PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #55

APRIL 4, 1987

Comprehensive Examination for Spring 1987

General Instructions

This Comprehensive Examination for Spring 1987 (#55) consists of eight problems of equal weight (20 points each). It has two parts. The first part (Problems 1-4) is handed out at 9:00 am (duration: 3 hours) and the second part (Problems 5-8) at 1:00 pm (duration: 3 hours).

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you may find useful. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solution. "Scratch" work will not be graded.

Physics Department Comprehensive Exam #55, April 4, 1987.

## PART I.

### PROBLEM #1.

The gravitational potential energy of a particle of mass m in the equatorial plane of the earth has the form:

$$V(r) = -\frac{k}{r} + \frac{c}{3r^3}$$

where k and c are constants. The second term is a small correction for the nonspherical shape of the earth.

Find the precessional angular velocity of the apogee (point of maximum r) for a nearly circular orbit in the equatorial plane in terms of the given constants and the radius, $r_o$, of the circular orbit.

### PROBLEM #2.

The Virial Theorem states that the time average of the total kinetic energy of N particles is equal to

$$-\frac{1}{2}\left\langle \sum_{\alpha=1}^{N} \vec{F}_\alpha \cdot \vec{r}_\alpha \right\rangle$$

where < > refers to a time average. $\vec{F}_\alpha$ is the net force on particle $\alpha$ at position $\vec{r}_\alpha$.

Using the Virial theorem derive the ideal gas law for a system of N particles enclosed in a container of volume V at an absolute temperature T.

PROBLEM #3.

The potentials in the radiation zone wave derived from a calculation using the retarded potentials of an oscillating magnetic dipole moment along the z axis, with $\vec{m} = \hat{k} m_o \cos\omega t$ are:

$$V(r,\theta,\phi,t) = 0$$

and $$\vec{A}(r,\theta,\phi,t) = -\frac{\mu_o m_o \omega}{(4\pi c)} \frac{\sin\theta}{r} \sin[\omega(t - r/c)]\hat{\phi}.$$

a. Assume that the dimensions of the dipole are of order s. In terms of s what approximations were used in the derivation of $\vec{A}$?

b. What are the electric and magnetic fields, $\vec{E}$ and $\vec{B}$ at $\vec{r}$?

c. What is the Poynting vector at $\vec{r}$ ?

d. What is the average intensity at $\vec{r}$ ?

The earth's magnetic north pole is $\psi$ degrees south of the geographic north pole. In terms of the earth's magnetic dipole moment M, $\psi$, and $\omega$:

e. What is the total power radiated by the earth's magnetic field?

f. If the earth's field at the equator is about 1/2 gauss, the earth's radius is about 6000 km, and the angle $\psi$ is 11°, what is the total power radiated in watts?

$\mu_o = 4\pi * 10^{-7}$ 1 gauss $= 10^{-4}$ tesla

PROBLEM #4.

The Hamiltonian for the $^2S_{1/2}$ ground state of the hydrogen atom in a magnetic field is given by;

$$H = H^\circ + \Delta E(\vec{I}\cdot\vec{J}) - \vec{\mu}\cdot\vec{B} = H^\circ + \Delta E(\vec{I}\cdot\vec{J}) - \mu_z B_z .$$

$\Delta E$ is the zero field hyperfine splitting and $\mu_z$ is the z component of the atomic magnetic moment which is the sum of the moments of the proton and the electron. These can be written as $g_n\mu_n I_z$ and $g_o\mu_o J_z$ respectively where $g_n\mu_n << g_o\mu_o$.

a. Calculate E(B), the magnetic field dependence of the appropriate energy levels to the lowest order in B relative to

   $E_o = \langle ^2S_{1/2}|H_o|^2S_{1/2}\rangle$ for $g_o\mu_o B_z >> \Delta E$.

b. Sketch E(B) vs B for the energy levels of (a).

c. Repeat (a) above for $\Delta E >> g_o\mu_o B_z$.

d. Sketch E(B) vs B for the energy levels of (c).

*Clebsch–Gordan Coefficients* $\langle j_1 j_2 m_1 m_2 | j_1 j_2 j m\rangle$.
$\langle j_1 j_2 m_1 m_2 | j_1 j_2 j m\rangle = (-1)^{j_1+j_2-j}\langle j_2 j_1 m_2 m_1 | j_2 j_1 j m\rangle$.

| $j_1 = \frac{1}{2}$ | $j_2 = \frac{1}{2}$ | $j = 1$ | | | $j = 0$ |
|---|---|---|---|---|---|
| $m_1$ | $m_2$ | $m = 1$ | $m = 0$ | $m = -1$ | $m = 0$ |
| 1/2 | 1/2 | 1 | | | |
| 1/2 | −1/2 | | $\sqrt{1/2}$ | | $\sqrt{1/2}$ |
| −1/2 | 1/2 | | $\sqrt{1/2}$ | | $-\sqrt{1/2}$ |
| −1/2 | −1/2 | | | 1 | |

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PART II.

PROBLEM #5.

A bead is constrained to slide along a smooth wire with the shape $z = Ax^2$ with the plane of the wire normal to the earth's surface.

a. What is meant by a holonomic constraint in classical mechanics? Give an example of a non-holonomic constraint.

b. Use a Lagrangian method to determine the equations of motion for the constrained bead.

c. Find the frequency of small-amplitude oscillations executed by the bead around the equilibrium position.

d. Use the Hamiltonian to find the equations of motion.

PROBLEM #6.

Nearly monochromatic light from some source is viewed through a polarizing analyzer A. There are orientations of A for which maximum intensity is transmitted but none which give zero intensity. A quarter-wave plate is appropriately installed and it is observed that there is still no orientation of A which gives zero intensity but a rotation of A by 30° from the position which formerly gave maximum intensity restores this maximum condition.

a. Describe in some detail how a polarizing analyzer and a quarter-wave plate work.

b. What is the position and orientation of the quarter-wave plate with respect to analyzer A?

c. From the information given what is your best conclusion about the polarization of the incident light? Explain your reasoning.

PROBLEM #7.

Develop the theory for the temperature dependence of the electron (and hole) carrier density, n (and p), and the mobility, $\mu$, for an intrinsic pure semiconductor. In deriving your equations, it is not necessary to keep track of all of the multiplication constants.

An intrinsic semiconductor is found to have a conductivity $\sigma = 0.010$/ohm-meter at $T_1 = 273K$. The gap width is 0.10 eV. Calculate the conductivity at $T_2 = 500K$. Assume $m_e^* = m_h^*$.

PROBLEM #8.

Show how from the observed reaction: (which takes place when slow negative pi mesons are captured in liquid deuterium)

$$\pi^- + \text{deuteron} \rightarrow 2 \text{ neutrons}$$

one can infer that the parity of the $\pi^-$ meson is odd. (The spin of the pion is 0 and the spin of the deuteron is 1.)

# 1

Effective potential

$$V_E = \frac{K}{r} + \frac{C}{3r^3} + \frac{L^2}{2mr^2}$$

$$\left.\frac{\partial V_E}{\partial r}\right)_{r_0} = -\frac{{}^{3}K}{r_0^{\cancel{2}3}} - \frac{{}^{3}C}{r_0^{\cancel{4}5}} - \frac{{}^{3}L^2}{m r_0^{\cancel{3}4}} = 0 \qquad \times \frac{3}{r_0}$$ and add to eliminate $L$.

$$m\omega_r^2 = \left.\frac{\partial^2 V_E}{\partial r^2}\right)_{r_0} = \frac{2K}{r_0^3} + \frac{4C}{r_0^5} + \frac{3L^2}{m r_0^4}$$

$$\omega_r^2 = \frac{1}{m}\left(-\frac{K}{r_0^3} + \frac{C}{r_0^5}\right) = \frac{-K}{m r_0^3}\left(1 - \frac{C}{K r_0^2}\right) \quad \approx (1+\Delta)$$

small

$$\omega_r = \sqrt{\frac{-K}{r_0^2}}\left(1 - \frac{C}{2K r_0^2}\right)$$

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$$m r_0 \dot{\theta}^2 = \frac{-K}{r_0^2} - \frac{C}{r_0^4} \qquad \dot{\theta} \equiv \omega_\theta$$

$$\omega_\theta^2 = \frac{-K}{m r_0^3}\left(1 + \frac{C}{K r_0^2}\right)$$

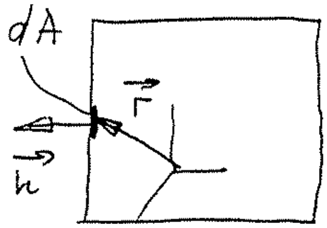
$$\omega_\theta = \sqrt{\frac{-K}{m r_0^3}}\left(1 + \frac{C}{2K r_0^2}\right)$$

$\therefore$ precessional angular velocity $\omega_p = \omega_\theta - \omega_r = \frac{1}{\sqrt{-K m r_0^3}}\left(\frac{C}{r_0^2}\right)$

#2

Solution.

For a monoatomic gas $\langle T \rangle = N\frac{3}{2}kT$




$d\vec{F}_\alpha = -P\vec{n}\,dA$.

assume internal forces are small.

$$\frac{1}{2}\sum_\alpha \vec{F}_\alpha \cdot \vec{r}_\alpha = -\frac{P}{2}\int \vec{n}\cdot\vec{r}\,dA$$

surface of container

apply gauss's theorem.

$$\int_A \vec{n}\cdot\vec{r}\,dA = \int_V \vec{\nabla}\cdot\vec{r}\,dV = 3V$$

$$\left(\vec{\nabla}\cdot\vec{r} = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z\right)$$

$$\frac{3}{2}\underbrace{Nk}_{nR}T = \frac{3}{2}PV$$

$n$ = number of moles
$R$ = gas constant

$$\boxed{PV = nRT}$$

#3 E & M

(a) 1) $s << r$ "perfect" multipole

2) $s << c/\omega = (1/k \approx \lambda$ non-relativistic or multipole

3) $kr >> 1$ or $r >> \frac{1}{k} \approx \lambda$ radiation zo

(b) $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} = -\frac{\partial A}{\partial t}$

$\vec{B} = \nabla \times \vec{A}$

use $\alpha = \omega(t - r/c)$ $\quad \frac{\partial \alpha}{\partial t} = \omega$ ; $\frac{\partial \alpha}{\partial r} = -\frac{\omega}{c}$

$$\vec{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega(t - r/c)\ \hat{\varphi}$$

$$\vec{B} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos\omega(t - r/c)\ \hat{\theta} \quad \left\{ \text{drop term of } O\left(\frac{1}{r^2}\right) \right.$$

$\therefore \quad \vec{B} = \frac{1}{c} \hat{r} \times \vec{E}$

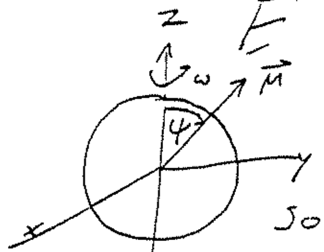
(c) $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{\mu_0 c} \vec{E} \times (\hat{r} \times \vec{E})$ $\left\{ \vec{E} \cdot \hat{r} = 0 \right.$

$$\vec{S} = \frac{E^2}{\mu_0 c} \hat{r} = \frac{\mu_0}{c}\left[\frac{m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega(t - r/c)\right]^2 \hat{r}$$

(d) $I = \langle \vec{S} \cdot \hat{r} \rangle = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \frac{\sin^2\theta}{r^2}$

as $\frac{1}{T}\int_0^T \cos^2\omega(t - r/c)\,dt = \frac{1}{2}$ $\quad \left\{ T = \frac{2\pi}{\omega} \right\}$

(c)




$\vec{M}(t) = M \sin\psi \,(\cos\omega t\, \hat{\imath} + \sin\omega t\, \hat{\jmath}) + M\cos\psi\, \hat{k}$

so oscillating magnetic dipoles along the x & y axis

Each dipole would produce fields $E^{(x)}$ and $E^{(y)}$ as in (b) with axis rotated by 90°

$$\vec{S} = \frac{1}{\mu_0 c}\left(E^{(x)}_{(t)} + E^{(y)}_{(t)}\right)^2 = \frac{1}{\mu_0 c}\left\{(E^{(x)})^2 + (E^{(y)})^2 + 2\vec{E}^{(x)}\cdot\vec{E}^{(y)}\right)$$

when the time average is taken to get $\langle S \rangle$. the $\vec{E}^{(x)}\cdot\vec{E}^{(y)}$ term will $\to 0$ as

$$\frac{1}{T}\int_0^T \sin\omega(t - r/c)\cos\omega(t - r/c)\, dt = 0$$

so $P = P^{(x)} + P^{(y)}$

$$P = \int \langle\vec{S}\rangle \cdot \hat{r}\, r^2 d\Omega \quad \text{and from (d)}$$

$$\int \sin^2\theta \sin\theta\, d\theta\, d\varphi = 2\pi\int_{-1}^{1}(1-u^2)\,du = \frac{8\pi}{3}$$

(e) $P = 2 \times \frac{8\pi}{3} \times \frac{\mu_0 M^2\omega^4}{32\pi^2 c^3} \times \sin^2\psi = \frac{\mu_0 M^2 \omega^4}{6\pi c^3}\sin^2\psi$

from static term $\vec{B} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$ $\quad \vec{m} = M\cos\psi\,\hat{k}$

at equator $\hat{k}\cdot\hat{r} = 0$ ; $\hat{k} = -\hat{\theta}$

$\therefore \vec{B} = \frac{\mu_0}{4\pi r^3} M\cos\psi\,\hat{\theta}$ $\quad \therefore M = B\cdot\frac{4\pi r^3}{\mu_0\cos\psi} = \frac{1}{2}\times 10^{-4} \times \frac{4\pi \times (6\times 10^6}{4\pi\times 10^{-7}\cos}$

$\cos(11°) \sim 1$

$M = 10^{23}\ A\text{-}m^2$

(f) $P = \frac{4\pi\times 10^{-7}\times(10^{23})^2 \times \left(\frac{2\pi}{24\times 3600}\right)^4}{6\pi\times(3\times 10^8)^3}\sin^2 11° = 4\times 10^{-5}$ watts

#4 Grad Atomic ...

(a) $H \cong H^0 + g_0 \mu_0 B_z J_z = H_0 + g_0 \mu_0 B_z S_z$

<u>for</u> 1 electron s-state $\vec{J} = \vec{S}$

$$E = E_0 + E(H) = E_0 + \langle |S_z| \rangle g_0 \mu_0 B_z$$

The states are decoupled in $I m_I S m_s$ when the $\Delta E(\vec{I} \cdot \vec{J})$ hfs term is neglected so writing states as $|I J m_I m_J\rangle$ there are 4 states

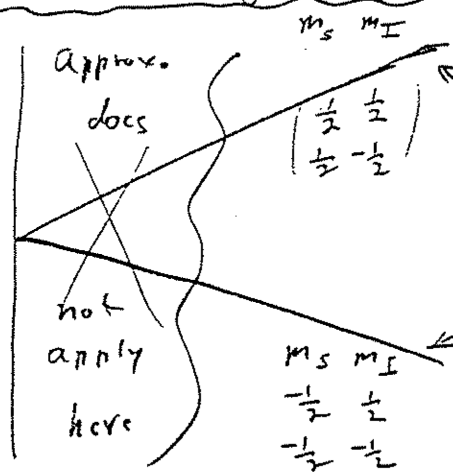
$I = \frac{1}{2}$, $J = S = \frac{1}{2}$, $m_I = \pm 1/2$, $m_J = m_S = \pm \frac{1}{2}$

since $S_z |I S m_I m_S\rangle = m_S |I S m_I m_S\rangle$

this set of states is diagonal in $S_z$

$\therefore E(B) = m_S g_0 \mu_0 B_z$

(b)




Two degenerate states $|\frac{1}{2}\ \frac{1}{2}\ \pm\frac{1}{2}\ \frac{1}{2}\rangle = |I J m_I m_J\rangle$

$|\frac{1}{2}\ \frac{1}{2}\ \pm\frac{1}{2}\ \frac{1}{2}\rangle$

(c) Use $g_0 \mu_0 B_z S_z$ as perturbation to $H^0 + \Delta E(\vec{I} \cdot \vec{J})$

States are coupled by $(\vec{I} \cdot \vec{J})_{op}$ to $F, m_F$ where $\vec{F} = \vec{J} + \vec{I}$

$$\therefore F_{op}^2 = J_{op}^2 + I_{op}^2 + 2(\vec{I} \cdot \vec{J})_{op}$$

$$(I \cdot J)_{op} = \frac{1}{2}\left(F_{op}^2 - J_{op}^2 - I_{op}^2\right)$$

states diagonal in $(\vec{J}\cdot\vec{I})$ are $|I\,J\,F\,m_F\rangle$

& $F_{op}^2\,|IJFm_F\rangle = F(F+1)\,|IJFm_F\rangle$

similarly for $J_{op}^2$ & $I_{op}^2$

since $I = J = \frac{1}{2}$ $\quad F = 1, 0$

$\therefore \langle I, J, F=1, m_F|\,\vec{I}\cdot\vec{J}\,|\quad, F=1, m_F\rangle = \frac{1}{2}\left[1\cdot 2 - \frac{1}{2}\,\frac{3}{2} - \frac{1}{2}\,\frac{3}{2}\right] = \frac{1}{4}$

$\langle I, J\; F=0, m_F|\,\vec{I}\cdot\vec{J}\,|\quad ; F=0, m_F\rangle = \frac{1}{2}\left[0\cdot 1 - \frac{1}{2}\cdot\frac{3}{2} - \frac{1}{2}\,\frac{3}{2}\right] = -\frac{3}{4}$

$\therefore E(0) = E_0(0) + \frac{\Delta E}{4}$ for states $F = 1$ $m_F = 1, 0, -1$

$E(0) = E_0(0) - 3\frac{\Delta E}{4}$ for state $F = 0$ $m_F = 0$

for $E(B)$ need to apply $g_0\mu_0 B_z S_z$ to eigenstates of $(\vec{I}\cdot\vec{J})$ expanded in terms of $I m_I J m_J$ state using C.G. coef

$$|IJ\,Fm_F\rangle = \sum_{m_I m_J} \langle IJ.m_I\, m_J|IJ.Fm_F\rangle\,|IJ\,m_J.m_I\rangle \quad \&$$

calculate $\langle IJFm_F|\,S_z\,|IJFm_F\rangle$

since $|\tfrac{1}{2}\tfrac{1}{2}1,1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\;\tfrac{1}{2}\;\tfrac{1}{2}\rangle$

$|\tfrac{1}{2}\,\tfrac{1}{2}1,0\rangle = \frac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\;\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + \frac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\,-\tfrac{1}{2}\;\tfrac{1}{2}\rangle$

$|\tfrac{1}{2}\tfrac{1}{2}1\,-1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\,-\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$

$|\tfrac{1}{2}, \tfrac{1}{2}0,0\rangle = \frac{1}{\sqrt{2}}\,|\tfrac{1}{2}\tfrac{1}{2}\;\tfrac{1}{2}\,-\tfrac{1}{2}\rangle - \frac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\,-\tfrac{1}{2}\;\tfrac{1}{2}\rangle$

$\langle\tfrac{1}{2}\,\tfrac{1}{2}\;1\,1|\,S_z\,|\tfrac{1}{2}\,\tfrac{1}{2}\;1\,1\rangle = \frac{1}{2}g\mu_0 B_z$

$\langle\tfrac{1}{2}\,\tfrac{1}{2}\;1\,0|\,S_z\,|\tfrac{1}{2}\,\tfrac{1}{2}\;1\,0\rangle = \frac{1}{2}\left(\tfrac{1}{2} - \tfrac{1}{2}\right)g_0\mu_0 B_z = 0$

$\langle\tfrac{1}{2}\,\tfrac{1}{2}\;1\,-1|\,S_z\,|\tfrac{1}{2}\,\tfrac{1}{2}\;1\,-1\rangle = -\tfrac{1}{2}\,g_0\mu_0 B_z$

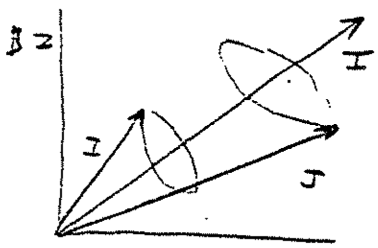
$\langle\tfrac{1}{2}\,\tfrac{1}{2}\;0\,0|\,S_z\,|\tfrac{1}{2}\,\tfrac{1}{2}\;0\,0\rangle = \frac{1}{2}\left(\tfrac{1}{2} - \tfrac{1}{2}\right)g_0\mu_0 B_z = 0$

$$\therefore E(B) = \frac{1}{2}\,g_0\,\mu_0\,B_z\;m_F \quad \{m_F = m_J + m_I\}$$

An alternate approach to (c)
uses vector model

If $\Delta E(I\cdot J) >> g_0\mu_0 B_z S_z$ "good" Q. No's are $I, J, F, m_F$

$\vec{I}$ + $\vec{J}$ couple strongly to $\vec{F} = \vec{I} + \vec{J}$




& precess

$$\therefore J_z = \frac{(\vec{J}\cdot\vec{F})}{F^2} F_z \; ; \; I_z = \frac{\vec{I}\cdot\vec{F}}{F^2} F_z$$

neglect $I_z$ term

$$\vec{J}\cdot\vec{F} = \vec{J}\cdot(\vec{J}+\vec{I}) = J^2 - \vec{J}\cdot\vec{I} = J^2 - \tfrac{1}{2}(F^2 - J^2 - I^2)$$

$$(\vec{J}\cdot\vec{F})_{op} = \tfrac{1}{2}(F_{op}^2 + J_{op}^2 - I_{op}^2)$$ use $F_{op}^2 |IJFm_F\rangle = F(F+1)|IJFm_F\rangle$ etc for $J, I$

$$g_0\mu_0 B_z J_z = g_0\mu_0 B_z \frac{1}{2}\left[\frac{F(F+1) + \cancel{J(J+1)} - \cancel{I(I+1)}}{F(F+1)}\right] F_z = \frac{1}{2} g_0\mu_0 B_z F_z$$

& $F_z |JIFm_F\rangle = m_F |JIFm_F\rangle$

$$\therefore E(B) = \frac{1}{2} g_0 \mu_0 B_z m_F$$

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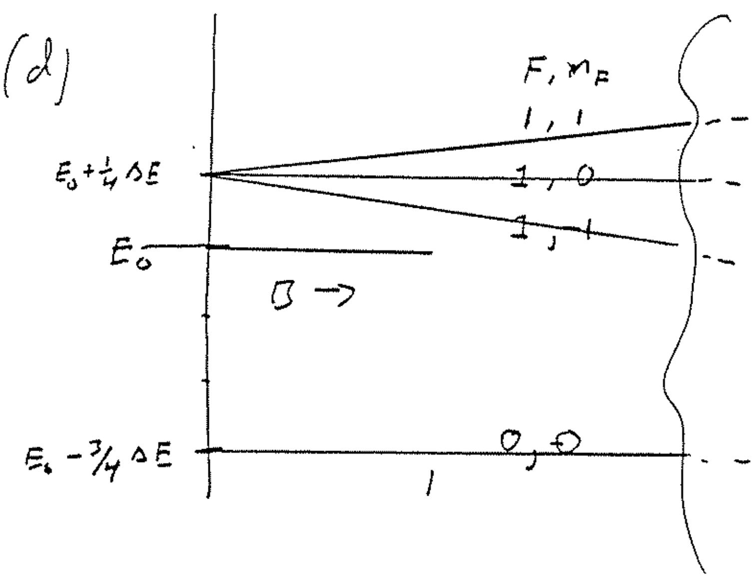
(d)




Approx not good here

#5

a. A holonomic constraint relates the coordinates and time with a functional equality. This functional must be differentiable with respect to the coordinates.

A non-holonomic constraint is one that does not relate the coordinates by an equality, but may do so by an inequality of the form:

$$f(x_1\ x_2\ x_3 \dots t) \geq 0$$

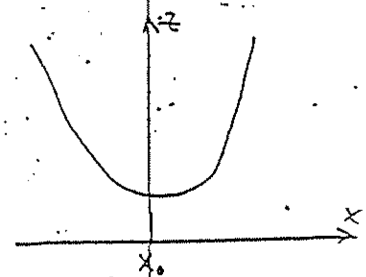
b. The Lagrangian may be written using the cartesian coordinates a

$$L = \frac{m}{2}(\dot{x}^2 + \dot{z}^2) - mgz$$

the constraint is holonomic and $f = Ax^2 - z = 0$

We can include the constraint directly or we can impose it by the method of Lagrange multipliers: the latter method we be pursued first, then

First Approach: $L = \frac{m}{2}(\dot{x}^2 + \dot{z}^2) - mgz + \lambda(Ax^2 - z)$




the Lagrange eqs + constraint condition are

$$\begin{cases} m\ddot{x} - 2A\lambda x = 0 & (1) \\ m\ddot{z} + mg + \lambda = 0 & (2) \\ Ax^2 - z = 0 & (3) \end{cases}$$

First we time differentiate (3) twice.

$$\frac{d}{dt}(Ax^2 - z) = 0 = 2Ax\dot{x} - \dot{z}$$

$$\frac{d}{dt}(2Ax\dot{x} - \dot{z}) = 0 = 2A(\dot{x}^2 + x\ddot{x}) - \ddot{z}$$

the (2) becomes

$$2Am(\dot{x}^2 + x\ddot{x}) + mg + \lambda = 0$$

now we replace $\lambda$ in (1)

$$m\ddot{x} + 2Ax[(2Am)(\dot{x}^2 + x\ddot{x}) + mg] = 0$$

$$\boxed{m\ddot{x} + 2Amgx + 4A^2mx(\dot{x}^2 + x\ddot{x}) = 0}$$

?

The last term is obviously proportional to the third power of the amplitude, while the first two are proportional to the first power so that in the limit of small oscillations we have

$$m\ddot{x} + 2Amgx = 0$$

$$\therefore \boxed{\omega^2 = 2Ag}$$

$$\therefore \lambda = -2Am(\dot{x}^2 + x\ddot{x}) - mg = \frac{m\ddot{x} + 2mAgx}{2Ax} - mg = \lim_{\text{small osc}} -mg$$

**Second Approach:**

Now if we use the constraint (3) to eliminate $z$ from $L$ we get

$$L = \frac{m}{2}\left[\dot{x}^2 + (2Ax\dot{x})^2\right] - mgAx^2$$

$$\frac{\partial L}{\partial x} = x\left[4A^2m\dot{x}^2 - 2mAg\right] \quad : \quad \frac{\partial L}{\partial \dot{x}} = \dot{x}\left[m + 4A^2mx^2\right]$$

Lagranges eq. is

$$m\ddot{x} + 4A^2m\left[\ddot{x}x^2 + 2\dot{x}^2x\right] - mx\left[4A^2\dot{x}^2 - 2Ag\right] = 0$$

$$\boxed{m\ddot{x} + 4mA^2\left[\ddot{x}x^2 + x\dot{x}^2\right] + 2mAgx = 0}$$ which is the same as we got before.

Alternate Way to treat small oscillations

We may define an effective force

$$f' = -4mA^2\left[x^2\ddot{x} + x\dot{x}^2\right] - 2mAgx$$

and $f' = m\ddot{x}$

expanding $f'$ about the equilibrium position $x_0$ (where $\dot{z} = 0$)

$$f'(x) \simeq f'(x_0) + \frac{\partial f'}{\partial x}(x - x_0) \quad : \text{ let } x - x_0 \equiv \beta \quad f'(x_0) = 0$$

$$\ddot{x} = \ddot{\beta}$$

$$m\ddot{x} = -2mAg(x - x_0)$$

$$m\ddot{\beta} = -2mAg\beta \quad \therefore \underline{\omega = 2Ag} \text{ as before.}$$

Taking the approach of Hamilton's equations

$$P_x = \frac{\partial L}{\partial \dot{x}} = \dot{x}\left[m + 4A^2mx^2\right] \quad ! \quad H = P_x\dot{x} - L:$$

Hamilton's equations are

$$\frac{\partial H}{\partial P_x} = \dot{x} \quad : \quad \frac{\partial H}{\partial x} = -\dot{P}_x$$

$$2mgAx + 4mA^2\dot{x}^2x = -\ddot{x}\left[m + 4A^2mx^2\right]$$

$$m\ddot{x} + 4mA^2\left[\ddot{x}x^2 + x\dot{x}^2\right] + 2mAgx = 0$$

as before, for small amplitudes

$$m\ddot{x} + 2mAgx = 0$$

$$\underline{\omega^2 = 2Ag}$$

$$H = \dot{x}^2(m + 4A^2mx^2) - \frac{m}{2}\left[\dot{x}^2 + (2Ax\dot{x})^2\right] + mg\ldots$$

$$= \frac{m}{2}\left[\dot{x}^2 + 4A^2x^2\dot{x}^2\right] + mgAx^2$$

$$H = T + V = E \quad \text{(conserved)}$$

$$H = \frac{\dot{x}}{2}\left[m\dot{x} + 4mA^2x^2\dot{x}\right] + mgAx^2$$

$$= \frac{\dot{x}}{2}P_x + mgAx^2 = ?$$

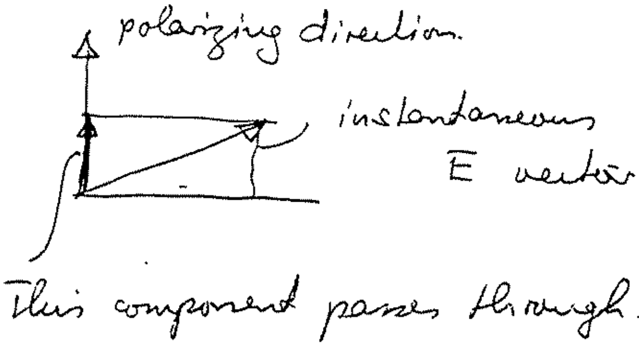
$$\frac{\partial H}{\partial P_x} = \dot{x}$$

$$\frac{\dot{x}}{2} = \dot{x} \quad ?$$

#6

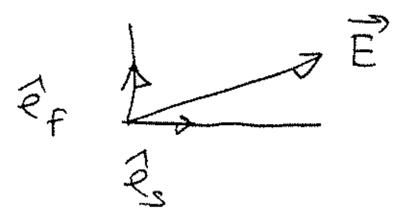
Solution:

a) polarizer

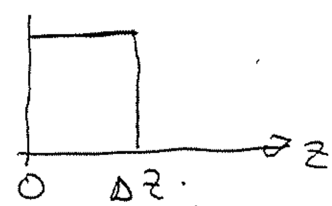



quarter wave plate:

has a fast axis (smaller index of refraction) and a slow axis (larger index of refraction). The two axes are perpendicular to each other.




$$E(0,t) = e^{i\omega t}\left[\hat{e}_s A_s e^{i\phi_s} + \hat{e}_f A_f e^{i\phi_f}\right]$$




$$E(z,t) = e^{i\omega t}\left[\hat{e}_s A_s e^{i(\phi_s - n_s \omega z/c)} + \hat{e}_f A_f e^{i(\phi_f - n_f \omega z/c)}\right]$$

phase retardation of $E_{slow}$ relative to $E_{fast}$.

$$\Delta\phi = (n_s - n_f)\frac{\omega \Delta z}{c} = (n_s - n_f)\, 2\pi \frac{\Delta z}{\lambda_{vac}}.$$

Solution.

For quarter-wave plate $\Delta\phi = \frac{\pi}{2}$.

Linearly polarized light is converted into elliptically polarized light or vice-versa.

b) quarter-wave plate in front of $\underline{A}$ with fast axis aligned with polarizing direction of A.

c) Since max is seen but no zero minimum light is either 1. partly plane polarized 2. elliptically polarized 3. partly elliptically polarized.

1. is excluded because quarter wave plate could not produce a change in position of max.
2. is excluded because rotation of $\underline{A}$ would produce zero intensity.
3 is correct. quarter-wave plate produces partly plane polarized light with max along fast axis which is rotated from semi-major axis of ellipse by 30°.

Answer: partly elliptically polarized.

# 7

a. $j = nev_d = ne\mu E \qquad v_d = \mu E$

$$= \sigma E$$

$$\sigma = \frac{ne^2 \tau}{m^*}$$

$$ne\mu = \sigma = \frac{ne^2\tau}{m^*}$$

$$\mu = \frac{e\tau}{m^*} \qquad \tau = \frac{l}{v_0} \qquad l \sim \frac{1}{T}$$

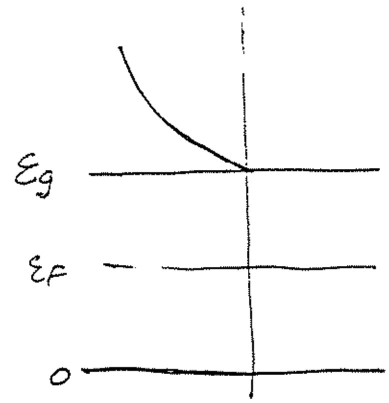
can treat e- classically, non-degenerate in conduction band (same for holes in valence band)

$$\frac{1}{2} m^* v_0^2 = \frac{3}{2} kT$$

$$v_0 = \sqrt{\frac{3kT}{m^*}}$$

so $\mu \approx \frac{e}{m^*}\sqrt{\frac{m^*}{3kT}} \cdot \frac{1}{T}$

$$\mu \sim \frac{1}{\sqrt{m^*}} \frac{1}{T^{3/2}}$$



$$g(\mathcal{E}) \sim (\mathcal{E}-\mathcal{E}_g)^{1/2}$$

$$f(\mathcal{E}) = \frac{1}{e^{\frac{(\mathcal{E}-\mathcal{E}_F)}{kT}}+1} \sim e^{-\frac{(\mathcal{E}-\mathcal{E}_F)}{kT}} \qquad \mathcal{E}-\mathcal{E}_F \gg kT$$

$$\mathcal{E}_F \approx \frac{\mathcal{E}_g}{2}$$

$$(= \text{for } m_e^* = m_h^*$$

$$n = \int_{\mathcal{E}_g}^{\infty} g(\mathcal{E})\, f(\mathcal{E})\, d\mathcal{E}$$

$$\sim \int (\mathcal{E}-\mathcal{E}_g)^{1/2}\, e^{-\frac{(\mathcal{E}-\mathcal{E}_g/2)}{kT}}\, d\mathcal{E}$$

$$\sim e^{-\mathcal{E}_g/2kT} \int_{\mathcal{E}_g}^{\infty} (\mathcal{E}-\mathcal{E}_g)^{1/2}\, e^{-\frac{(\mathcal{E}-\mathcal{E}_g)}{kT}}\, d(\mathcal{E}-\mathcal{E}_g)$$

let $x = \frac{\mathcal{E}-\mathcal{E}_g}{kT}$ then $\int_{\mathcal{E}_g}^{\infty} \rightarrow \int_0^{\infty}$ pick up $(kT)^{3/2}$

$$\sim e^{-\mathcal{E}_g/2kT}\, (kT)^{3/2} \underbrace{\int_0^{\infty} x^{\frac{1}{2}} e^{-x}\, dx}_{\text{const.}}$$

$$n \sim (T)^{3/2} e^{-\mathcal{E}_g/2kT} \qquad \text{(same for holes)}$$

$$\sigma = ne\mu \sim \frac{(T)^{3/2}\, e^{-\mathcal{E}_g/2kT}}{\cancel{T^{3/2}}}$$

$$\frac{\sigma(500)}{\sigma(273)} = \frac{\sigma(T_2)}{\sigma(T_1)} = \frac{e^{-\mathcal{E}_g/2kT_2}}{e^{-\mathcal{E}_g/2kT_1}} = e^{-\frac{\mathcal{E}_g}{2}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = e^{+\frac{\mathcal{E}_g}{2T_1T_2}(T_2-T_1)}$$

$$\frac{\sigma}{.01} = e^{+\frac{0.1}{2k = 1.38\times10^{-23}}\left(\frac{500-273}{500\times273}\right)1.6\times10^{-19}} = e^{+.964} = 2.6$$

$$\sigma(500) = 0.026/\Omega m$$

\# 8

(1) <u>Angular Momentum</u>: $\pi^-$'s are captured primarily in an S-state (i.e. orbital ang. mom. about deuteron $l=0$). This follows from $a_{0\pi} = a_{0e} \cdot \frac{m_e}{m_\pi} \approx 100 \times R_{deuteron}$. Intrin ang. mom. $S_\pi = 0$ ∴ States of 2 neutrons allowed by conservation of ang. mom. are : $^3S_1$ ; $^1P_1$ ; $^3P_1$ ; $^3D_1$.
$J=1$ states with $S=0,1$, $\vec{J} = \vec{L} + \vec{S}$

(2) <u>Exclusion Principle</u>: Two neutrons can be in only the states : $^1S_0$ ; $^3P_{0,1,2}$ ; $^1D_2$ : .... Combining (1) and (2) only $^3P_1$ is allowed for 2 neutrons in final state.
$\chi_{spin}\,\phi_{space}$ must be anti-symmetric.

(3) <u>Parity</u>: Final state : Two neutrons in $^3P_1$ state have <u>odd</u> parity (or any 2 identical particles in odd orbital angular momentum state.)
Initial state: the intrinsic parity of the deuteron is the same as that of two neutrons (even), since it consists of two nucleons in an S-state, and neutrons and protons are defined to have the same intrinsic parity. The orbital motion of the $\pi^-$ in the S-state has even parity also. Thus for the initial state to have odd parity, to conserve parity on going to the final state, the $\pi^-$ meson must have <u>odd</u> intrinsic parity.
$\pi^-$, $J^\pi = 0^-$ is a pseudoscalar meson.