PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #54

JAN. 10, 1987

Comprehensive Examination for Winter 1987

General Instructions

This Comprehensive Examination for Winter 1987 (#54) consists of eight problems of equal weight (20 points each). It has two parts. The first part (Problems 1-4) is handed out at 9:00 am (duration: 3 hours) and the second part (Problems 5-8) at 1:00 pm (duration: 3 hours).

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you *may* find useful. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solution. "Scratch" work will *not* be graded.

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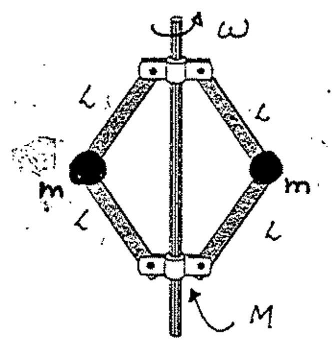
PART I.

PROBLEM #1.

A "flyball governor" is a device used to control the speed of an engine. It consists (rf. figure) of two balls each of mass $\underline{m}$, attached by means of four hinged arms, each of length $\underline{L}$, to sleeves which slide on a vertical rod. The upper sleeve is fastened to the rod; the lower sleeve of mass $\underline{M}$ is free to slide up and down the rod as the balls move out from or toward the rod. The rod-ball system rotates with constant angular velocity, $\underline{\omega}$, about the vertical rod.

a) Set up the Lagrangian equation of motion neglecting the weight of the arms and vertical rod. Find the effective potential and discuss the motion.

b) Determine the value of the height $\underline{z}$ of the lower sleeve above its lowest point as a function of $\underline{\omega}$ for steady rotation of the balls, and find the frequency of small oscillations of $\underline{z}$ about this steady state value.



A flyball governor.

PROBLEM #2.

Two physics students are in a space ship which moves with speed v in the positive x-direction of an inertial frame of reference (Assume $v = 0.5c$). Student A is at $x' = -L$ and student B at $x' = +L$, respectively. Student A sends a light signal to student B. As soon as B receives the signal he sends it back to A.

a) Construct a space-time diagram of these events.

b) According to an observer C at $x = 0$ in the intertial frame of reference, when does B receive the signal and when does A get the reply? [set $t=t'=0$ when the origins of the two coordinate systems coincide]

c) Observer C uses the interval Δt [for signal to travel A→B→A] to determine the distance Δx between A and B. Compare his measurement with a similar measurement by students A and B.

PROBLEM #3.

A spherical shell of radius R having a uniform charge density σ is spinning around an axis with angular velocity ω. Find the following quantities:

a) the exact vector potentials $\vec{A}(r,\theta,\phi)$ both inside and outside the sphere. The polar axis is along the axis of rotation,

b) the exact magnetic fields, $\vec{B}(r,\theta,\phi)$ inside and outside the sphere,

c) the magnetic moment, $\vec{m}$, of the rotating shell,

d) the first non-zero term in a magnetic multipole expansion of $\vec{A}(r,\theta,\phi)$ for $r > R$,

e) the gyromagnetic ratio of the system, assuming a uniform mass density.

Possibly useful integrals:

$$\int \frac{x\,dx}{(ax^2 + bx + c)^{1/2}} = \frac{(ax^2 + bx + c)^{1/2}}{a}$$

$$\int \frac{x\,dx}{(ax + b)^{1/2}} = \frac{2(ax - 2b)}{3a^2}(ax + b)^{1/2}$$

$$\int \frac{dx}{(ax + b)^{1/2}} = \frac{2(ax + b)^{1/2}}{a}$$

PROBLEM #4.

Find the distribution of magnetization current corresponding to a uniformly magnetized sphere with magnetization M. Find also the density of magnetic poles corresponding to a uniform magnetization.

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PART II.

PROBLEM #5.

Calculate γ, the ratio of the heat capacity at constant pressure to the heat capacity at constant volume using classical concepts for:

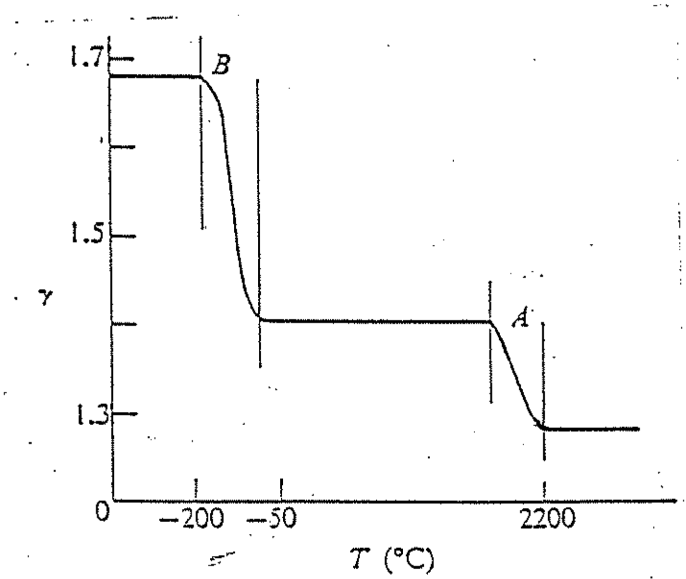
(a) an ideal monatomic gas,

(b) an ideal diatomic gas.

Specify the physical assumptions made in the model used for your calculation.

Real monatomic gases such as He, Ne, etc. agree well with (a) over a large temperature range. However, as shown below, the experimental value for γ vs. T for the diatomic gas H_2 does not agree at all with (b) if based on a simple classical model.

Explain in 250 words or less.



Experimental values of γ for hydrogen as a function of temperature. The temperature is plotted on a logarithmic scale.

PROBLEM #6.

The groundstate of the hydrogen atom is split into two hyperfine states (F=0 and F=1) by the electron spin-nuclear spin interaction. The atom is placed in a constant magnetic field. The perturbation matrix is

$$V = \begin{pmatrix} 0 & V_{01} \\ V_{10} & 0 \end{pmatrix}$$

with $V_{10} = V_{01}^*$ (* = complex conjugate). If at t=0 the hydrogen atom is in the F=0 state, determine the amplitudes for finding it in either the F=0 or F=1 state, respectively, at any later time.

PROBLEM #7.

A quantum mechanical particle of mass m moves in a potential $V(x) = mgx$.

a) Evaluate $\frac{d}{dt}\langle x\rangle$ and $\frac{d}{dt}\langle p\rangle$.

b) Solve the Schroedinger equation for the state function ϕ in momentum space

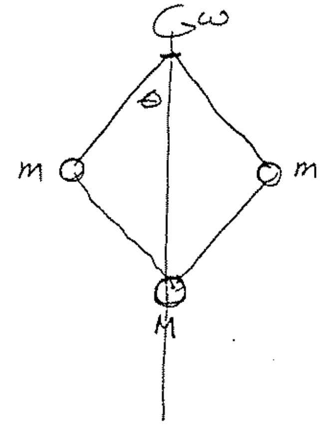
$[H(p,x)\phi(p) = E\phi(p)]$.

What energy values are allowed?

PROBLEM #8.

Correlate and discuss your knowledge of the effects of atomic electrons on nuclear transitions and measurements. Specifically, what would be the direction and approximate magnitude of the disagreements between two experimental nuclear physicists, one of whom made measurements on normal atoms as we know them, while his colleague could make measurements only on nuclei which possessed no atomic electrons, but were the same in all other respects. Consider observations on nuclear decay schemes and life-times and measurements of nuclear charge, size, mass, and moments. Discussions should not be too wordy. Make brief statements, and support them when needed by pertinent calculations and sketches.

#1



$$T = m\ell^2\sin^2\theta\,\dot\phi^2 + m\ell^2\dot\theta^2 + 2M\ell^2\sin^2\theta\,\dot\theta^2$$
$$= m\ell^2\sin^2\theta\,\dot\phi^2 + (2M\sin^2\theta + m)\ell^2\dot\theta^2$$

$$V = -2(M+m)g\ell\cos\theta$$

(a) $$L = T - V = (2M\sin^2\theta + m)\ell^2\dot\theta^2 + m\ell^2\sin^2\theta\,\dot\phi^2 + 2(M+m)g\ell\cos\theta$$

$$\frac{\partial L}{\partial\dot\theta} = 2(2M\sin^2\theta + m)\ell^2\dot\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = (4M\sin\theta\cos\theta\,\dot\theta)2\ell^2\dot\theta + 2(2M\sin^2\theta + m)\ell^2\ddot\theta$$

$$\frac{\partial L}{\partial\theta} = 2m\ell^2\sin\theta\cos\theta\,\omega^2 + 4M\ell^2\dot\theta^2\sin\theta\cos\theta - 2(M+m)g\ell\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = 2(2M\sin^2\theta + m)\ell^2\ddot\theta + 4M\ell^2\sin\theta\cos\theta\,\dot\theta^2 - 2m\ell^2\sin\theta\cos\theta\,\omega^2 + 2(M+m)g\ell\sin\theta = 0$$

$$(2M\sin^2\theta + m)\ell^2\ddot\theta + 2M\ell^2\sin\theta\cos\theta\,\dot\theta^2 - m\ell^2\sin\theta\cos\theta\,\omega^2 + (M+m)g\ell\sin\theta = 0$$

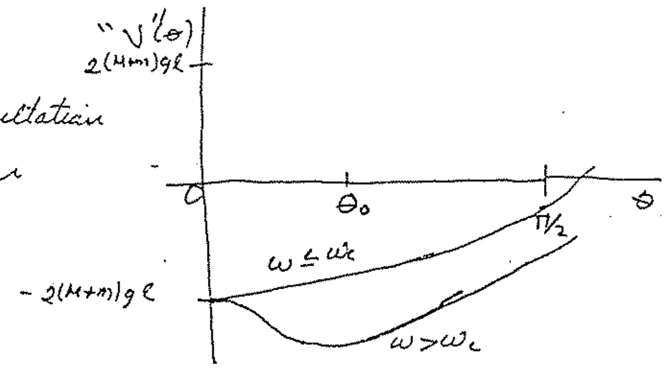
$$\dot\theta\frac{\partial L}{\partial\dot\theta} - L = (2M\sin^2\theta + m)\ell^2\dot\theta^2 - m\ell^2\sin^2\theta\,\omega^2 - 2(M+m)g\ell\cos\theta = E$$

$$\text{"}V\text{"} = -m\ell^2\sin^2\theta\,\omega^2 - 2(m+M)g\ell\cos\theta$$
$$\frac{\partial\text{"}V\text{"}}{\partial\theta} = -2m\ell^2\sin\theta\cos\theta\,\omega^2 + 2(m+M)g\ell\sin\theta = 0$$

$$\cos\theta_c = \frac{(m+M)g}{m\ell\omega^2}, \quad \omega_c = \sqrt{\frac{(m+M)g}{m\ell}}$$

$$0 \le \theta \le \pi/2$$

If $\omega \le \omega_c$ the governor will rotate without oscillation for all values of "E" between $-2(M+m)gl$ and $-ml^2\omega^2$. If $\omega > \omega_c$ and "E" = "V"(θ_0) the governor will rotate without oscillation. If "E" > "V"(θ_0) the governor will oscillate.



(b) If $\ddot{\theta} = \dot{\theta} = 0$,

$$ml\cos\theta_0\omega^2 = (M+m)g \qquad \cos\theta_0 = 1 - z/2l$$

$$\therefore z_0 = 2l\left(1 - \frac{(M+m)g}{ml\omega^2}\right)$$

$$\left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\theta=\theta_0} = -2ml^2\cos^2\theta_0\,\omega^2 + 2ml^2\sin^2\theta_0\,\omega^2 + 2(m+M)gl\cos\theta_0$$

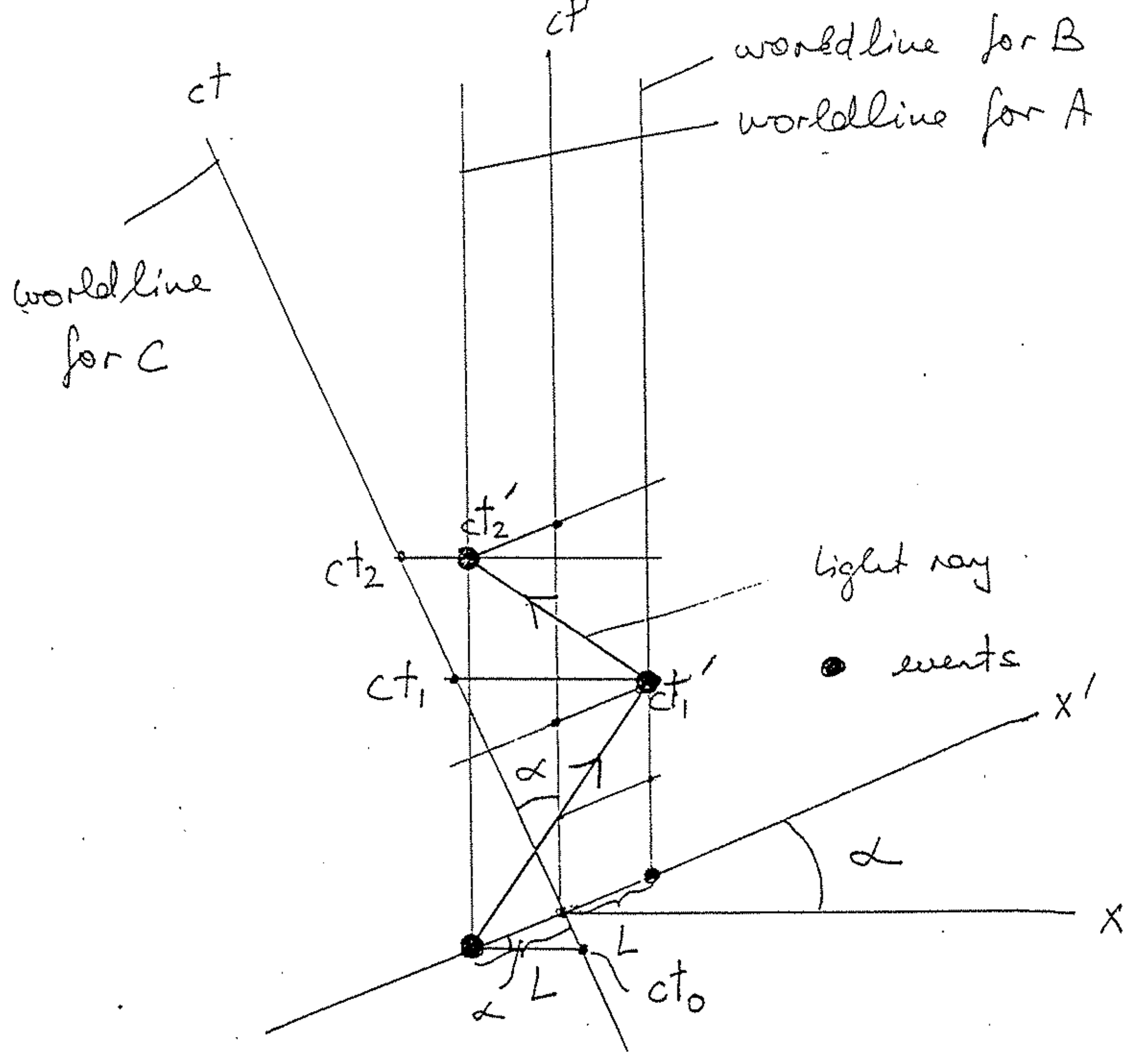
$$M\omega_\theta^2 = 2ml^2(1 - 2\cos^2\theta_0)\,\omega^2 + 2(m+M)gl\cos\theta_0$$

$$\omega_\theta^2 = \frac{2ml^2(1-2\cos^2\theta_0)\dfrac{(m+M)g}{ml\cos\theta_0} + 2(m+M)gl\cos\theta_0}{2(2M\sin^2\theta_0 + m)l^2}$$

$$= \frac{2gl(m+M)\sin^2\theta_0}{2\cos\theta_0(2M\sin^2\theta_0 + m)l^2} = \frac{g(m+M)\sin^2\theta_0}{l\cos\theta_0(2M\sin^2\theta_0 + m)} = \omega_\theta^2$$

Relativity Problem : Solution #2

1) a) Loedel diagram



b) from Loedel diagram.

A sends photon at $t' = 0$

$$ct_0 = - L \sin\alpha / \cos\alpha$$

$$\boxed{t_0 = - \frac{L}{c} \frac{v}{c} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

B receives photon at $t_1' = \frac{2L}{c}$

$$ct_1 \cos\alpha = ct_1' + L\sin\alpha$$

$$ct_1 = \frac{L(2 + \sin\alpha)}{\cos\alpha}$$

$$\boxed{t_1 = \frac{L}{c}\frac{\left(2 + \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

A receives photon at $t_2' = \frac{4L}{c}$.

$$ct_2 \cos\alpha = ct_2' - L\sin\alpha.$$

$$= L(4 - \sin\alpha)$$

$$\boxed{t_2 = \frac{L}{c}\frac{\left(4 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

c) for observer C $\Delta x = c\Delta t$

$\Delta t = t_2 - t_0$

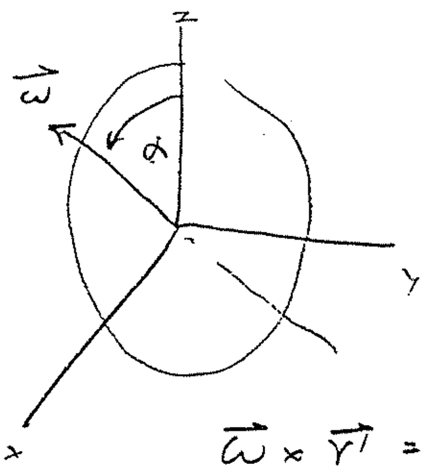
for observers A and B $\Delta x' = c\Delta t' = 4L$

$$\Delta x = \frac{4L}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note $\Delta x > \Delta x'$ unlike standard lengths comparisons where $\Delta x < \Delta x'$

#3 E & M

Consider $\vec{r}$ along the z axis with $\vec{\omega}$ in the x–z plane at an polar angle α.



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}\,')\,da'}{|\vec{r}-\vec{r}\,'|}$$

as $\vec{J} = \vec{K}\,\delta(r'-R)$

$\vec{r} = \hat{k}\,r$

$\vec{K} = \sigma\vec{v} = \sigma(\vec{\omega}\times\vec{r}\,')$

$da' = R^2\sin\theta'\,d\theta'\,d\varphi'$ $\quad |\vec{r}-\vec{r}\,'| = (R^2+r^2-2Rr\cos\theta')$

$$\vec{\omega}\times\vec{r}\,' = (\omega\sin\alpha\,\hat{\imath} + \omega\cos\alpha\,\hat{k})\times R(\hat{\imath}\sin\theta'\cos\varphi' + \hat{\jmath}\sin\theta'\sin\varphi' + \hat{k}\cos\theta')$$

$$= R\omega\big[-\hat{\imath}(\cos\alpha\sin\theta'\sin\varphi') + \hat{\jmath}(\cos\alpha\sin\theta'\cos\varphi' - \sin\alpha\cos\theta') + \hat{k}(\sin\alpha\sin\theta'\sin\varphi')\big]$$

$\therefore$ Since $\int_0^{2\pi}\cos\varphi'\,d\varphi' = \int_0^{2\pi}\sin\varphi'\,d\varphi' = 0$ only $\neq 0$

$$\vec{A}(\vec{r}) = \frac{-\mu_0 R^3\omega\sigma\sin\alpha}{4\pi}\times 2\pi\int_0^{\pi}\frac{\cos\theta'\sin\theta'\,d\theta'}{(R^2+r^2-2Rr\cos\theta')^{1/2}}\;\hat{\jmath}$$

$\cos\theta' = u$

$$I = \int_{-1}^{1}\frac{u\,du}{(a+bu)^{1/2}} \qquad a = R^2+r^2,\quad b = -2Rr$$

$$= -\frac{R^2+r^2+Rr}{3R^2r^2}|R-r| + \frac{R^2+r^2-Rr}{3R^2r^2}(R+r)$$

for $r > R$ outside $= 2R/3r^2$

for $r < R$ inside $= 2r/3R^2$

$$\therefore\ \vec{A} = -\frac{\mu_0 R^4\omega\sigma\sin\alpha}{3\,r^2}\,\hat{\jmath} \qquad r > R$$

$$\vec{A} = -\frac{\mu_0 R\omega\sigma}{3}\,r\sin\alpha\,\hat{\jmath} \qquad r < R$$

but since $\vec{r} = \hat{k}r$ & $\vec{\omega}$ has $\hat{\imath}$ + $\hat{k}$ component

$$r\omega \sin\alpha\, \hat{\jmath} = -\vec{r} \times \vec{\omega}$$

$$\vec{A} = \begin{cases} \dfrac{\mu_0 R^4 \sigma}{3r^3}(\vec{\omega}\times\vec{r}) & r > R \\ \dfrac{\mu_0 R \sigma}{3}(\vec{\omega}\times\vec{r}) & r < R \end{cases}$$

Now let $\vec{\omega} = \omega\hat{k}$ & $\vec{r} = \vec{r}(r,\theta,\varphi)$

(a)
$$\vec{A}(r,\theta,\varphi) = \begin{cases} \dfrac{\mu_0 R^4 \sigma\omega}{3}\dfrac{\sin\theta}{r^2}\,\hat{\varphi} & r > R \\ \dfrac{\mu_0 R\sigma\omega}{3} r\sin\theta\,\hat{\varphi} & r < R \end{cases}$$

$$\vec{B} = \nabla\times\vec{A}$$

(b)
$$\vec{B}(r,\theta,\varphi) = \begin{cases} \dfrac{\mu_0 R^4 \omega\sigma}{3r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) \\ \dfrac{2\mu_0 R\,\omega\sigma}{3}(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) = \dfrac{2\mu_0 R\omega\sigma}{3}\hat{k} = \dfrac{2}{3}\mu_0 R\sigma\,\vec{\omega} \end{cases}$$

(c)
$$\vec{m} = \frac{1}{2}\int (\vec{r}\times\vec{J})\, d\tau$$

$$\vec{J} = \rho\vec{v} = \sigma\,\delta(r-R)(\vec{\omega}\times\vec{r}) \quad ; \omega = \hat{k}\omega$$

$$\vec{m} = \frac{\sigma\omega R^4}{2}\int \hat{r}\times(\hat{k}\times\hat{r})\sin\theta\, d\theta\, d\varphi$$

$$\vec{m} = \frac{\sigma\omega R^4}{2}\int [\hat{k} - \hat{r}(\hat{r}\cdot\hat{k})]\sin\theta\, d\theta\, d\varphi \qquad \hat{r}\cdot\hat{k} = \cos\theta$$

in $\hat{r}$ term $\hat{\imath}$ & $\hat{\jmath}$ components $\to 0$ for $\int_0^{2\pi}\sin\varphi\, d\varphi = \int_0^{2\pi}\cos\varphi\, d\varphi = 0$

$$\vec{m} = \frac{\sigma\omega R^4}{2}\hat{k}\,2\pi\int[1-\cos^2\theta]\sin\theta\, d\theta = \frac{\sigma\omega R^4}{2}2\pi\frac{4}{3} = \frac{4\pi}{3}\sigma\omega R^4\,\hat{k}$$

(d) $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$

$\vec{A}(r,\theta,\varphi) = \frac{\mu_0}{4\pi} \frac{4\pi}{3} \sigma \omega R^4 \frac{\hat{k} \times \hat{r}}{r^2} = \frac{\mu_0 \sigma \omega R^4}{3} \frac{\sin\theta}{r^2} \hat{\varphi}$

<u>Same</u> as (a) so is exact!

(e) $g = m/L$

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = \int \left[\vec{r} \times \rho_g(\vec{r}) \vec{v}\right] d\tau$

ρ_g = mass density $= \sigma_g \delta(r-R)$; $\vec{v} = \vec{\omega} \times \vec{r}$

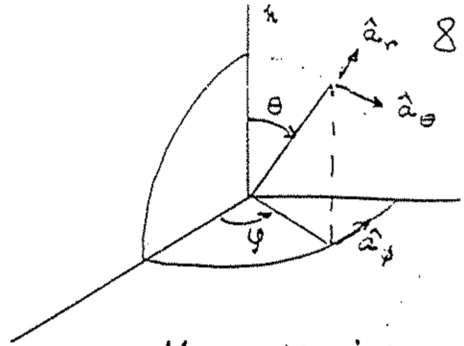
$= \sigma_g R^4 \omega \int \hat{r} \times (\hat{\omega} \times \hat{r}) \sin\theta \, d\theta \, d\varphi$

$\int$ same as in (c) $= \frac{8\pi}{3} \hat{k}$

$\vec{L} = \sigma_g R^4 \omega \frac{8\pi}{3} \hat{k}$

$g = \frac{\frac{4\pi}{3} \sigma \omega R^4}{\frac{8\pi}{3} \sigma_g \omega R^4} = \frac{4\pi R^2 \sigma \cdot \cancel{\omega} \, \cancel{R^2}}{2 \cdot 4\pi R^2 \sigma_g \cdot \cancel{\omega} \, \cancel{R^2}} = \frac{Q}{2M}$

Problem #4



$$M = M_0 \hat{z}$$

$$J_M = \text{curl } \vec{M} = 0$$

$$= \hat{a}_\phi \frac{1}{r}\left[\frac{\partial(rM_\theta)}{\partial r} - \frac{\partial M_r}{\partial \theta}\right]$$

$$= \hat{a}_\phi \frac{1}{r}\left[-M\sin\theta + M\sin\theta\right] = 0$$

$M_\theta = -M\sin\theta$
$M_r = M\cos\theta$
$\vec{M} = M_r\hat{a}_r + M_\theta\hat{a}_\theta$

the volume magnetization current $= 0$
next consider the surface magnetization current

$$\vec{j}_M = \vec{M} \times \hat{n} = M \times \hat{a}_r$$

$$= M_0 \hat{z} \times \hat{a}_r = \underline{M_0 \sin\theta}\ \hat{a}_\phi$$

which has a maximum value around the equator and is zero around the poles.

Now, we compute the volume density of magnetic poles

$$-\rho_M = \text{div } M(\vec{r}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 M_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(M_\theta \sin\theta)$$

$$= \frac{2rM\cos\theta}{r^2} - \frac{M}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta)$$

$$= \frac{2M\cos\theta}{r} - \frac{M\,2\cancel{\sin\theta}\cos\theta}{r\cancel{\sin\theta}} = 0$$

and the surface density of magnetic poles

$$\sigma_M = \vec{M}\cdot\hat{n} = (M_r\hat{a}_r + M_\theta\hat{a}_\theta)\cdot\hat{a}_r = M_r = \underline{M\cos\theta}$$

the only magnetic poles are distributed around the geometric poles ($\hat{z}$) as $\cos\theta$

#5 Thermo

ideal gas - point, elastic, isotropic interaction

(a) 3 degrees of freedom v_x, v_y, v_z

(b) 7 degrees of freedom $v_x\, v_y\, v_z$

+ 2 rotation angles as $I = 0$ along internuclear axis

+ 2 vibration degrees of freedom r & $\dot{r}$ where r is distance between atoms

Each degree of freedom provides $\frac{1}{2}kT$ of internal energy / molecule.

$$C_V = \left.\frac{dQ}{dT}\right|_V = \frac{dU}{dT} + P\left.\cancel{\frac{dV}{dT}}\right|_V$$

1st Law $\Delta U = \Delta Q - \Delta W$

$\Delta W = P\Delta V$ work out

$$C_P = \left.\frac{dQ}{dT}\right|_P = \frac{dU}{dT} + P\left.\frac{dV}{dT}\right|_P$$

$U = U(T)$ only

but $PV = nkT$ or $PdV = nk\,dT$

$$\therefore C_P = C_V + nk$$

$$\gamma = C_P/C_V = 1 + \frac{nk}{C_V}$$

(a) $U = 3/2\, nkT \quad \therefore C_V = 3/2\, nkT$

$\gamma = 1 + 2/3 = 1.67$

(b) $U = 7/2\, nkT \quad C_V = 7/2\, nk$

$\gamma = 1 + 2/7 = 1.29$

Since Rotation + Vibration are quantized a minimum energy/molecule $kT \sim$ 1st excited state is required to excite the respective motions

Below $T \approx B$ neither vibration nor rotation is excited so gas acts like a monotomic gas with only 3 translational degrees of freedom. $\therefore \gamma = 1.67$

For $A \lesssim T \lesssim B$ one of either vibration or rotation is excited. (We know from other considerations that it is rotation that has the lowest threshold.) so have $\underline{5}$ degrees of freedom

$\therefore \gamma = 1 + 2/5 = 1.40$

For $T > A$ both vibration + rotational levels are excited so $\gamma = 1.29$ as predicted by the classical theory

4 6.

Solution : QM Problem. 11

Let the amplitudes of the $F=0$ and $F=1$ states be a_0 and a_1 respectively.

$$i\frac{da_0}{dt} = H_{00}a_0 + H_{01}a_1 = E_0 a_0 + V_{01} a_1$$

$$i\frac{da_1}{dt} = H_{10}a_0 + H_{11}a_1 = V_{01}^* a_0 + E_1 a_1$$

if $a_0 = A_0 e^{-i\omega t}$ and $a_1 = A_1 e^{-i\omega t}$ then

$$(\omega - E_0)A_0 - V_{01}A_1 = 0$$
$$V_{01}^* A_0 + (E_1 - \omega)A_1 = 0$$

$$\begin{vmatrix} \omega - E_0 & -V_{01} \\ V_{01}^* & E_1 - \omega \end{vmatrix} = 0$$

$$\omega^2 - (E_0+E_1)\omega + E_0E_1 - V_{01}^2 = 0$$

$$\omega_{+,-} = \frac{E_0+E_1 \pm \sqrt{(E_0-E_1)^2 + 4V_{01}^2}}{2}.$$

substitute into algebraic eq.

let $a_0 = A_0 e^{-i\omega_+ t} + B_0 e^{-i\omega_- t}$

$a_1 = A_1 e^{-i\omega_+ t} + B_1 e^{-i\omega_- t}$

$$\frac{A_0}{A_1} = \frac{2V_{01}}{(E_1 - E_0) + \sqrt{\quad}}$$

$$\frac{B_0}{B_1} = \frac{2V_{01}}{(E_1 - E_0) - \sqrt{\quad}}$$

from initial conditions $\quad A_0 + B_0 = 1$

$$A_1 + B_1 = 0.$$

Probability requirements

$$A_0^2 + B_0^2 + A_1^2 + B_1^2 = 1 \qquad A_0 B_0 + A_1 B_1 = 0$$

Solve for A_0, B_0, A_1, B_1

Use $\quad B_0 = 1 - A_0 \qquad B_1 = -A_1$

substitute into $\quad \dfrac{B_0}{B_1} = \dfrac{1 - A_0}{-A_1} = \dfrac{2V_{01}}{(E_1 - E_0) - \sqrt{\quad}}$

solve for A_1 and substitute into $\quad \dfrac{A_0}{A_1} = \dfrac{2V_{01}}{E - E_0 + \sqrt{\quad}}$

yields

$$A_0 = \frac{1}{2}\left[\sqrt{1 + \frac{4V_{01}^2}{(E_1 - E_0)^2}} - 1\right]$$

$$B_0 = \frac{1}{2}\left[\sqrt{1 + \frac{4V_{01}^2}{(E_1 - E_0)^2}} + 1\right]$$

$$A_1 = \frac{V_{01}}{E_1 - E_0} = -B_1$$

substitute into

$$a_0 = A_0 e^{-i\omega_+ t} + B_0 e^{-i\omega_- t}$$

$$a_1 = A_1 e^{-i\omega_+ t} + B_1 e^{-i\omega_- t}.$$

#7

$$\hat{H} = \frac{p^2}{2m} + V = \frac{p^2}{2m} + mgx \qquad \hat{p} = \frac{\hbar}{i}\frac{d}{dx}$$

$$\frac{d\langle x\rangle}{dt} = +\frac{i}{\hbar}\langle[H, x]\rangle$$

$$= \frac{i}{\hbar}\langle\left[\left(\frac{p^2}{2m} + mgx\right), x\right]\rangle$$

$$= \frac{i}{2m\hbar}\left[p^2, x\right] + \frac{i}{\hbar}mg\underbrace{[x, x]}_{0} = \frac{\hbar}{mi}\langle\,|\tfrac{d}{dx}|\,\rangle$$

$$[p^2, x] = -\hbar^2\left[\frac{d^2}{dx^2}, x\right] = -2\hbar^2\frac{d}{dx} \qquad = \frac{\langle p\rangle}{m}$$

(Ehrenfest)

$$\frac{d^2}{dx^2}(xf) - x\frac{d^2f}{dx^2}$$

$$\frac{d}{dx}(xf) = f + x\frac{df}{dx}$$

$$\frac{d^2}{dx^2}(\,) = \frac{df}{dx} + \frac{df}{dx} + x\frac{d^2f}{dx^2}$$

$$\frac{d\langle p\rangle}{dt} = \frac{i}{\hbar}\langle[H, p] = \frac{i}{\hbar}\langle\left[\left(\frac{p^2}{2m} + mgx\right), p\right]$$

$$[p^2, p] = 0$$

$$[x, p] = -\frac{\hbar}{i} \qquad \frac{d\langle p\rangle}{dt} = -mg = \langle -\frac{\partial V}{\partial x}\rangle$$

b)

$$\hat{H} = \frac{p^2}{2m} + mgx$$

$$\hat{p} = p$$

$$\hat{x} = -\frac{\hbar}{i}\frac{\partial}{\partial p}$$

$$H\varphi = E\varphi$$

$$\frac{p^2}{2m}\phi - \frac{mg\hbar}{i}\frac{d\phi}{dp} = E\phi$$

$$\frac{d\phi}{dp} = \frac{-i}{mg\hbar}\left[\frac{E}{} - \frac{p^2}{2m}\right]\phi$$

$$\phi(p) = A\,e^{\frac{i}{mg\hbar}\left[Ep - \frac{p^2}{6m}\right]}$$

apparently all energy values are allowed.

#8

If no electron, then

no internal conversion

no electron capture transitions

isomers would be longer lived

there would be more stable isobars, e.g. Li^7 & Be^7

mean life for α decay would be increased

" " " β^+ " " "

" " " β^- " " decreased

kinetic energy of emitted α rays would be increased

" " " β^+ " "

cross sections for (α, n) etc would be changed

there would be no screening corrections

there would be no compton collisions

" " " photoelectric abs. of photons

With respect to determination of nuclear charge

there would be no characteristic x-rays

" " Moseley's law

" " Auger effect

" " chemistry

" " Compton scattering

" " Mesic Atoms

With respect to determination of radius

there would be no optical isotope shift

radii deduced from α decay would appear smaller

" " anomalous scattering would appear smaller

With respect to determinations of mass:
there would be no mass-spectroscopic doublets
the Q values for nuclear reactions would shift
With respect to determination of nuclear moments:
there would be no hyperfine structure
" " " band spectra
" " electric quadrupole interactions
Conclusion: "Electrons are the nuclear physicists' best friend."