PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #57

APRIL 2, 1988

Comprehensive Examination for Winter 1988

General Instructions

This Comprehensive Examination for Winter 1988 (#5 7) consists of eight problems of equal weight (20 points each). It has two parts. The first part (Problems 1-4) is handed out at 9:00 am (duration: 3 hours) and the second part (Problems 5-8) at 1:00 pm (duration 3 hours).

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solutions. "Scratch" work will not be graded.

Physics Department Comprehensive Exam #57, April 2, 1988

PART I

Problem # 1

Calculate the correction to the ground state of a hydrogenic atom due to the finite extension of the nucleus to the lowest non-zero order.

(a) Assume that the nuclear charge is uniformly distributed over the surface of a sphere of radius R_{nuc}

(b) Repeat the preceding problem for a model in which the nuclear charge is uniformly distributed throughout the volume of the spherical nucleus of radius R_{nuc}.

(c) From the results of (a) and (b), what can be said concerning the effect of the choice the radial charge distribution of a spherically symmetric nucleus?

Note 1: The radial part of the hydrogenic ground state wave function is

$$R(r) = (\pi a^3)^{-1/2} \exp(-r/a) \qquad \text{where } a = a_\infty/Z.$$

Note 2: The ground state energy may be taken to be

$$E_o = -(1/2)\, m_e c^2\, Z^2 \alpha^2 \qquad \text{where } \alpha = e^2/(\hbar c).$$

Note 3: $1\ F = 1 \times 10^{-15}$ m.

Hint: Mathematical approximations appropriate to the accuracy of the physical approximations may be used.

Problem #2

A electromagnetic plane wave of frequency ν travelling in free space is incident normally upon the flat surface of a conductor. Assume that the electric and magnetic fields inside the conductor, $\vec{E}_c$ and $\vec{B}_c$, are of the form

$$e^{i(\vec{k}_c \cdot \vec{r} - \omega t)} ,$$

where $\omega = 2\pi\nu$ and $\vec{k}_c$ is to be determined. Make the following assumptions:

i) $\vec{k}_c$ is normal to the surface.
ii) Ohm's law is obeyed.
iii) The charge density is zero everywhere.
iv) The conductor is nonmagnetic ($\mu = \mu_o$) and its electric permittivity is ϵ.

(a) Show that the current density, $\vec{J}_c$, inside the conductor, can be eliminated from Maxwell's equations, which implies that the wave equations are homogeneous. Hint: Utilize the special properties of the form of traveling waves.

(b) Express $|\vec{k}_c|$ in terms of μ_o, ϵ, and the conductivity σ; and give a qualitative description of wave propagation in poor conductors.

(c) Determine the electric vector amplitudes of the transmitted and reflected waves in terms of the amplitude of the incident wave and appropriate constants.

Problem #3

(a) An ideal gas of N fermions of mass m occupies a cubical volume of side L at a temperature of absolute zero. Calculate the fermi energy of this system. Hint: The density of standing wave modes in k-space $= L^3/\pi^3$.

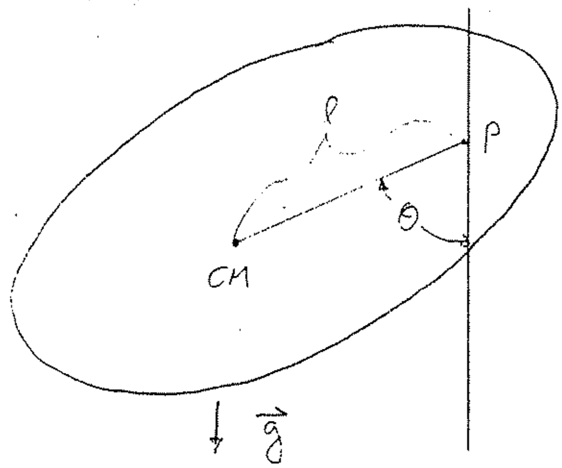
(b) Suppose the N conduction electrons in a cubical conductor of side L are replaced with N non-interacting negative pions whose decay is forbidden inside the conductor. Thus, we have an ideal gas of pions. Determine the thermal energy of this system at absolute zero. Hint: Examine the spectrum of single particle energy states; and use the fact that pions are bosons.

(c) Suppose the pions of part (b) were spin one particles like photons. Determine the configuration entropy, S, of this system of fictitious spin one pions at absolute zero. Obtain a simple expression for S by using the approximation appropriate for large values of N.

(d) A system of N spin 1/2 particles with a g value of two are contained at a temperature T in zero magnetic field. The magnetic field is increased along an isotherm to a final value of B tesla. B/T is on the order of 0.1 tesla/K. Find the entrophy change of the process.

Problem #4

A physical pendulum of mass M is suspended from a point P. The axis of rotation is perpendicular to the plane of the paper.



The moment of inertia with respect to a parallel axis through the center of mass (CM) is equal to Mk^2 where k is the radius of gyration.

(a) Displace the pendulum slightly from its equilibrium position and determine the period of oscillation by solving Lagrange's equation of motion.

(b) It turns out that if the pendulum is suspended from another point P' whose distance from the center of mass is ℓ' the period is the same as calculated in (a). Show that this period is equal to the period of a simple pendulum of length $\ell + \ell'$.

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PART II

Problem #5

A thin uniform circular metal disk of radius R and mass M lies on an infinite conducting plane. A uniform gravitational field is oriented normal to the plane. Initially the disk and plane are uncharged. Charge is slowly added. What value of charge density is required to cause the disk to leave the plate?

Problem # 6

For each of the following processes, calculate the entropy change, ΔS, and indicate whether it is positive or negative.

(a) M grams of a liquid are heated from T_1 to T_2. The substance has a constant specific heat of c.

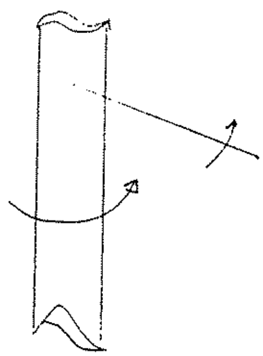
(b) A beaker contains a mixture of m_i grams of ice and m_w of water in equilibrium at normal pressure. Heat is applied until m_x grams of ice melt.

(c) A system with a constant molar heat capacity of c initially at a temperature of T_1 is put into thermal contact with a heat reservoir at a a temperature T_o. Consider both $T_1 > T_o$ and $T_1 < T_o$ and show $\Delta S > 0$ for both cases.

(d) A thermally isolated volume V is divided into two section of volume V_1 and V_2 connected by a closed valve. The two sections are not thermally isolated from each other. V_1 contains N molecules of a perfect gas at temperature T_1 while V_2 is a vacuum. The valve is then opened and the system comes to equilibrium.

(e) Two different perfect gases at the same temperature and pressure occupy volumes V_1 and V_2. A partition between the two volumes is removed and the two gases come to equilibrium uniformly mixed.

Problem #7



A flexible string of Length L is fastened to a rotating axle, so that it whirls around the axle in a horizontal plane. Suppose that the angular velocity ω of the string and axle are large enough so that gravity can be neglected. As the string whirls it is struck a vertical blow so that it oscillates above and below the plane of steady motion.

Derive the differential equation for the wave motion.

What are the boundary conditions on the solutions which fix the natural frequencies for standing waves on the string?

Problem #8

A positively charged Muon (spin 1/2) is brought to rest in a block of carbon. The Muon spin is polarized in the positive x direction while a magnetic field $\vec{B}$ is applied in the positive z direction.

(a) Calculate the probability after time t of measuring the Muon's spin and finding it in the positive x direction.

(b) Describe a possible experiment to determine the muon g-value.

#1

(a) $V = -\dfrac{Ze^2}{r} \quad r \geq R_{nuc}$
$V = -\dfrac{Ze^2}{R_{nuc}} \quad r \leq R_{nuc}$ } add & subtract $\dfrac{Ze^2}{r}$; $r \leq R_{nuc}$

$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$ where $\mathcal{H}_0 = \dfrac{-\hbar^2}{2m}\nabla^2 - \dfrac{Ze^2}{r} \quad 0 \leq r \leq \infty$

$$\mathcal{H}' = \begin{cases} +\dfrac{Ze^2}{r} - \dfrac{Ze^2}{R_{nuc}} & r \leq R_{nuc} \\ 0 & r \geq R_{nuc}. \end{cases}$$

1st O. Pert. Th   Ground State : ψ_{100}

$$E^{(1)}_{100} = \int \psi_{100}^* \mathcal{H}' \psi_{100} \, d\tau$$

$$= \frac{Ze^2}{\pi a^3} \int_0^{all} \int_0^{R_{nuc}} e^{-r/a} \left(\frac{1}{r} - \frac{1}{R_{nuc}} \right) e^{-r/a} r^2 dr \, d\Omega$$

$$= \frac{4\pi Ze^2}{\pi a^3} \left[\int_0^{R_{nuc}} r e^{-2r/a} dr - \frac{1}{R_{nuc}} \int_0^{R_{nuc}} r^2 e^{-2r/a} dr \right]$$

for $R_{nuc} \approx 1F \simeq 10^{-15}$ m while $a \approx \frac{a_0}{Z} \simeq \frac{1}{Z} \times 10^{-11}$ m

$r/a \ll 1$ for all values of $r \leq R_{nuc}$.

$\therefore \; e^{-2r/a} \simeq 1 - 2r/a$

$$E_1^{(1)} = 4\frac{Ze^2}{a^3} \left[\int_0^{R_{nuc}} r \, dr - \frac{1}{R_{nuc}} \int_0^{R_{nuc}} r^2 dr \right] \simeq 4\frac{Ze^2}{a^3} \left[\frac{R_{nuc}^2}{2} - \frac{R_{nuc}^2}{3} \right]$$

$$= \frac{4Ze^2}{a^3} \frac{R_{nuc}^2}{6} = \frac{2}{3} \frac{Ze^2}{a^3} R_{nuc}^2 = \frac{2}{3} \frac{Ze^2}{a} \left(\frac{R_{nuc}}{a} \right)^2$$

(2)

$$\frac{Zc^2}{a} = \frac{Z^2 c^2}{a_0} = \frac{Z^2 m_e c^2 e^4}{\hbar^2 c^2} = m_e c^2 Z^2 \alpha^2$$

$$\therefore E^{(1)}_{100} = +\frac{4}{3} \frac{m_e c^2}{2} Z^2 \alpha^2 \left(\frac{R_{nuc}}{a}\right)^2 - \frac{4}{3} E^{(0)}_{100}$$

$$\therefore E_{100} = E^{(0)}_{100} + E^{(1)}_{100} = -E^{(0)}_{100}\left(1 - \frac{4}{3}\left(\frac{R_{nuc}}{a}\right)^2\right)$$

(b) As (a) except:

$$\nabla \cdot \vec{E} = -4\pi\rho$$

From Gauss' Th

R_{nuc}, ρ_0

$$\int_A \vec{E} \cdot d\vec{a} = -4\pi \int \rho \, d\tau = 4\pi\, q(r) \quad r < R_{nuc}$$

$$q(r) = \frac{4}{3}\pi r^3 \rho_0 = \frac{4\pi}{3} r^3 \frac{Ze^3}{\frac{4}{3}\pi R_{nuc}^3} = Zc^2 \frac{r^3}{R_{nuc}^3}$$

$$E_r 4\pi r^2 = 4\pi Zc^2 \frac{r^3}{R_{nuc}^3} \quad ; \quad \vec{E} = \hat{r} \frac{Zc^2}{R_{nuc}^3} r \qquad 0 \le r \le R_{nuc}$$

$$V(r) = \int \vec{E}' \cdot \hat{r}' dr' + C = \frac{Zc^2}{R_{nuc}^3} \frac{r^2}{2} + C$$

$$V(r) \quad r > R_{nuc} = -\frac{Zc^2}{r} \qquad V(R_{nuc}) = -\frac{Zc^2}{R_{nuc}} = \frac{Zc^2}{2R_{nuc}} + C$$

$$\therefore C = -\frac{3}{2} \frac{Zc^2}{R_{nuc}}$$

$$V(r) = \begin{cases} -\dfrac{Zc^2}{r} & r \geq R_{nuc} \\ -\dfrac{Zc^2}{R_{nuc}}\left(\dfrac{3}{2} - \dfrac{1}{2}\dfrac{r^2}{R_{nuc}^2}\right) & r \le R_{nuc} \end{cases} \qquad \int \pm \frac{Zc^2}{r}; \; r \le R_{nuc}$$

$$\mathcal{H}' = \begin{cases} 0 & r \geq R_{nuc} \\ -\dfrac{Zc^2}{R_{nuc}}\left(\dfrac{3}{2} - \dfrac{1}{2} \dfrac{r^2}{R_{nuc}^2}\right) + \dfrac{Zc^2}{r} & r \le R_{nuc} \end{cases}$$

$$E^{(1)}_{100} = \int \psi^{(0)}_{100} \mathcal{H}' \psi^{(0)}_{100} \, r^2 dr \, d\Omega = \frac{4\pi Zc^2}{\pi a^3} \int e^{-2r/a} \mathcal{H}' r^2 dr$$

as in (a) $e^{-2r/a} \simeq 1$

$$E^{(1)}_{100} = \frac{4Zc^2}{a^3 R_n}\left[-\frac{3}{2}\int_0^{R_{nuc}} r^2 dr + \frac{1}{2R_{nuc}^2}\int_0^{R_{nuc}} r^4 dr + R_{nuc}\int r \, dr\right]$$

3

$$E_{100}^{(1)} = \frac{4ZC^2}{a^3 R_{nuc}}\left[-\frac{1}{3}R_{n}^3 + \frac{1}{10}R_{nuc}^3 + \frac{R_{nuc}^3}{2}\right]$$

$$= \frac{2}{5}\frac{ZC^2 R_{nuc}^2}{a^3} = \frac{2}{5}\frac{ZC^2}{a}\left(\frac{R_{nuc}}{a}\right)^2 = \frac{4}{5}\frac{m_e c^2}{2}\times Z^2\alpha^2\left(\frac{R_{nuc}}{a}\right)$$

$$E_{100} = -E_{100}^{(0)}\left(1 - \frac{4}{5}\left(\frac{R_{nuc}}{a}\right)^2\right)$$

(c) both (a) & (b) $E_{100} = -E_{100}^{(0)}\left(1 - K\left(\frac{R_{nuc}}{a}\right)^2\right)$

where K is a factor $\gtrsim 1$ so one would expect any resonable distribution of $\rho(r)$ in a spherical nucleus of R_{nuc} will produce a correction to $E_{100}^{(0)} \approx \left(\frac{R_{nuc}}{a}\right)^2$

where $\left(\frac{R_{nuc}}{a}\right) \approx Z\frac{10^{-15}}{5\times10^{-11}} \sim 10^{-5} Z$

#2

A plane electromagnetic wave of frequency ν travelling in free space i. incident normally upon the flat surface of a conductor. Assume that the electric and magnetic fields inside the conductor, $\vec{E}_c$ and $\vec{B}_c$, are of the form

$$e^{i(\vec{K}_c\cdot\vec{r}-\omega t)}, \quad \text{where } \omega = 2\pi\nu$$

and $\vec{K}_c$ is to be determined. Make the following assumptions:

i) $\vec{K}_c$ is normal to the surface.
ii) Ohm's Law is obeyed.
iii) The charge density is zero everywhere.
iv) The conductor is nonmagnetic ($\mu=\mu_0$) and its electric permittivity is ϵ.

a) Show that the current density, $\vec{J}_c$, inside the conductor, can be elliminated from Maxwell's equations, which implies that the wave equations are homogeneous. *

b) Express $|\vec{K}_c|$ in terms of μ_0, ϵ, and the conductivity σ, and give a qualitative description of wave propagation in poor conductors.

c) Determine the electric vector amplitudes of the transmitted and reflected waves in terms of the ~~incident~~ amplitude of the incident wave and appropriate constants

*Hint: Utilize the special properties of the form of travelling waves.

#12

(a) Since $\vec{E}_c = \vec{E}_{oc}\, e^{i(\vec{K}_c\cdot\vec{r}-\omega t)}$, $\frac{\partial \vec{E}_c}{\partial t} = -i\omega \vec{E}_c$. The only one of Maxwell's equations which differs in form from the free space expressions is Ampere's Law:

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}_c = \vec{J}_c + \epsilon\frac{\partial \vec{E}_c}{\partial t}. \quad \text{But } \vec{J}_c = \sigma\vec{E};$$

$$\therefore \quad \vec{J}_c = -\frac{\sigma}{i\omega}\frac{\partial \vec{E}_c}{\partial t} = i\frac{\sigma}{\omega}\frac{\partial \vec{E}_c}{\partial t}, \text{ and we have}$$

(b)

$$\vec{\nabla}\times\vec{B}_c = \mu_0\left(\epsilon + \frac{i\sigma}{\omega}\right)\frac{\partial \vec{E}_c}{\partial t} = \mu_0\epsilon'\frac{\partial \vec{E}_c}{\partial t}. \text{ Thus,}$$

we have a homogeneous wave equation with

$$K_c^2 = \mu_0\epsilon'\omega^2, \quad K_c = \sqrt{\mu_0}\,\omega\left(\epsilon + \frac{i\sigma}{\omega}\right)^{1/2}$$

For poor conduction $\left(1+\frac{i\sigma}{\omega\epsilon}\right)^{1/2} \simeq 1 + \frac{i\sigma}{2\omega\epsilon}$; $\therefore$

$K_c \simeq \sqrt{\mu_0\epsilon}\,\omega + i\frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon}}$. The transmitted waves are of the form $e^{-k'x}\, e^{i(k_0 x-\omega t)}$, where the direction of propagation is taken to be the x-direction, $k' = \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon}}$, and $k_0 = \sqrt{\mu_0\epsilon}\,\omega = \frac{\omega}{v}$. Thus, we have damped travelling waves.

c) Continuity of parallel $\vec{E}$: $E_{i0} + E_{r0} = E_{t0}$

Continuity of parallel $\vec{B}$: $\frac{k_i}{\omega} E_{i0} - \frac{k_i}{\omega} E_{r0} = \frac{k_c}{\omega} E_{t0}$

Combining these, we obtain $E_{r0} = \frac{k_i - k_c}{k_i + k_c} E_{i0}$, and

$$E_{t0} = \frac{2k_i}{k_i + k_c} E_{i0} .$$

a) $\epsilon_f = \frac{\hbar^2}{2m} k_f^2$, where k_f is the wave vector of the highest energy filled level. Using the Pauli principal,

$$N = 2 \cdot \frac{1}{8} \int_0^{k_f} 4\pi k^2 dk \left(\frac{V}{\pi^3}\right),$$

$$N = \frac{\pi k_f^3}{3} \cdot \frac{V}{\pi^2}, \quad k_f^3 = \frac{3\pi^2 N}{V}; \therefore,$$

$$\epsilon_f = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

b) Single particle energies: $\epsilon_{\vec{n}} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (\vec{n})^2$, where $(\vec{n})^2 = n_x^2 + n_y^2 + n_z^2$. The smallest allowed value of $(\vec{n})^2$ is three. All pions will occupy this state at $T=0$; $\therefore$

$$U(0) = N \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 \cdot 3 = \frac{3\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 N$$

c) Each magnetic sub-state is equally likely to be occupied; $\therefore$, the number of arrangements is

$$W = \frac{N!}{(N/3!)^3}, \text{ and } S = k \ln W.$$ Using Stirling's approximation, $\frac{S}{k} \simeq N \ln N - N - 3\left[\frac{N}{3}\ln\frac{N}{3} - \frac{N}{3}\right]$,

$$S \simeq NK \ln 3$$

d) $S = k\ln(Z) + \frac{U}{T}$, $\ln Z = N\ln\left[e^{\mu B/kT} + e^{-\mu B/kT}\right]$,

and $U = kT^2\frac{\partial}{\partial T}\left[\ln Z\right]$. Setting $x = \frac{\mu B}{kT}$)

we have $\frac{S}{Nk} = \ln(e^{x} + e^{-x}) - \frac{x(e^{x} - e^{-x})}{e^{x} + e^{-x}}$

For small x, $S \simeq 2Nk\left(\frac{\mu B}{kT}\right)e^{-2\mu B/kT}$

For $B/T \simeq .1$, $\mu B/kT \simeq 5 \cdot 10^{-2}$

F T comp exam

Solution

$$I_{cm} = Mk^2$$

a) apply parallel axis theorem

$$I = Mk^2 + M\ell^2$$

$$L = T - U = \tfrac{1}{2} I \dot{\theta}^2 - Mg\ell(1-\cos\theta)$$

Lagrange's eq.

$$-\frac{\partial L}{\partial \theta} + \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = I\ddot{\theta} + Mg\ell\sin\theta = 0$$

for small displacements $\sin\theta \simeq \theta$.

$$\ddot{\theta} + \underbrace{\frac{g\ell}{k^2+\ell^2}}_{\omega^2}\theta = 0$$

$$\theta(t) = A\cos(\omega t + \delta)$$

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{k^2+\ell^2}{g\ell}}$$

b) with respect to P'

$$\tau = 2\pi\sqrt{\frac{k^2+\ell'^2}{g\ell'}}$$

Solution cont.

thus
$$\frac{k^2+l'^2}{g\,l'} = \frac{k^2+l^2}{g\,l} \rightarrow k^2 = l'l.$$

and
$$T = 2\pi\sqrt{\frac{l'l+l^2}{g\,l}} = 2\pi\sqrt{\frac{l'+l}{g}}.$$

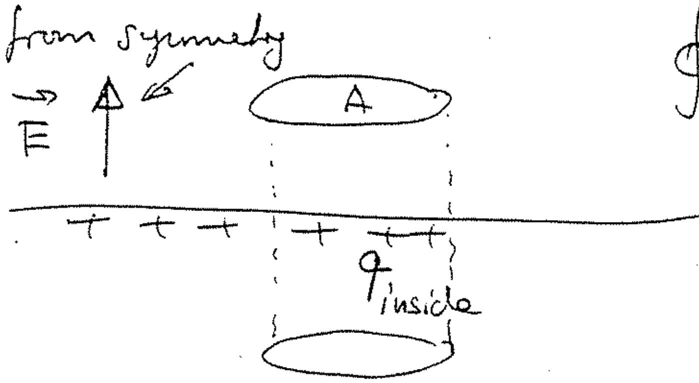
the period of a simple pendulum of length $l'+l$.

#5

E+M Solution

E field due to infinite plane

from Gauss's law



$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{inside}}{\varepsilon_0}$$

$$= \frac{\sigma A}{\varepsilon_0}$$

$$E(A) = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}.$$

When dish rises an infinitesimal distance dz. the energy of the field is decreased by an amount equal to the volume of the excluded space times the electrostatic energy density:

$$dU = -\frac{1}{2}\varepsilon_0 E^2 (\pi R^2 dz)$$

thus

$$F = -\frac{dU}{dz} = \frac{1}{2}\varepsilon_0 E^2 \pi R^2$$

$$= \frac{\pi R^2 \sigma^2}{2\varepsilon_0}$$

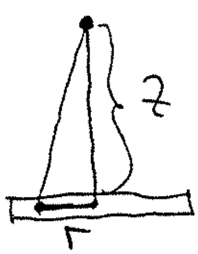
#5 E+M Solution

cont.

Alternate method.

The field has two sources – the charge on the dish and the charge on the plane. The former cannot give rise to a net force on the dish. Calculate the field due to the charge on the dish and subtract from the total field $\left(E = \frac{\sigma}{\varepsilon_0}\right)$.

Potential at z due to charge on dish:



$$V(z) = \frac{\sigma}{4\pi\varepsilon_0} \int_0^R \frac{2\pi r\,dr}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[(R^2+z^2)^{\frac{1}{2}} - z\right].$$

$$E(0) = -\left.\frac{dV}{dz}\right|_{z=0} = -\frac{\sigma}{2\varepsilon_0}\left.\left[\frac{z}{\sqrt{R^2+z^2}} - 1\right]\right|_{z=0}$$

$$= \frac{\sigma}{2\varepsilon_0}$$

The force on the dish is $q(\text{disk})\,E(0)$

$$F = \frac{\sigma \pi R^2 \sigma}{2\varepsilon_0} = \underline{\frac{\pi R^2 \sigma^2}{2\varepsilon_0}}$$

7.1 6

(1)

(a) $\Delta S = \int_1^2 \frac{dQ}{T}$; $mc = \frac{dQ}{dT}$

$\Delta S = mc \int_{T_1}^{T_2} dT/T = mc \ln T_2/T_1$

$\Delta S > 0$ if $T_2 > T_1$

(b) $\Delta S = \int_1^2 \frac{dQ}{T}$ $\Delta Q = m_x l_1$

l_1 = latent heat of fusion

$\Delta S = \frac{1}{T} \Delta Q = \frac{m_x l_1}{T}$; $\Delta S > 0$

(c) Sys $\Delta S_s = nc \int_{T_1}^{T_0} \frac{dT}{T} = nc \ln \frac{T_0}{T_1}$ $\underline{n = \text{no. of moles}}$

Reservoir $\Delta S_R = \int \frac{dQ}{T_0} = \int_{T_1}^{T_0} nc\, dT$ $nc(T_0 - T_1)$

for $T_1 > T_0$ $\begin{cases} \Delta S_s < 0 = -nc \ln \frac{T_1}{T_0} \\ \Delta S_R > 0 = nc \frac{(T_1 - T_0)}{T_0} \end{cases}$

$\underline{\Delta S = \Delta S_s + \Delta S_R = nc \frac{(T_1 - T_0)}{T_0} - nc \ln \frac{T_1}{T_0}} = nc\left[\frac{T_1}{T_0} - 1 - \ln \frac{T_1}{T_0}\right]$

for $T_1 < T_0$ $\Delta S_s > 0$ $+ nc \ln \frac{T_0}{T_1}$

$\Delta S_R = -nc \frac{(T_0 - T_1)}{T_0}$

$\Delta S = nc\left[\ln \frac{T_0}{T_1} - \frac{(T_0 - T_1)}{T_0}\right] = nc\left[\ln \frac{T_0}{T_1} + \frac{T_1}{T_0} - 1\right]$

#6 (C) conclude (2)

$T_1 > T_0 \quad \Delta S = nc\, f(x) \quad$ where $x = \frac{T_1}{T_0} > 1$

and $f(x) = x - 1 - \ln x$

$$\frac{\partial f}{\partial x} = 0 = 1 - \frac{1}{x} \quad . \quad x = 1 \quad \text{inflection point}$$

$$\frac{\partial^2 f}{\partial x^2} = +\frac{1}{x^2}\Big|_{x=1} = 1 \quad \therefore \quad x = 1 \text{ is } \underline{\text{a minimum}}$$

So for $\underline{\text{all}}$ $x > 1$ or $T_1 > T_0 \quad \Delta S > 0$

$T_1 < T_0 \quad \Delta S = nc\, f(x)\, ; \, x = \frac{T_0}{T_1} > 1$

$$f(x) = \ln x + \frac{1}{x} - 1$$

$$f'(x) = \frac{1}{x} - \frac{1}{x^2} = 0 \quad \text{inflection point at } x = 1$$

$$f'' x = -\frac{1}{x^2} + 2/x^3\Big|_{x=1} = 1 \quad \therefore \quad \text{minimum at } x = 1$$

so for all $x > 1 \quad \Delta S > 0$

(d) $\Delta S = \int_1^2 \frac{dQ}{T} \quad ; \quad dU = dQ - dW$

$dQ = 0$ and $dW = 0$

$\therefore dU = 0$ & $U = U(T)$ so $\Delta T = 0$ $\underline{\text{isotherm}}$

so consider a reversable isothermal process between the same initial & final states i.e. an expansion in contact with a resivoir at a temperature T

$\therefore \; dQ = dW = p\,dV$ & $pV = NkT$

$$\Delta S = \int_{V_1}^{V_2} \frac{p\,dV}{T} = Nk \int_{V_1}^{V} \frac{dV}{V} = Nk \ln \frac{V}{V_1}$$

#6 (c) can be considered as two free expansions (3/
as (d)

$$\Delta S = N_1 k \ln \frac{V}{V_1} + N_2 k \ln \frac{V}{V_2}$$

but. $P_1 V_1 = N_1 k T_1$ $\quad P_1 = P_2 \quad T_1 = T_2$ before mixing

So $\frac{V_1}{N_1} = \frac{kT}{P} = \frac{V_2}{N_2}$ $\quad V = V_1 + V_2, \quad N = N_1 + N_2$

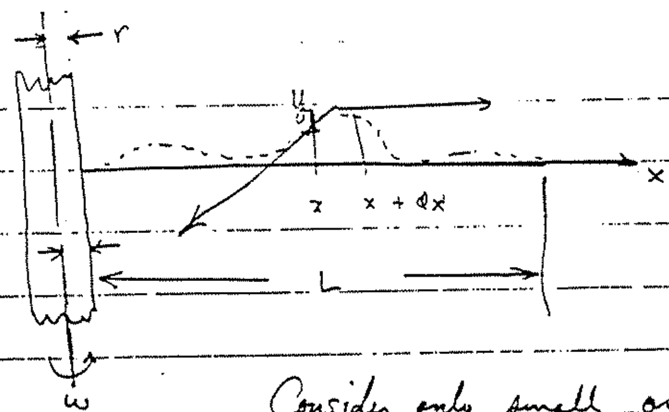
$$V = V_1 + \frac{N_2}{N_1} V_1 = V_1 \left(1 + \frac{N_2}{N_1}\right) = \frac{V_1}{N_1} N \quad \therefore \frac{V_1}{N_1} = \frac{V}{N}$$

Similarly $P = P_1 + P_2$ after mixing

$\therefore N_1 = N \frac{V_1}{V} = \frac{P V_1}{kT}$ also for N_2

$$\Delta S = \frac{P}{T} \left[V_1 \ln \frac{V}{V_1} + V_2 \ln \frac{V}{V_2} \right]$$

Question #7



r = radius of axel

Call the Tension at x, $T(x)$.

ρ = density (mass per unit length)

y = displacement from steady position

Consider only small oscillations, neglect air resistance and gravity. the vertical component of the net force acting on an element of string dx is

$$-T(x)\,y'(x) + T(x+dx)\,y'(x+dx) \quad \text{where } y' = \partial y/\partial x.$$

thus by Newton's Second Law

$$\rho\, dx \frac{\partial^2 y}{\partial t^2} = -Ty' + (T + T'dx)(y' + y''dx) = T'y''dx^2 + T'y'dx + Ty''dx$$

By dropping the terms in d^2x we obtain the differential eq

$$\rho \frac{\partial^2 y}{\partial t^2} = T'y' + Ty''$$

the tension at x must balance the centrifugal force on the string from x to L thus

$$T(x) = \int_x^L \rho\, dx\, \omega^2 (x + r) = \rho\omega^2 \left\{ r(L - x) + \frac{1}{2}(L^2 - x^2) \right\}$$

For simplicity we assume $r \ll L$ so that

$$T(x) = \frac{\rho\omega^2}{2}(L^2 - x^2) \quad \text{and} \quad T'(x) = -\rho\omega^2 x$$

the final form of the diff. eq is then

$$\rho \frac{\partial^2 y}{\partial t^2} = \frac{\rho\omega^2}{2}(L^2 - x^2)\frac{\partial^2 y}{\partial x^2} - \rho\omega^2 x \frac{\partial y}{\partial x}$$

the boundry conditions are

$$y(x=0,t) = \frac{\partial y}{\partial t}(x=0,t) = 0$$

QUESTION 8

We can expand the $+x$-direction state in terms of $\pm z$-direction states at $t=0$, say $\chi = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} = C_+ \alpha + C_- \beta$

where $\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $C_+ = \frac{1}{\sqrt{2}}$, $C_- = \frac{1}{\sqrt{2}}$

The time dependence of such a state is given by:

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} C_+ \\ C_- \end{pmatrix} = H'\begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \mu_B \sigma \cdot B \begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \mu_B B_z \sigma_z \begin{pmatrix} C_+ \\ C_- \end{pmatrix}$$

where $C_+(t) = C_+(0)\, e^{i\frac{\mu B}{\hbar}t}$

$C_-(t) = C_-(0)\, e^{-i\frac{\mu B}{\hbar}t}$

Now $\chi(t) = C_+(t)\alpha + C_-(t)\beta$

$$\langle \sigma_x \rangle = \langle \chi(t) | \sigma_x | \chi(t) \rangle$$

$$= \frac{1}{2} e^{i\frac{\mu B}{\hbar}t} + \frac{1}{2} e^{i\frac{\mu B}{\hbar}t} = \cos\frac{\mu B}{\hbar}t$$

Probability of eigenstate $+1$ in x-direction $= \cos^2\frac{\mu B}{\hbar}t$

Polarized muons are produced from $\pi \to \mu$ decay, and the angular asymmetry with respect to the muon spin direction in the emission of electrons from $\mu \to e$ decay indicate the direction of the muon spin. The angular distribution of the decay electrons with respect to the muon spin direction is

$$N(\theta) \propto 1 + a\cos\theta \qquad a = \text{const.}$$

A measurement of the decay rate in the x-direction leads to information on μ. By measuring the decay rate as a function of time relative to muon stopping times and comparing to the function $\cos^2 \mu B t/\hbar$ you can determine μ. Another method is to measure the Larmor precession of muons in a magnetic field or from measurements of induced magnetic resonance transitions between different energy levels of muons in a magnetic field.