PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #51

April 6, 1985

General Instructions

This Comprehensive Examination for Spring 1985 (#51) consists of six problems of equal weight (20 points each). Please check that you have all of them.

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you *may* find useful. Please return it at the end of the exam.

Use the last pages of your bluebooks for "scratch" work separated by at least one empty page from your solution. "Scratch" work will not be graded.

PROBLEM #1.

A projectile is fired vertically upwards from a point on the surface of the earth at a northern latitude λ. The initial speed is v_0. The earth is rotating with angular velocity $\vec{\omega}$. The maximum height of the projectile is much less than the radius of the earth.

a) In a local frame of reference with the x-axis to the East, the y-axis to the North and the z-axis in the vertical direction find the components of the net acceleration acting on the projectile.

b) Assuming $v_z >> v_x, v_y$ in the equations of motion, calculate the maximum height of the projectile.

c) At its maximum height, what are the x- and y-coordinates of the projectile?

PROBLEM #2.

X-rays with angular frequency ω are impinging on a metal plate. The X-rays make an angle θ with respect to the normal of the plate. Assume that there are N electrons per unit volume in the metal. The electrons are weakly bound to the ions and execute simple harmonic motion with the angular frequency ω of the X-rays.

a) Calculate the X-ray induced polarizability of the metal.

b) This is a case where there can be total external reflection. Calculate the dependence of the critical angle on the angular frequency ω.

c) For what frequency range does one obtain total reflection at all incident angles?

PROBLEM #3.

The classical canonical partition function for an isothermal system of N non-interacting particles is given by

$$Q = \frac{1}{N!h^{3N}} \int e^{-\beta H} d\Gamma$$

where h is Planck's constant, $\beta = 1/kT$, H is the hamiltonian, and $d\Gamma$ is an element of phase space.

a) Evaluate Q for a gas of monatomic atoms contained in a cylinder of height L and volume $V = LD^2$ in a gravitational field of acceleration $\vec{g}$, downward.

b) Compute the heat capacity at constant volume for the two cases of thermal energy kT much greater than and much less than the maximum gravitational energy of an atom.

PROBLEM #4.

Given a one-dimensional potential:

$$V(x) = \begin{cases} \frac{\hbar^2}{2m}\left[\alpha^2 x^2 + \frac{\beta(\beta+1)}{x^2}\right] & x \geq 0 \\ \infty & x \leq 0 \end{cases} \qquad \text{where } \beta \text{ is a + integer}$$

a) Use a power series solution of the Schrödinger equation of the form $u(\eta) = \eta^\gamma \sum_{i=0}^{\infty} a_i \eta^{2i}$ where γ is a + integer related to β and $\eta = \sqrt{\alpha}\, x$. Develop a recursion relationship for the expansion coefficients, and from this show the energy Eigenvalues are given by:

$$E_n = \frac{\alpha^2 \hbar^2}{2m}[4n + 2\beta + 3]$$

b) By redefining α, compare the answer of part a) with an ordinary harmonic oscillator as to level spacing and ground state energies.

c) For $\beta=0$, how do the levels compare with the simple harmonic oscillator? Why are certain levels missing?

PROBLEM #5.

A uniform disc of mass M and Radius R has a rod (considered massless) passing through its center; the rod is at an angle α to the normal of the disc surface (see figure below). In the following all vectors to be calculated are with respect to the body axes of the disc, 1, 2, and 3 shown in the figure. Axes 1 and 2 are in the plane of the disc and 3 is along the normal. The rod and disc rotate at a constant angular angular velocity $\vec{\omega}$ shown in the figure. Choose the orientation of the 2-body axes so $\vec{\omega}$ lies in the 2,3-plane.

a) Find the angular momentum of the disc (magnitude and direction).

b) Find the torque (magnitude and direction) which must be exerted on the rod to maintain the rotation.

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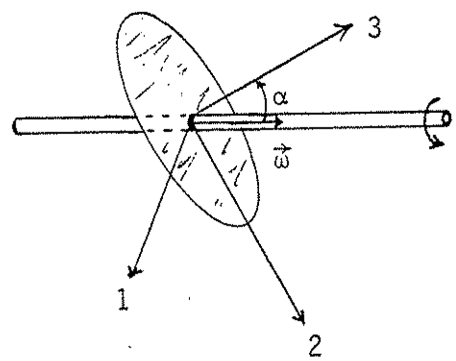
Show the angular momentum and torque vectors on a sketch and discuss what happens to them during the rotation.



PROBLEM #6.

The hamiltonian for helium is

$$-\frac{\hbar^2}{2m}(\nabla_1^{\,2}+\nabla_2^{\,2}) - e^2\left(\frac{2}{r_1}+\frac{2}{r_2}-\frac{1}{r_{12}}\right)$$

where $r_{12} = |\vec{r}_1 - \vec{r}_2|$. Hartree's method for multielectron atoms treats the electrons as moving independently in the field of the nucleus plus the spherically symmetric average field of the other electrons. The wavefunction is the product of hydrogen-like functions $\psi_i \propto R_{n\ell}(r_i)\, Y_{\ell m}(\theta_i,\phi_i)$. For helium, the electron-electron interaction energy is then found as that of one charge distribution, $\rho_1 = -e|\psi_1|^2$, interacting with charged shells containing

$$dq_2 = \left[\int (-e|\psi_2|^2)\, d\Omega_2\right](r_2^{\,2} dr_2) \quad .$$

a) Assume the wavefunction for the ground state is proportional to $e^{-\alpha(r_1+r_2)}$, where α is a constant to be used for variation. Normalize the wavefunction and obtain ψ_1, ψ_2.

b) Recall that the potential of a uniform spherical shell is constant inside the shell and is equivalent to the potential of a point charge at points outside the shell. Set up (but do not perform) the integration for the interaction potential energy of the electrons in helium.