— Reading Room —

PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #53

April 5, 1986

Comprehensive Examination for Spring 1986

General Instructions

This Comprehensive Examination for Spring 1986 (#53) consists of eight problems of equal weight (20 points each). It has two parts. The first part (Problems 1-4) is handed out at 9:00 a.m. (duration: 3 hours) and the second part (Problems 5-8) at 1:00 p.m. (duration: 3 hours)

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you may find useful. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solution. "Scratch" work will not be graded.

Physics Department Comprehensive Exam #53, April 5, 1986

PART I.

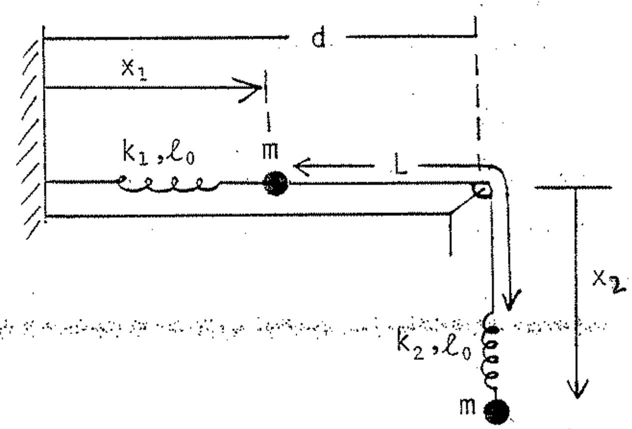
PROBLEM #1.

A spherical satellite of mass m and radius a moves in a circular orbit about the earth with speed v. It moves through an atmosphere of constant density ρ.

a) Find the frictional force (which can be considered small in comparison to the gravitational force). Assume the average velocity of the air molecules can be neglected in comparison to the velocity of the ~~rocket~~ satellite and each molecule when struck becomes embedded in the skin of the satellite.

b) If the orbit is 400 km above the earth (earth radius = 6400 km) where $\rho = 10^{-11}$ kg/m^3, and a = 1 m, m = 100 kg, calculate (to one sig. fig.) the change in altitude and the change in period of revolution in one week of orbiting.

PROBLEM #2.



In the system shown the two masses are equal and the undistorted lengths of the springs are both ℓ_0, but $k_1 \neq k_2$. A string of length L connects the mass at x_1 with the top of the spring connected to the mass at x_2. The masses move in straight lines.

a) Find the kinetic and potential energies of the system in terms of x_1, x_2.

b) Reformulate in terms of $\eta_1 = x_1 - x_{10}$, $\eta_2 = x_2 - x_{20}$, where x_{10}, x_{20} are equilibrium positions, and obtain the Lagrangian and the equations of motion.

c) Find the normal mode (angular) frequencies for small oscillations and approximate expressions for these frequencies for $k_2 \gg k_1$.

PROBLEM #3.

The wave equation is

$$\nabla^2\psi(x,y,z,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi(x,y,z,t) = 0$$

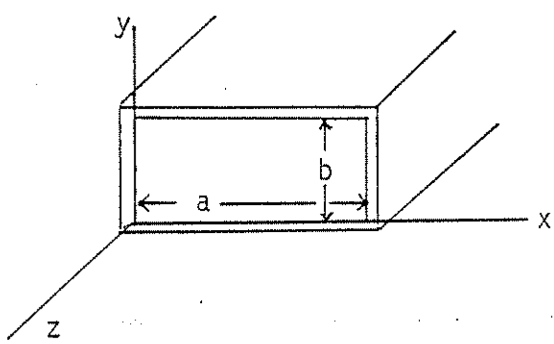
where ψ is any rectangular component of $\vec{E}$ or $\vec{B}$.

Calculate for a rectangular metal wave guide with air dielectric as shown below:

a) the cut-off frequency and the mode designation, i.e. TE_{mn} or TM_{mn}, of the lowest frequency mode. (TE and TM refer to transverse electric fields or transverse magnetic fields, respectively.)

b) the mode designation and cut-off frequency of the next highest frequency mode.

c) the cut-off frequency of the lowest frequency TM mode if (a) is a TE or the lowest TE if (a) is a TM mode.



$a = \sqrt{2}\, b$

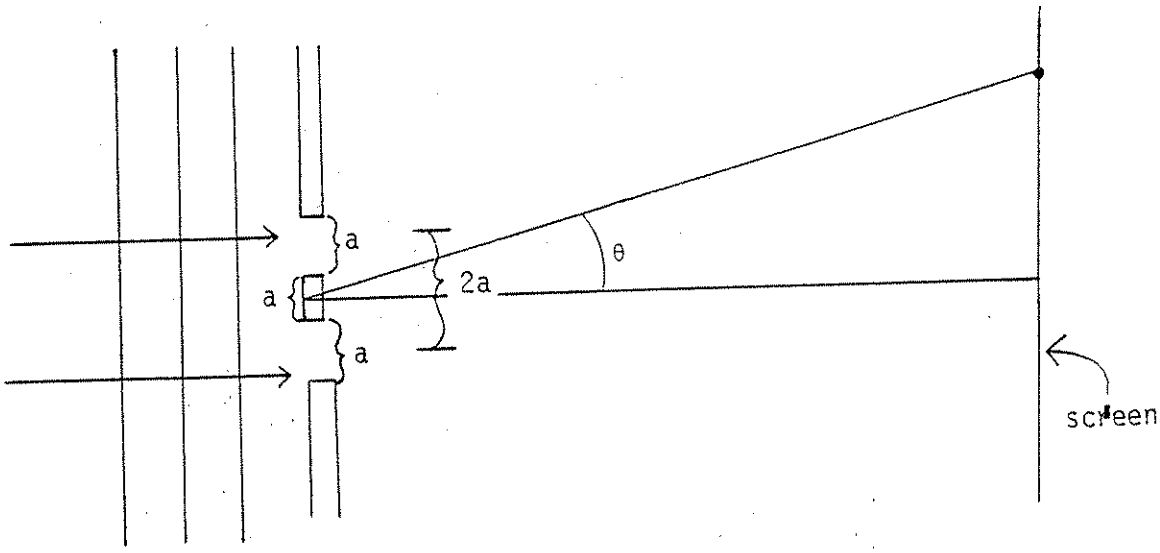
Use the following notation: $k_o = \omega/c$; k_g = guide propagation constant; $k_c{}^2 = k_o{}^2 - k_g{}^2$.

Hint: The boundary condition for a TE wave is $\partial B_z/\partial n\big|_S = 0$, for a TM mode $E\cancel{B}_z\big|_S = 0$.

NOTE: FORMULA(S) USED TO DETERMINE (a), (b), OR (c) MUST BE DERIVED FROM THE INFORMATION GIVEN AND FROM YOUR KNOWLEDGE OF THE PHYSICS OF THE SYSTEM.

PROBLEM #4.

Two slits of width a are separated by a distance 2a. A monochromatic plane wave with wavelength λ impinges normally on the slits from the left. A screen is placed a large distance away to the right of the slits.



a) Cover up one slit. Calculate the relative light intensity distribution $I(\theta)$ on the screen.

b) With both slits open calculate the number of interference maxima within the central diffraction envelope. Make a sketch of the intensity distribution.

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PART II.

PROBLEM #5.

a) Using first order perturbation theory obtain an approximate expression for the energy shift of the hydrogen atom due to the finite size of the proton, assuming that the proton is a uniformly charged sphere of radius $R = 10^{-13}$ cm.

b) Compare the shift with the ground state energy of the hydrogen atom.

The groundstate wave function for the hydrogen atom with a point proton is

$$\psi = \frac{1}{\sqrt{\pi}\, a_o^{3/2}} e^{-\frac{r}{a_o}} . \quad (a_o \text{ is the Bohr radius})$$

PROBLEM #6.

A well-collimated beam of 1 MeV neutrons is incident on a target containing 10^{23} nuclei per cm^2. With a detector placed behind the target we find that only 83% of the incident neutrons remain. A measure of the angular distribution of the scattered neutrons reveals that the distribution is isotropic in the center-of-mass system. We assume neutrons are lost only through elastic scattering.

a) What is the total cross section, σ_0, of the target nuclei for 1 MeV neutrons?

b) The scattering amplitude in a partial wave expansion in the presence of a short range, spin-independent scattering potential is given by

$$f(\theta) = \sum_{\ell=0}^{\infty} \left(\frac{2\ell+1}{k}\right) P_\ell(\cos\theta)\, e^{i\delta_\ell} \sin\delta_\ell$$

where the δ_ℓ are the phase shifts, k is the wave number, and the $P_\ell(x)$ are the Legendre polynomials. Find the expression for the total cross section, $\sigma_0(E)$.

c) What are the values of the lowest three phase shifts?

The orthogonality relationship for the $P_\ell(x)$'s is

$$\int_{-1}^{1} P_\ell(x)\, P_{\ell'}(x)\, dx = [2/(2\ell+1)]\delta_{\ell\ell'}.$$

NOTE: The answers in (c) may be approximate to ≈ 50%.

PROBLEM #7.

A system of N nuclei of spin I = 1 is embedded in a crystalline lattice where they may be considered as distinguishable, owing to their unique locations. A magnetic field $\vec{B} = B_0\hat{k}$ is applied to the crystal, which is at temperature T. Each nucleus has a magnetic moment $\vec{\mu}$ whose maximum component in the z-direction is μ_z.

a) Find the partition function for spin orientation. (Note that the volume V will not enter, as the nuclei cannot move in translation.)

b) Find the Helmholz free energy and, thus, the entropy of the system. Expand S to find its dependence on $\xi = \mu_z B_0/kT$ for high temperature.

PROBLEM #8.

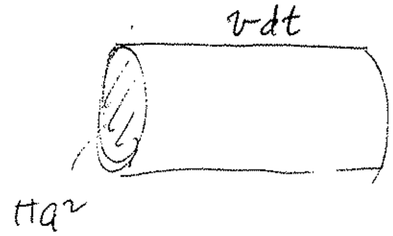
$^{27}_{14}Si$ decays to its "mirror" nucleus $^{27}_{13}Al$ by <u>positron</u> emission: the maximum positron energy is 3.5 MeV. (Notation: $^{A}_{Z}X$ where A is the mass number and Z is the number of protons). Nuclear radii depend only upon the mass number (i.e., number of nucleons) as $R = r_o A^{1/3}$. Use the above data to estimate the constant r_o.

In solving this problem you will need the neutron-hydrogen mass difference which is $m(^{1}_{0}n) - m(^{1}_{1}H) = 0.78$ MeV. Also, in considering Coulomb effects within the nucleus, assume the nuclear charge, Ze, to be uniformly spread throughout the volume of the spherical nucleus.

1

Mech (UG)

a)



$$dm = \rho\, dV = \rho \pi a^2 v\, dt$$

$$\frac{dm}{dt} = \rho \pi a^2 v$$

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

(Small compared to second term)

So $\boxed{F = \rho \pi a^2 v^2}$ (drag force)

b) $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$ Change in height

$$v = \sqrt{\frac{GM}{r}}, \qquad \frac{dv}{dt} = -\frac{1}{2}\frac{\sqrt{GM}}{r^{3/2}}\frac{dr}{dt}$$

$$\rho \pi a^2 v^2 = m\frac{dv}{dt}$$ (WHY ? THIS ISN'T RIGHT

$$\pi a^2 \rho\left(\frac{GM}{r}\right) = -\frac{m}{2}\frac{\sqrt{GM}}{r^{3/2}}\frac{dr}{dt}$$

$$dr = -\frac{2\pi a^2 \rho\left(\sqrt{GM}\right)}{m} r^{1/2}\, dt$$

$dr \ll R$ (earth radius)

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$GM = gR^2$$

$$|\Delta r| \approx \frac{2\pi a^2 \rho R}{m}\sqrt{gR}\ \Delta t$$

$$= \underbrace{\frac{6 \times 1^2 \times 10^{-11} \times 6 \times 10^6}{10^2}}\ \underbrace{\sqrt{9.8 \times 6 \times 10^6}}_{8\times10^3}\ \underbrace{(7 \times 24 \times 3600)}_{7\times10^5} = 2000 \times 10^1$$

$\Delta R = 200\,km$

change in period

$$m\omega^2 r = \frac{GMm}{r^2}$$

$$\omega = \frac{2\pi}{\tau} = \sqrt{\frac{GM}{r^3}}$$

$$\tau = \frac{2\pi}{\sqrt{GM}}\, r^{3/2}$$

$$\frac{d\tau}{\cancel{dt}} = \frac{2\pi}{\sqrt{GM}} \cdot \frac{3}{2}\, r^{1/2}\, \frac{dr}{\cancel{dt}}$$

$$\Delta\tau \approx \frac{3\pi}{\sqrt{g}\,R}\, R^{\frac{1}{2}}\, \Delta R = \frac{3\pi}{\sqrt{gR}}\, \Delta R$$

$$= \frac{10 \quad 2\times10^4}{\sqrt{10\times 6\times10^6}} = \frac{2\times10^5}{8\times10^3} = 30 \text{ sec}$$

(2)

a) $T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2$

$$V = \frac{1}{2} k_1 (l_1 - l_0)^2 + \frac{1}{2} k_2 (l_2 - l_0)^2 - m g x_2$$

$$l_1 = x_1 \quad , \quad x_1 + L + l_2 = d + x_2 ,$$

$$\therefore V = \frac{1}{2} k_1 (x_1 - l_0)^2 + \frac{1}{2} k_2 \left[(d + x_2 - x_1 - L) - l_0 \right]^2 - m g x_2$$

b) $\frac{\partial V}{\partial x_1} = k_1 (x_1 - l_0) - k_2 \left[-x_1 + x_2 + d - L - l_0 \right]$

$$\frac{\partial V}{\partial x_2} = k_2 \left[-x_1 + x_2 + d - L - l_0 \right] - m g$$

$$\frac{\partial^2 V}{\partial x_1^2} = k_1 + k_2 \quad , \quad \frac{\partial^2 V}{\partial x_1 \partial x_2} = -k_2 \quad , \quad \frac{\partial^2 V}{\partial x_2^2} = k_2$$

$$\eta_1 = x_1 - x_{10} \quad , \quad \dot{\eta}_1 = \dot{x}_1 \quad , \quad \eta_2 = x_2 - x_{20} \quad , \quad \dot{\eta}_2 = \dot{x}_2$$

$$T = \frac{1}{2} m (\dot{\eta}_1^2 + \dot{\eta}_2^2)$$

$$V = V(x_{10}, x_{20}) + \cancel{\left(\frac{\partial V}{\partial x_1} \right)_0} \eta_1 + \cancel{\left(\frac{\partial V}{\partial x_2} \right)_0} \eta_2$$

$$+ \frac{1}{2} \left(\frac{\partial^2 V}{\partial x_1^2} \right)_0 \eta_1^2 + \left(\frac{\partial^2 V}{\partial x_1 \partial x_2} \right)_0 \eta_1 \eta_2 + \frac{1}{2} \left(\frac{\partial^2 V}{\partial x_2^2} \right)_0 \eta_2^2$$

$$= V_0 + \frac{1}{2} (k_1 + k_2) \eta_1^2 - k_2 \eta_1 \eta_2 + \frac{1}{2} k_2 \eta_2^2$$

$$L = T - V = \frac{m}{2} (\dot{\eta}_1^2 + \dot{\eta}_2^2) - \frac{1}{2} (k_1 + k_2) \eta_1^2 + k_2 \eta_1 \eta_2 - \frac{1}{2} k_2 \eta_2^2 + V_0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\eta}_1} = m \ddot{\eta}_1 = \frac{\partial L}{\partial \eta_1} = -(k_1 + k_2) \eta_1 + k_2 \eta_2$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\eta}_2} = m \ddot{\eta}_2 = \frac{\partial L}{\partial \eta_2} = k_2 \eta_1 - k_2 \eta_2$$

c). To find normal mode frequencies assume time dependence $e^{-i\omega t}$ for both η_1 and η_2:

$$\eta_1 = C_1 e^{-i\omega t}, \quad \eta_2 = C_2 e^{-i\omega t}$$

$$\begin{cases} -m\omega^2 C_1 = -(k_1+k_2) C_1 + k_2 C_2 \\ -m\omega^2 C_2 = k_2 C_1 - k_2 C_2 \end{cases}$$

Secular determinant:

$$\begin{vmatrix} \omega^2 - \frac{k_1+k_2}{m} & k_2/m \\ k_2/m & \omega^2 - \frac{k_2}{m} \end{vmatrix} = 0$$

$$\omega^4 - \left(\frac{k_1+2k_2}{m}\right)\omega^2 + \frac{k_1 k_2}{m^2} = 0$$

$$\omega^2 = \frac{1}{2}\left(\frac{k_1+2k_2}{m}\right) \pm \frac{1}{2}\sqrt{\left(\frac{k_1^2+4k_1k_2+4k_2^2}{m^2}\right) - \frac{4k_1k_2}{m^2}}$$

$$= \frac{k_2}{m}\left(1+\frac{k_1}{2k_2}\right) \pm \frac{1}{2}\,\frac{2k_2}{m}\sqrt{1+\left(\frac{k_1}{2k_2}\right)^2}$$

$$= \frac{k_2}{m}\left\{\left(1+\frac{k_1}{2k_2}\right) \pm \sqrt{1+\left(\frac{k_1}{2k_2}\right)^2}\right\}$$

$$\doteq \frac{k_2}{m}\left\{\left(1+\frac{k_1}{2k_2}\right) \pm \left(1+\cancel{\frac{k_1^2}{8k_2^2}}\right)\right\}$$

neg

$$\doteq \begin{cases} 2k_2/m \\ k_1/2m \end{cases}$$

E & M (1)

(3) The solutions for propagation are

$$\psi_0(x,y)\, e^{i(k_g z - \omega t)}$$ for a ω frequency, monochromatic wave or Fourier amplitude

put in D.E.

$$\left\{\nabla_{t\,(x,y)}^2 + \left(-k_g^2 + \omega^2/c^2\right)\right\}\psi_0(x,y) = 0 \qquad \nabla_t \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$k_0^2 \equiv \omega^2/c^2 \ ; \ k_c^2 = k_0^2 - k_g^2$$

For propagation k_g must be real or $k_g^2 > 0$

$$k_g^2 = k_0^2 - k_c^2 \quad \therefore k_0^2 > k_c^2$$

$$k_{cut\text{-}off}^2 \equiv k_0^2 = k_c^2$$ for limiting case

Solve D.E. $\psi_0(x,y) = X(x)\,Y(y)$

put in D.E. & (x) $\frac{1}{\psi_0}$

$$\frac{X''(x)}{X} + \frac{Y''(y)}{Y} + k_c^2 = 0$$

Since $X''/X = f(x)$ & $Y''/Y = g(y)$ both must be constants i.e. $-k_1^2$ & $-k_2^2$

$X''(x) + k_1^2 X(x) = 0$ Similarly for Y S.H.O. sol's

$$X(x) = A\cos k_1 x + B\sin k_1 x$$
$$Y(y) = C\cos k_2 y + D\sin k_2 y$$

(2)

TM mode $E_z = E_{z0}(x,y)\, e^{i(k_g z - \omega t)}$, $B_z = 0$ always

TE $B_z = B_{z0}(x,y)\, e^{i(k_g z - \omega t)}$, $E_z = 0$ always.

TM.

TM $E_{z0}\Big|_{\substack{x=0\\x=a\\y=0\\y=b}} = 0 \quad \therefore \quad A = 0 \; ; \; C = 0$

$$k_1 = \frac{m\pi}{a} \qquad m = 0, 1, 2, \dots$$
$$k_2 = \frac{n\pi}{b} \qquad n = 0, 1, 2 .$$

$BD \Rightarrow E_0$

$$E_{z0}(x,y) = E_0 \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \quad \therefore \text{ can } n$$

no TM_{00} or TM_{01} or TM_{10} modes

TE $\frac{\partial B_{z0}}{\partial x}\Big|_{\substack{x=0\\x=a}} = 0$ & $\frac{\partial B_{z0}}{\partial y}\Big|_{\substack{y=0\\y=b}} = 0$

$$\therefore \; B = D = 0 \qquad k_1 = \frac{m\pi}{a}, \; k_2 = \frac{n\pi}{b}$$

$$B_{z0}(x,y) = -B_0 \cos \frac{m\pi}{a} x \cos \frac{n\pi}{b} y$$

no TE_{00} mode can have TE_{10} & TE_{01}

$$k_0^2 = k_1^2 + k_2^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\frac{\omega_{c0}}{c} = k_{c0} = \pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{c0} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Sol

$a = \sqrt{2}\, b$ or $b^2 = a^2/2$

$$f_{co} = \frac{c}{2a}\sqrt{m^2 + 2n^2}$$

(a) Lowest mode $m=1$ $n=0$ or TE_{10} $\boxed{f_{co}^{TE_{10}} = c/2a}$

(b) Next lowest mode $m=0$ $n=1$ or TE

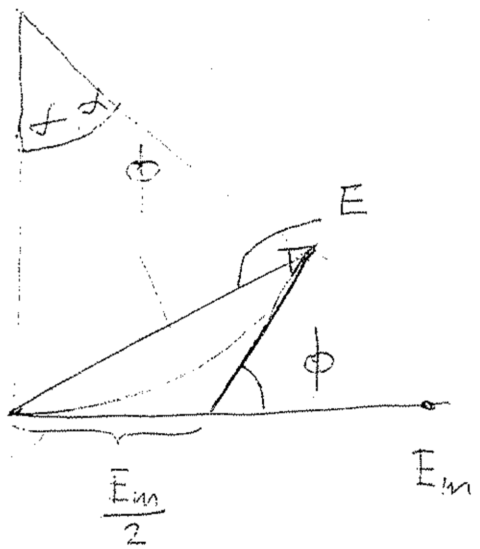
$$\boxed{f_{co}^{TE_{01}} = \frac{c}{\sqrt{2}\,a}}$$

(c) Lowest TM $n=1, m=1$

$$\boxed{f_{co}^{TM_{11}} = \frac{c}{a}\frac{\sqrt{3}}{2}}$$

(4) Optics (4.6) Solution

a) Use phasor diagram.



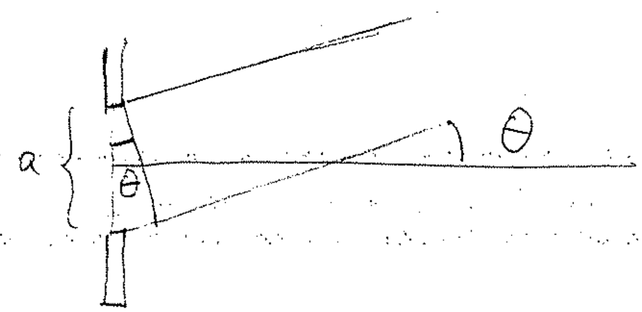
ϕ = phase difference between light ray at top and light ray at bottom of slit.

$$E = E_m \frac{\sin\alpha}{\alpha}$$

$$\frac{\text{phase diff}}{2\pi} = \frac{\text{path diff}}{\lambda}$$

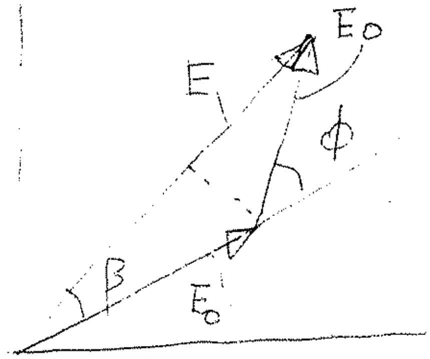
$$\phi = \frac{2\pi}{\lambda}(a \sin\theta)$$

$$I = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \alpha = \frac{\pi a \sin\theta}{\lambda}$$



b) Calculate interference of two rays; one from each slit.

Phasor diagram



$$E = 2 E_0 \cos\beta$$

$$\phi = \frac{2\pi}{\lambda}\left(2a \sin\theta\right)$$

$$\beta = \frac{1}{2}\phi$$

$$\beta = \frac{\pi}{\lambda}\left(2a \sin\theta\right)$$

$$E = E_{max} \cos\beta$$

$$I = I_m \cos^2\beta$$

combine with single slit diffraction result.

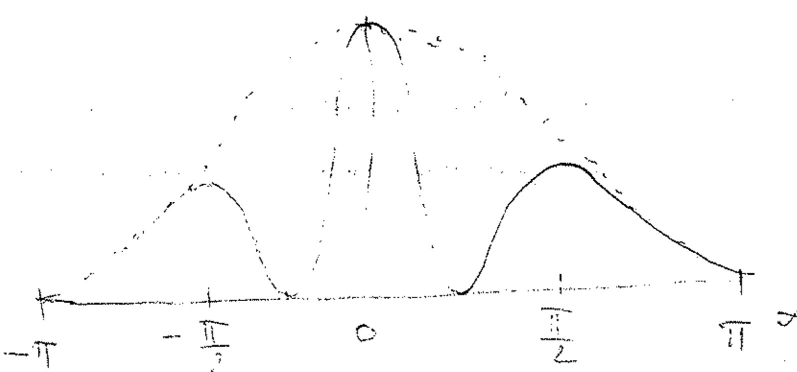
$$I = I_0 (\cos\beta)^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

constructive interference for $\beta = m\pi$ $\quad m = 1, 2 \ldots$

$$\lambda = \frac{2a\sin\theta}{m}$$

Here $\beta = 2\alpha$

$$I = I_0 (\cos 2\alpha)^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$



The minimum of the envelope is at $\alpha = \pi$

Interference maximas are at $\alpha = 0, \frac{\pi}{2}, \pi$

There are $\underline{3}$ interfer. fringes in the central envelope.

⑤ Atomic Physics Sol.

For a uniform charge distribution ($\rho =$ const)
the electrostatic PE is

$$U = -eV = -e \begin{cases} \dfrac{e}{r} & r > R \\ \dfrac{3}{2}\dfrac{e}{R}\left(1 - \dfrac{r^2}{3R^2}\right) \; * & r < R \end{cases}$$

$*$ Calc $V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$ $E_{inside} = \dfrac{er}{R^3}$

[apply gauss's law]

change in PE

$$\Delta U = \begin{cases} 0 & r > R \\ \dfrac{e^2}{r} - \dfrac{3}{2}\dfrac{e^2}{R}\left(1 - \dfrac{r^2}{3R^2}\right) & r < R \end{cases}$$

$$\Delta E = \langle 1s | \Delta U | 1s \rangle$$

$$= \frac{e^2}{\pi a_0^3} \int_0^{\infty} e^{-2r/a_0} \left(\frac{1}{r} - \frac{3}{2}\frac{1}{R} + \frac{1}{2}\frac{r^2}{R^3} \right) 4\pi r^2 dr$$

$\int d\Omega$

a) $$\Delta E = \frac{2e^2}{a_0^3}\left(R^2 - R^2 + \frac{1}{5}R^2\right) = \frac{2}{5}\left(\frac{e^2}{a_0}\right)\left(\frac{R}{a_0}\right)^2$$

$$\boxed{\Delta E = \frac{2}{5}\left(\frac{e^2}{a_0}\right)\left(\frac{R}{a_0}\right)^2}$$

b) $$\frac{\Delta E}{\frac{e^2}{2a_0}} = \frac{4}{5}\left(\frac{R}{a_0}\right)^2 = \underline{3.2 \times 10^{-10}}$$

ground state
energy of H

QM

(6)

(a) .17 of particles scatter out of beam

$$.17 = \sigma_0 \times 10^{23}/\text{cm}^2$$

$$\sigma_0 = 1.7 \times 10^{-24}\ \text{cm}^2$$

(b) $\sigma(E) = \int |f(\theta)|^2 d\Omega = \int_0^{\pi}\int_0^{2\pi} |f(\theta)|^2 \sin\theta\, d\theta\, d\varphi$

$$f(\theta) = \sum_{l=0}^{\infty} \frac{2l+1}{k} P_l(\cos\theta)\cos\delta_l \sin\delta_l + i\sum_{l=0} \frac{2l+1}{k} P_l(\cos\theta)\sin^2\delta_l$$

$$\sigma(E) = \frac{2\pi}{k^2}\int_{-1}^{1} \sum_{l,l'} (2l+1)(2l'+1) P_l(\cos\theta) P_{l'}(\cos\theta)\left[\cos\delta_l \cos\delta_{l'} \sin\delta_l \sin\delta_{l'} + \sin^2\delta_l \sin^2\delta_{l'}\right] d(\cos\theta)$$

$\int_{-1}^{1}\sum_{l,l'} \to \sum_{l,l'}\int_{-1}^{1}$ & use $\int_{-1}^{1} P_l(x) P_{l'}(x)\, dx = \frac{2}{2l+1}\delta_{ll'}$

$$\sigma(E) = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\left[\cos^2\delta_l \sin^2\delta_l + \sin^4\delta_l\right] =$$

$$\sigma(E) = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

(c) From isotropy $|f(\theta)|^2 \neq$ function of θ

$\therefore$ all δ_l $(l \neq 0) = 0$

$$\sigma(E) = \frac{4\pi}{k^2}\sin^2\delta_0$$

$$k^2 = (p/\hbar)^2 = \frac{2mT}{\hbar^2} = \frac{2(mc^2)T}{(\hbar c)^2} = \frac{2 \times 940 \times 1}{(197.3)^2}\ F^{-2}$$

$$\sin \delta_0 = \sqrt{\frac{\sigma_0 k^2}{4\pi}} \simeq \sqrt{\frac{1.7\times 10^{-24} \times 2 \times 940 \times \frac{1}{2}}{(197.3)^2 \, 4\pi} \times \frac{1}{10^{-?}}}$$

$$\sin \delta_0 = .651$$

$$\delta_0 = \sin^{-1}(.651) = .94 \text{ rad}$$

$$\delta_1 = 0$$

$$\delta_2 = 0$$

or $$\sqrt{\frac{\cancel{2}\times 10^{-24} \times \cancel{2} \times 1000 \times 1}{(200)^2 \times \cancel{4}\pi} \left(\frac{1}{10^{-13}}\right)^2}$$

$$\sqrt{2.5/\pi} \simeq \sqrt{.8} \simeq .9$$

$$\delta_0 \simeq \sin^{-1} .9 \simeq 1 \text{ rad or } 60^\circ$$

(7)

a) For $I = 1$ there will be $2I+1 = 3$ states, and these will have energy $0, \pm \mu_z B_0$.

$$Z = z^N = \left(e^{-\frac{\mu_z B_0}{kT}} + 1 + e^{\frac{\mu_z B_0}{kT}}\right)^N$$

because the states are not degenerate.

b) $$A = -kT \ln Z = -NkT \ln\left(e^{-\mu_z B_0/kT} + 1 + e^{\mu_z B_0/kT}\right)$$

$$S = -\left.\frac{\partial A}{\partial T}\right)_V = Nk\left\{\ln z + T\frac{1}{z}\frac{\partial z}{\partial T}\right\}$$

$$= Nk\left\{\ln z + \frac{T}{z}\frac{dz}{d\xi}\left(-\frac{\xi}{T}\right)\right\}$$

$$= Nk\left\{\ln z - \frac{\xi}{z}\left(-e^{-\xi} + e^{\xi}\right)\right\}$$

$$= Nk\left\{\ln(1 + 2\cosh\xi) - \frac{\xi\,(2\sinh\xi)}{1 + 2\cosh\xi}\right\}$$

For $\xi << 1$ $\quad (kT >> \mu_z B_0)$,

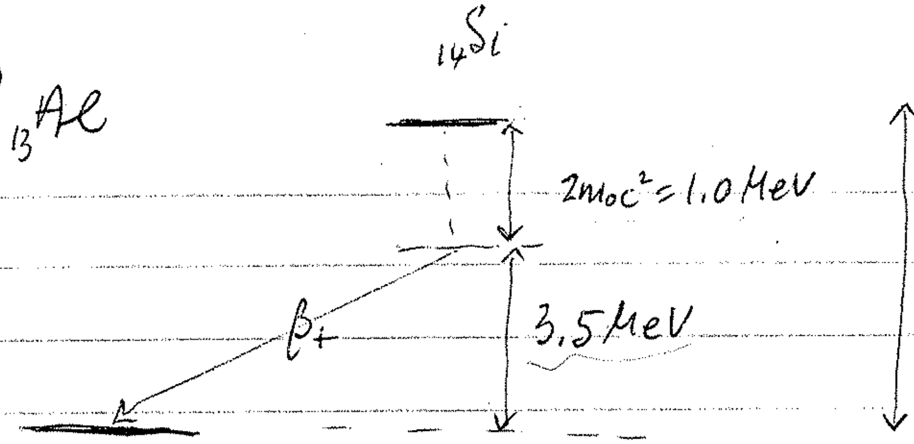
$$\frac{1}{Nk}S \doteq \ln\left\{1 + \left[\left(1+\xi+\tfrac{1}{2}\xi^2\right) + \left(1-\xi+\tfrac{1}{2}\xi^2\right)\right]\right\} - \frac{\xi\left[\left(1+\xi+\tfrac{1}{2}\xi^2\right) - \left(1-\xi+\tfrac{1}{2}\xi^2\right)\right]}{1 + \left(1+\xi+\tfrac{1}{2}\xi^2\right) + \left(1-\xi+\tfrac{1}{2}\xi^2\right)}$$

$$\doteq \ln\left\{3\left(1 + \tfrac{1}{3}\xi^2\right)\right\} - \frac{2\xi^2}{3 + \xi^2}$$

$$\doteq \ln 3 + \ln\left(1 + \tfrac{1}{3}\xi^2\right) - \tfrac{2}{3}\xi^2\left(1 - \tfrac{1}{3}\xi^2\right)$$

$$\doteq \underline{\ln 3 - \tfrac{1}{3}\xi^2}$$

(8) $_{13}Al$



$\Delta M = 4.5\,MeV$ (due *only* to Coulomb energy difference).

$$(Si):\quad M(Z+1,A) = (Z+1)m_H + (A-Z-1)m_n + E_c(Z+1) - BE\ (\text{nuclear})$$
$$(Al)\quad M(Z,A) = Z\,m_H + (A-Z)m_n + E_c(Z) - BE\ \text{"}$$

$$\Delta M = 4.5\,MeV = \underbrace{(m_H - m_n)}_{-.78\,MeV} + \underbrace{[E_c(Z+1) - E_c(Z)]}$$

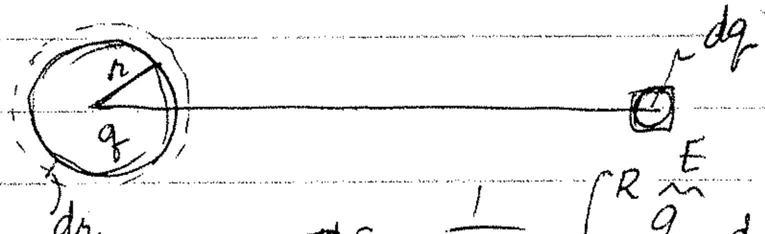
E_c = Coulomb energy

BE = nuclear binding energy (assumed to be the same for both)

need coulomb energy of a uniformly charged sphere of radius R

charge density $\rho = \dfrac{Ze}{\frac{4}{3}\pi R^3}$ $\quad 4\pi\rho = \dfrac{3Ze}{R^3}$

$$q = \rho\cdot\tfrac{4}{3}\pi r^3$$
$$dq = \rho\cdot 4\pi r^2\,dr$$



$$E_c = \frac{1}{4\pi\varepsilon_0}\int_\infty^R \frac{q}{r}\,dq = \frac{1}{4\pi\varepsilon_0}\,\frac{\rho^2(4\pi)^2}{3}\underbrace{\int_\infty^R \frac{r^3}{r}\,r^2\,dr}_{R^5/5}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{9Z^2e^2}{3R^6}\cdot\frac{R^5}{5} = \frac{1}{4\pi\varepsilon_0}\,\frac{3}{5}\,\frac{Z^2e^2}{R} = E_c$$

$$\Delta E_c = E_c(Z+1) - E_c(Z) = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{R}\underbrace{\left[(Z+1)^2 - Z^2\right]}_{\cancel{Z^2}+2Z+1-\cancel{Z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{5}\frac{(2Z+1)}{R}$$

$$\Delta E_c = \Delta M + (m_n - m_H) = 5.3\,\text{MeV} * 1.6\times10^{-13}\frac{\text{J}}{\text{MeV}} \approx 8.5\times10^{-13}\,\text{J}$$

$$= 4.5 + .8$$

$$R = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{5}\frac{(2Z+1)}{\Delta E_c} = \frac{9\times10^{9}\times \overset{.6}{\cancel{3}} \times \overset{1.6}{2.6}\times10^{-38}\,(27)}{\cancel{5}\times \cancel{8.5}\times10^{-13}}$$

$$A = 27$$

$$Z = 13$$

$$= 4.0\times10^{-15}\,\text{m} = r_0 A^{1/3} = 3r_0$$

$$\therefore\ r_0 = 1.3\times10^{-15}\,\text{m}$$