculate the energy of each state, only the relative energy (identify the lowest, next lowest, etc).

Hint: You may assume the nucleon has a mass of 1000 Mev/c^2.

Problem #6

An electric field of amplitude E_0 is described by the equation

$\vec{E}(\vec{r},t) = E_0\,\hat{k}\,\{\cos[\omega t - (k_x x - k_y y)] + \cos[\omega t - (k_x x + k_y y)]\}$,

where k_x and k_y are constant, $k_x^2 + k_y^2 = \omega^2/c^2$, and the ratio $k_y/k_x = \tan\varphi$ is such that $0 < \varphi < \pi/2$. Also, $\nu = \omega/2\pi$ is an optical frequency.

(a)(10) Describe the intensity of this wave field in a plane parallel to the y-z plane.

(b)(5) A thin plane photographic film is placed with its plane parallel to the y-z plane of part (a). After exposure and development, a hologram is formed. Assuming the density of film exposure to be related linearly to light intensity, describe quantitatively the pattern of light intensity transmitted by this hologram when it is illuminated normally by a laser beam of frequency ν.

(c)(5) If the wave field of part (a) is established in a photorefractive medium, conditions can be met such that the index of refraction of the photorefractive medium varies in the same way as does the exposure of the photographic film of part (b), except that the effect occurs throughout the volume of the photorefractive medium. Describe quantitatively how you would detect this photorefractivity using a weak laser beam "probe wave" of frequency ν.

Problem #7

A hypothetical one-electron atom with a spinless electron has a set of excited states given by $|n+1,\ \ell=1,\ m\rangle$ and is in a uniform moderate magnetic field, $\vec{B}_o$ which defines a z-axis. It radiates to a lower state given by $|n,\ \ell=0,\ m=0\rangle$. A detector of the radiation is available which can resolve the Zeeman splitting at the radiations corresponding to

$$|n,0,0\rangle \leftarrow |n+1,\ 1,\ m\rangle$$

The (complex)electromagnetic fields of the radiation may be calculated by classical E&M from electric dipole and magnetic dipole charge and current distributions respectively and are given by:

$$\vec{B}(\vec{r}) = \exp[-i(\omega t - kr)]/(kr)\sum_{m} [A_E(\ell,m)\ \vec{X}_{\ell,m}(\theta,\phi) + A_M(\ell,m)\ \hat{r} \times \vec{X}_{\ell,m}(\theta,\phi)$$

$$\vec{E}(\vec{r}) = \vec{B}(\vec{r}) \times \hat{r}.$$

The $\vec{X}$'s are the vector spherical harmonics:

$$\vec{X}_{1,0} = [3/(8\pi)]^{1/2}\ i\ \sin(\theta)\ \hat{\phi}$$

$$\vec{X}_{1,1} = -1/4[3/\pi]^{1/2}\ \exp[+i\phi]\ [\hat{\theta} - i\ \cos(\theta)\ \hat{\phi}]; \qquad \vec{X}_{1,-1} = (\vec{X}_{1,1})^* .$$

$A_E(\ell=1,m) = K\ k^3\ Q_{1,m}$ where K is a constant, k is the wave vector, and $Q_{1,m}$ is the electric dipole transition moment. Similarly $A_M(\ell=1,m) = K\ k^3\ M_{1,m}$ for the magnetic dipole transition.

(a) Calculate the angular distributions of the resolved radiations.

(b) Calculate the polarization of each of the above radiations observed when the detector is at:

(1) $\theta = 0°$ and
(2) $\theta = 90°$

Note: Right-circular polarization is defined as $\vec{E}$ rotating clock-wise when viewed from the detector looking back toward the source.

Problem #8

Consider a simple cubic solid, e.g. all atoms are located on the corners of little cubes. There are two different types of atoms, A and B, in this alloy and they are randomly distributed over all possible sites. The concentration of A-atoms is c. The total number of atoms is N and this number is very large, hence we ignore all surface effects. The only energies which play a role are nearest neighbor interactions and the total energy of the solid is

$$n_{AA}\ E_{AA} + n_{AB}\ E_{AB} + n_{BB}\ E_{BB}$$

with n_{ij} the number of nearest neighbor combinations i-j and E_{ij} the corresponding energy. Define $L = 2E_{AB} - E_{AA} - E_{BB}$.

(a)(4) Express n_{ij} in terms of the concentration c.

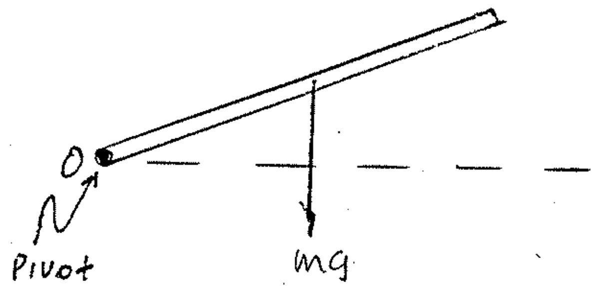
(b)(4) As a reference system we use two pieces of solid, one in which all the A-atoms are located in a simple cubic lattice, and one in which all the B-atoms are located in a simple cubic lattice. Determine the energy and the entropy of this reference system.

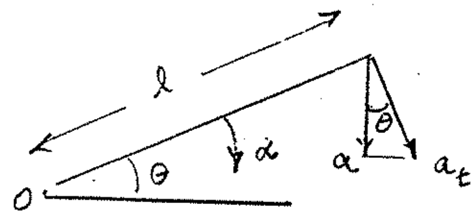
(c)(4) Find the difference in free energy, ΔF, between the alloy and the reference system. Use the appropriate formulas for large values of N.

(d)(4) Differentiating ΔF with respect to concentration yields the equilibrium concentration c_e as a function of temperature. The inverse of this relation gives a critical temperature T_c as a function of concentration. Find $T_c(c)$ and sketch this in a graph.

(e)(4) How do you interpret this T_c?

Problem # 1





Assuming the particle can slide initially, the downward acceleration at the end of the rod must be greater than g. m = mass of rod with length l

Taking torques about pivot at O

$$mg\frac{l}{2}\cos\theta_c = I\alpha = I\frac{a_t}{l/2}\ (\text{center})$$

$$= \frac{I}{l}a_t\ (\text{end})$$

$$= \frac{ml^2}{3l}\frac{a}{\cos\theta_c}$$

so

$$a = \frac{3}{2}g\cos^2\theta_c$$

then require $a \geq g$

$$\frac{3}{2}g\cos^2\theta_c \geq g$$

$$\boxed{\cos^2\theta_c \geq \frac{2}{3}}$$

$$\theta_c \geq \cos^{-1}\sqrt{\frac{2}{3}} = 35.2°$$

Relations among accelerations

Define a = downward comp. of accel.

a_t = tangential comp. of accel.

α = angular accel. about O

I = moment of inertia about O.
$= \frac{1}{3}ml^2$

$\alpha = \frac{a_t}{l}$ (at end of rod)

$a = a_t\cos\theta_c$

$a_t(\text{end}) = 2a_t(\text{center})$

Problem #2

$$S^{gas}(T) = \int_0^{T_t} C_p^{solid} \frac{dT}{T} + \frac{\Delta H_t}{T_t}$$

$$+ \int_{T_t}^{T} C_p^{gas} \frac{dT}{T}$$

$$= 5 \times 300 \text{ mcal / degree mole} + \frac{150}{300} \text{ cal / degree mole}$$

$C_p = C_v + R$

$C_v = \frac{f}{2} R$

$$+ \int_{T_t}^{T} R\left(1 + \frac{f}{2}\right) \frac{dT}{T}$$

f: number of degrees of freedom per molecule.

$$= \left[1.5 + 0.5 + 2\left(1 + \frac{f}{2}\right) \ln 2\right] \text{ cal / degree mole}$$

$f = 3 \qquad 2 + 5 \ln 2 \approx 5.5$

$f = 6 \qquad 2 + 8 \ln 2 \approx 6.2$

Problem #3

a) For large r, $V(r) \longrightarrow -E_0 r\cos\theta$ and the contribution to $\vec{E}$ from the charge induced on the sphere is that of a dipole; $\therefore$, we try $V(r,\theta) = A_1 r\cos\theta + \frac{B_1}{r^2}\cos\theta$.

Then, $A_1 = -E_0$; and, at $r = R$, $V(R,\theta) = 0 = -E_0 R\cos\theta + \frac{B_1}{R^2}\cos\theta$.

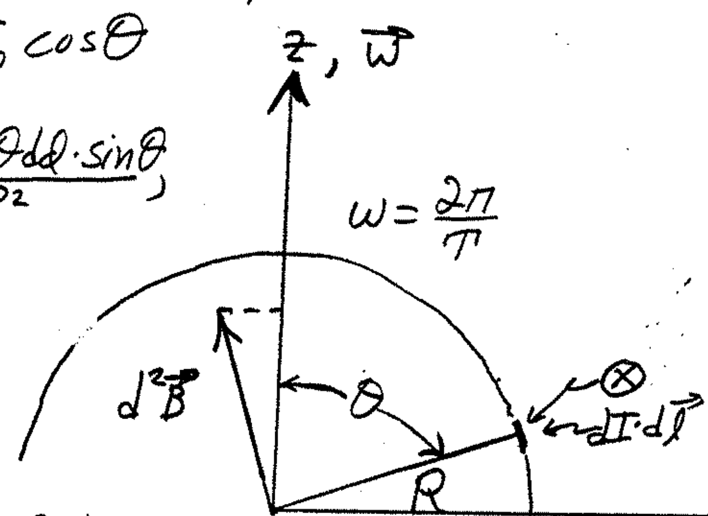
Thus, $B_1 = E_0 R^3$. $E_r = -\frac{\partial V}{\partial r} = E_0\cos\theta + \frac{2E_0R^3}{r^3}\cos\theta$.

$\sigma(\theta) = \epsilon_0 E_r(R,\theta) = 3\epsilon_0 E_0\cos\theta$

b) $d^2B_z = |d^2\vec{B}|\sin\theta = \frac{\mu_0}{4\pi}\,\frac{dI\,R\sin\theta d\phi\cdot\sin\theta}{R^2}$,

where $dI = \sigma\, dA/T$, or,

$$dI = \frac{\sigma_0\cos\theta\cdot 2\pi R\sin\theta\cdot R\,d\theta}{T}$$



Thus, $dB_z = \frac{\mu_0\sigma_0\omega R}{2}\sin^3\theta\cos\theta\, d\theta$

$$B_z(0) = \frac{\mu_0\sigma_0\omega R}{2}\int_0^{\pi/2}\sin^3\theta\cos\theta\, d\theta = \frac{\mu_0\sigma_0\omega R}{2}\left.\frac{\sin^4\theta}{4}\right|_0^{\pi/2}$$

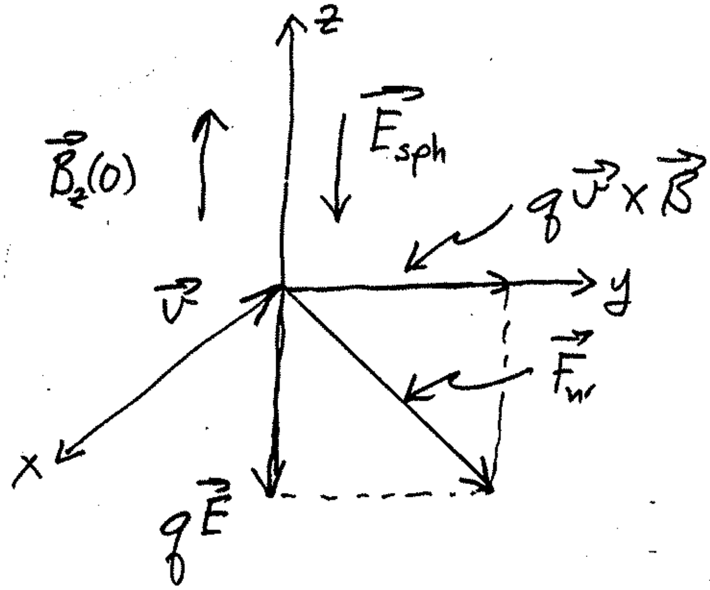
$\vec{B}_z(0) = \frac{3\mu_0\epsilon_0 E_0\omega R}{8}\hat{k}$ (c) Inside the spherical shell, $\vec{E}_\sigma = -\vec{E}_{applied} = -E_0\hat{k}$.

$E_0 = \frac{3\mu_0\epsilon_0 E_0\omega R}{8}$, $\omega = 8/3\mu_0\epsilon_0 R$.

d) $|\vec{F}_E| = qE_0$, $|\vec{F}_B| = qvE_0$.

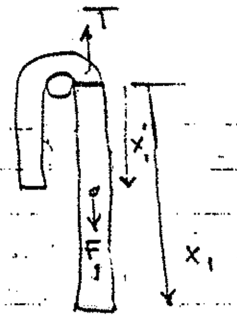
$F_B/F_E = v = 1$; $\therefore$,

$\vec{F}_{NET}$ is in y-z plane along the line $\theta = 3\pi/4$, as shown.



Problem # 4

(a) Using Newtonian Method



$F_1 = mg$

$m_1 = \frac{m}{l} x_1 \quad ; \quad m_2 = \frac{m}{l} x_2$

$m_1 \ddot{x}_1 = m_1 g - T \qquad (1)$

$m_2 \ddot{x}_2 = m_2 g - T \qquad (2)$

$x_1 + x_2 = l \qquad (3)$

subtracting (1) − (2)

$m_1 \ddot{x}_1 - m_2 \ddot{x}_2 = (m_1 - m_2) g$

$\frac{m}{l}\left(x_1 \ddot{x}_1 - x_2 \ddot{x}_2\right) = \frac{mg}{l}(x_1 - x_2)$ coupled eq.

use (3)

$\frac{m}{l}\left[x_1 \ddot{x}_1 + (l - x_1)\ddot{x}_1\right] = \frac{mg}{l}(2x_1 - l)$

$$\boxed{m \ddot{x}_1 = \frac{mg}{l}(2x_1 - l)}$$

the solution is

$x_1(t) = A e^{\sqrt{\frac{2g}{l}} t} + B e^{-\sqrt{\frac{2g}{l}} t} + \frac{l}{2}$

the initial conditions $x_1(0) = \frac{2}{3} l$ and $\dot{x}_1(0) = 0$

result in

$A + B = \frac{l}{6}$ and $A - B = 0 \quad \therefore \quad A = B = \frac{l}{12}$

$x_1(t) = \frac{l}{12}\left(e^{\sqrt{\frac{2g}{l}} t} + e^{-\sqrt{\frac{2g}{l}} t}\right) + \frac{l}{2} = \frac{l}{2}\left(\frac{1}{3}\cosh\sqrt{\frac{2g}{l}}\, t + 1\right) \xrightarrow[t \to 0]{\lim} \frac{2}{3} l$

to find the time required for the rope to slip off the peg

set $x_1 = l$

$l = \frac{l}{2}\left(\frac{1}{3}\cosh\sqrt{\frac{2g}{l}}\, t + 1\right) \quad \therefore \quad t = \sqrt{\frac{l}{2g}}\, \cosh^{-1}(3) = 1.25\sqrt{\frac{l}{g}}$

(b)

To obtain the Tension:

From (1)

$T = -m_1 \ddot{x}_1 + \frac{m}{l} x_1 g$

$= -\frac{m}{l} x_1 \ddot{x}_1 + \frac{m}{l} g\, x_1$

$= -\frac{m}{l} x_1 \left(\frac{g}{l}(2x_1 - l) - g\right)$

$= -\frac{mg}{l} x_1 \left(\frac{2x_1 - l}{l} - 1\right)$

$T = -\frac{mg}{l} x_1 \left(\frac{1}{3}\cosh\sqrt{\frac{2g}{l}}\, t - 1\right)$

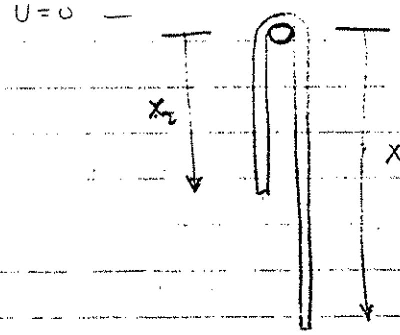
at $t = 0$

$T(x_1, 0) = -\frac{mg}{l} x_1$

the Tension along the rope is proportional to $(x_1 - x_1')/x_1$

$T' = \frac{T(x_1 - x_1')}{x_1} = -\frac{mg}{l}(x_1 - x_1')$

Lagrangian Approach



$$T = \frac{1}{2}\frac{m}{l}\left[x_2 \dot{x}_2^2 + x_1 \dot{x}_1^2\right]$$

$$U = -\frac{mg}{l}\left[x_2 \frac{x_2}{2} + x_1 \frac{x_1}{2}\right]$$

constraint : $x_2 + x_1 = l$

$$L = \frac{m}{2l}\left[x_2 \dot{x}_2^2 + x_1 \dot{x}_1^2\right] + \frac{mg}{2l}\left[x_2^2 + x_1^2\right]$$

using the constraint to eliminate x_2

$$L = \frac{m}{2l}\left[(l - x_1)\dot{x}_1^2 + x_1 \dot{x}_1^2\right] + \frac{mg}{2l}\left[(l-x_1)^2 + x_1^2\right]$$

$$L = \frac{m}{2l} l \dot{x}_1^2 + \frac{mg}{2l}(l^2 - 2lx_1 + 2x_1^2) \qquad \frac{\partial L}{\partial x_1} = \frac{mg}{l}(2x_1 - l)$$

Lagrange's Equation $\qquad \frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1$

$$\boxed{m\ddot{x}_1 - \frac{mg}{l}(2x_1 - l) = 0}$$

$\ddot{x}_1 - \frac{2g}{l}x_1 + g = 0$, the solution is $x_1(t) = Ae^{\sqrt{\frac{2g}{l}}t} + Be^{-\sqrt{\frac{2g}{l}}t} + \frac{l}{2}$

the initial conditions $x_1(0) = \frac{2}{3}l$ and $\dot{x}_1(0) = 0$

result in $A + B = \frac{l}{6}$ $\therefore$ $A = B = \frac{l}{12}$

$A - B = 0$

$$\boxed{x_1(t) = \frac{l}{12}\left(e^{\sqrt{\frac{2g}{l}}t} + e^{-\sqrt{\frac{2g}{l}}t}\right) + \frac{l}{2} = \frac{l}{2}\left(\frac{1}{3}\cosh\sqrt{\frac{2g}{l}}t + 1\right)} \xrightarrow[t \to 0]{\lim} \frac{2}{3}l$$

To find the time required for the rope to slip off the peg we set $x_1 = l$

$$l = \frac{l}{2}\left(\frac{1}{3}\cosh\sqrt{\frac{2g}{l}}t + 1\right) \quad \therefore \quad t = \sqrt{\frac{l}{2g}}\cosh^{-1}(3) = 1.25\sqrt{\frac{l}{g}}$$

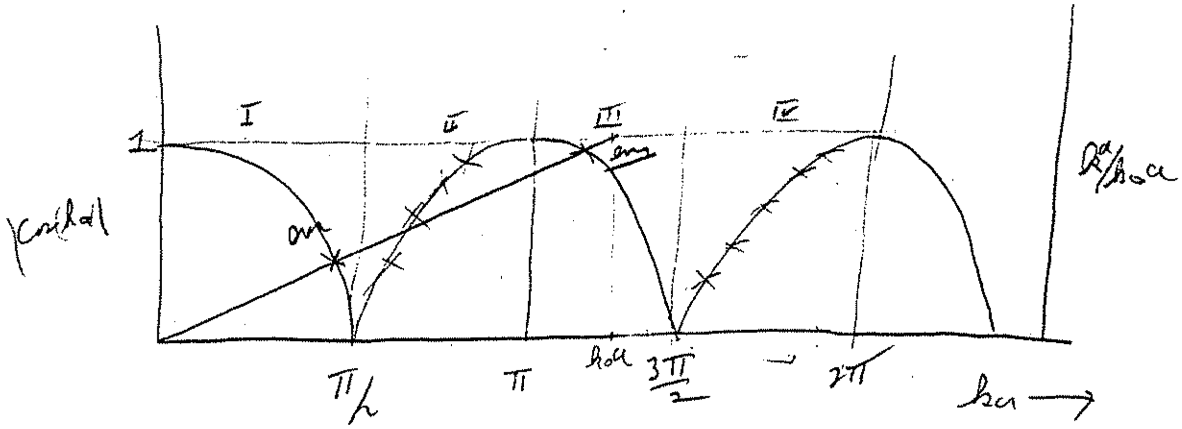
Square Well

Problem # 5

even for $\tan ka > 0$

ka 1st & 3rd quadrant

Plot $|\cos ka|$ vs ka and ka/k_0a vs ka

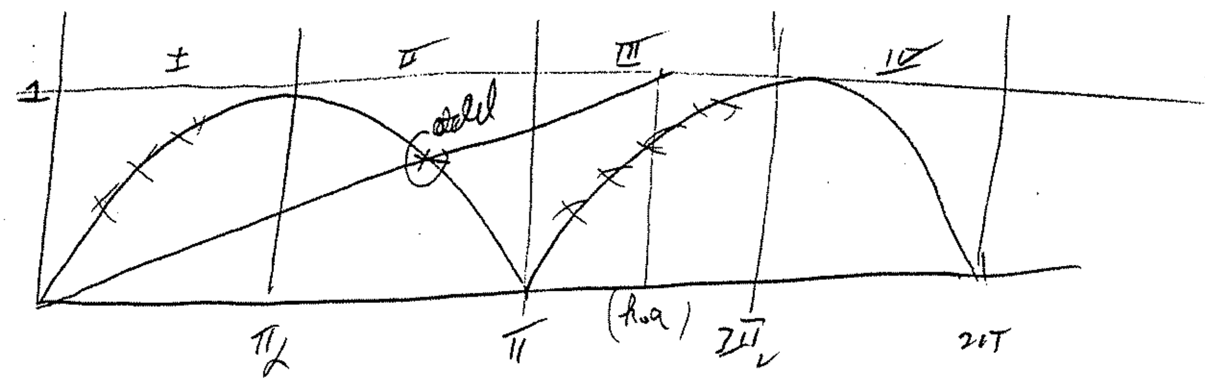


$$k_0a = \sqrt{(\kappa a)^2 + (ka)^2} = \frac{\sqrt{2m(|V|-|E|) + 2m|E|}\; a}{\hbar} = \frac{\sqrt{2m|V|a^2}}{\hbar}$$

$$k_0a = \frac{\sqrt{2mc^2|V|}\; a}{\hbar c} = \frac{\sqrt{2\times 940\times 80}\times 2 \text{ MeV-fm}}{197 \text{ MeV-fm}} = 3.94 = \underline{1.25\pi}$$

odd for $\tan ka < 0$ 2nd & 4th Quadrant

Plot $|\sin ka|$ vs ka



3 levels
lowest even parity
next " odd "
highest even parity

Problem #6

a) The only varying term in the intensity goes like $\langle \cos\theta_+ \cos\theta_- \rangle$, where $\theta_+ = \omega t - \vec{k}_+\cdot\vec{r}$, $\theta_- = \omega t - \vec{k}_-\cdot\vec{r}$, and $\langle\,\rangle$ is a time average over several optical periods.

$$\cos\theta_+\cos\theta_- = \frac{\cos(\theta_+-\theta_-)+\cos(\theta_++\theta_-)}{2},$$

and $\langle\cos(\theta_++\theta_-)\rangle = 0$. Thus,

$$2\langle\cos\theta_+\cos\theta_-\rangle = \langle\cos(2k_y y)\rangle = \cos(2k_y y).$$

$k_y y = (k\sin\phi)y$, where $k = \omega/c$, Therefore,

$$I(y) = \frac{E_0^2}{2}\left\{1+\cos[(2k\sin\phi)y]\right\}.$$

b) This hologram is a sinusoidal diffraction grating with a grating spacing, d, determined by

$$(2k\sin\phi)d = 2\pi,$$

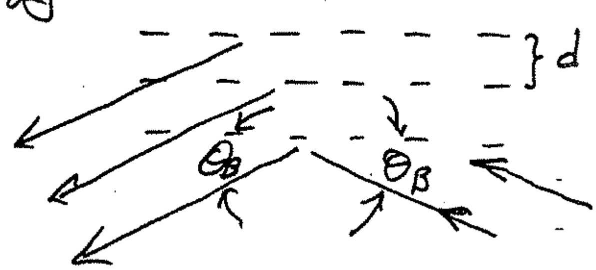
or, $d = \pi/k\sin\phi = \lambda/2\sin\phi$, where $\lambda = c/\nu$.

With normal incidence, 3 diffracted beams are formed: One at normal transmission, one at $\tan(\theta_{trans}) = \lambda/d$ and one at $\tan(\theta_{trans}) = -\lambda/d$.

c) Observe Bragg reflection from the index planes. The Bragg condition is

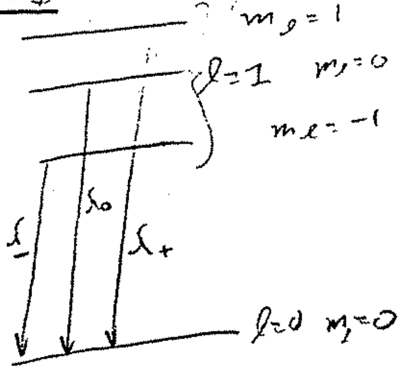
$$\lambda = 2d\sin\theta_{Bragg},$$

where d is given in part (b).



Grad QM

Problem #7



$\sigma_+ \to m=0 \leftarrow m=1 \quad \Delta m = -1 \quad \Delta l = 1$

$\sigma_0 \to m=0 \leftarrow m=0 \quad \Delta m = 0 \quad \Delta l = 1$

$\sigma_- \to m=0 \leftarrow m=-1 \quad \Delta m = 1 \quad \Delta l = 1$

Electric dipole transitions for $\Delta l = 1$

$\sigma_+ \propto |Q_{1-1}|^2 \quad \sigma_0 \propto |Q_{10}|^2 \quad \sigma_- \propto |Q_{11}|$

$E(nlm) = E(nl0) + g\mu_0 m_l B$

since $Q_{1m} = \langle l_f m_f | Y_{1m} | l_i m_i \rangle$

$\phi \; m_i + m = m_f = 0$

$\therefore m = -m_i$

$Q_{1-1} = \langle 0 0 | Y_{1-1} | 1 1 \rangle$

$Q_{10} = \langle 0 0 | Y_{1,0} | 1 0 \rangle$

$Q_{1+1} = \langle 0 0 | Y_{11} | 1 -1 \rangle$

$I = \langle |\vec{S} \cdot \hat{r}| \rangle \propto \langle (\vec{E} \times \vec{B}) \cdot \hat{r} \rangle = \langle ((\vec{B} \times \hat{r}) \times \vec{B}) \cdot \hat{r} \rangle = \langle [\vec{B} \times (\hat{r} \times \vec{B})] \cdot \hat{r} \rangle$

$= \langle \hat{r} \cdot (\vec{B} \cdot \vec{B})\hat{r} - \vec{B}\underbrace{\cancel{(\vec{B} \cdot \hat{r})}}_{\vec{B} \perp \hat{r}} \rangle = \langle \vec{B} \cdot \vec{B} \rangle = \frac{1}{2} \vec{B} \cdot \vec{B}^*$

$I \propto \frac{1}{k r^2} A_E(1,m) A_E^*(1,m) \vec{X}_{1m} \cdot \vec{X}_{1m}^* = \frac{k^4}{r^2} |Q_{1m}|^2 (\vec{X}_{11} \cdot \vec{X}_{11}^*)$

(a)

$\sigma_+ \; (m=-1) \quad I \propto \frac{k^4}{r^2} |Q_{1-1}|^2 (\hat{\theta} + i\cos\theta\, \hat{\phi})(\hat{\theta} - i\cos\theta\, \hat{\phi}) = k^4 |Q_{1-1}|^2 \frac{1 + \cos^2\theta}{r^2}$

$\therefore I(\theta, \varphi) \propto 1 + \cos^2\theta$

$\sigma_- \; (m=+1)$ Similarly - same as σ_+

$I(\theta, \varphi) \propto 1 + \cos^2\theta$

$\sigma_0 \; (m=0) \quad I(\theta,\varphi) \propto \frac{k^4}{r^2} |Q_{10}|^2 (\vec{X}_{10} \cdot \vec{X}_{10}^*) = k^4 |Q_{10}|^2 \frac{\sin^2\theta}{r^2}$

$\therefore I(\theta,\varphi) \propto \sin^2\theta$

(b) Find $\mathrm{Re}\,[\vec E(0,0)]$ for each radiation ②

$$\vec E = \vec B \times \hat r \;\propto\; e^{-i\omega t} A_{lm} (\vec X_{lm} \times \hat r)$$

$\Lambda_+\ (m=-1)$ $\quad \vec E_{-1} \propto e^{-i\omega t}\, e^{-i\varphi}(\hat\theta + \hat\varphi\, i\cos\theta)\times\hat r$

$$= e^{-i\omega t} e^{i\varphi}(-\hat\varphi + \hat\theta\, i\cos\theta)$$

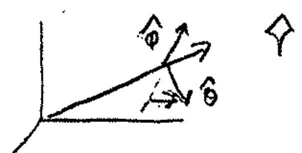
$\Lambda_0\ (m=0)$ $\quad E_0 \propto e^{-i\omega t}\, i\sin\theta\,\hat\varphi\times\hat r = e^{-i\omega t}\, i\sin\theta\,\hat\theta$

$\Lambda_-\ (m=+1)$ $\quad E_1 = -e^{-i\omega t}\, e^{-i\varphi}(\hat\varphi + \hat\theta\, i\cos\theta)$

For $\theta=0$ φ is arbitrary - choose $\varphi=0$

Λ_+ $\quad \vec E \propto e^{-i\omega t}(-\hat\varphi + i\hat\theta)$

$\mathrm{Re}\,\vec E \propto -\cos\omega t\,\hat\varphi + \sin\omega t\,\hat\theta$



$\therefore$ CCW - or Left Cir. Pol.

Λ_0 $\quad E \propto 0$ $\quad$ No Λ_0 radiation

Λ_- $\quad \vec E \propto -e^{-i\omega t}(\hat\varphi + \hat\theta\, i)$

$\mathrm{Re}\,\vec E \propto -\cos\omega t\,\hat\varphi - \sin\omega t\,\hat\theta$

CW or Right Cir Pol

(6) For $\theta = 90$ Choose $\varphi = \frac{\pi}{2}$ (arbitrary) ③
i.e. detector on y axis

(Λ_+) $\vec{E} \propto e^{-i\omega t} e^{-i\pi/2} (-\hat{\varphi})$; $\mathrm{Re}\,\vec{E} \propto \sin\omega t\ \hat{\varphi}$

Linear Polarization in X-Y plane i.e. $\perp$ to $\vec{B}_0$

(Λ_0) $\vec{E} \propto e^{-i\omega t}\, i\,\hat{\theta}$; $\mathrm{Re}\,\vec{E} \propto \sin\omega t\ \hat{\theta}$

Linear Polarization along Z-axis i.e. $\parallel$ to $\vec{B}_0$

Λ_- Same as Λ_+

Linear Polarization $\perp$ to $\vec{B}_0$

Problem #8

(a) N atoms $\Rightarrow$ $3N$ bonds

$$n_{AA} = 3N\, c^2$$
$$n_{AB} = 3N\, 2c(1-c)$$
$$n_{BB} = 3N\, (1-c)^2$$

(b) $E_{ref} = 3N\left[c\,E_{AA} + (1-c)\,E_{BB}\right]$ $\qquad S_{ref} = 0$

(c) $$\Delta S = k \ln \frac{N!}{(cN)!\,((1-c)N)!}$$

$$= k\,\{ N \ln N - cN \ln cN - (1-c)N \ln (1-c)N \}$$

$$= -kN\{ c \ln c + (1-c) \ln (1-c) \}$$

$$\Delta E = 3N\,\{ E_{AA}(c^2 - c) + 2c(1-c)\,E_{AB}$$
$$+ [(1-c)^2 - (1-c)]\,E_{BB} \}$$

$$= 3N\,c(1-c)\,\{ -E_{AA} + 2E_{AB} - E_{BB} \}$$

$$= 3NL\,c(1-c)$$

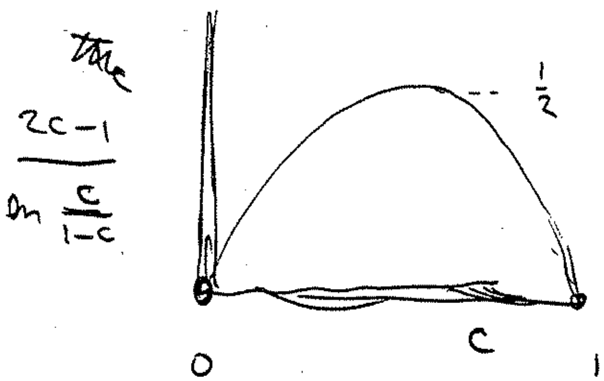
(d) $\Delta F = \Delta E - T\Delta S$

$$= 3NL\, c(1-c) + NkT \{ c \ln c + (1-c)\ln(1-c) \}$$

$$\frac{\partial \Delta F}{\partial c} = 3NL(1-2c) + NkT\{\ln c - \ln(1-c)\}$$

$$= 0$$

$$\Rightarrow \quad T_c(c) = \frac{3L(2c-1)}{k \ln c/(1-c)}$$



$c \to \frac{1}{2} + x$

$\ln \frac{c}{1-c} = \ln \frac{\frac{1}{2}+x}{\frac{1}{2}-x} \approx 4x$

(e) $L > 0 \qquad T_c > 0 \quad \Rightarrow$ phase separation

$L < 0 \qquad T_c < 0 \quad \Rightarrow$ all mixtures are stable.