PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #45

April 2, 1983

General Instructions

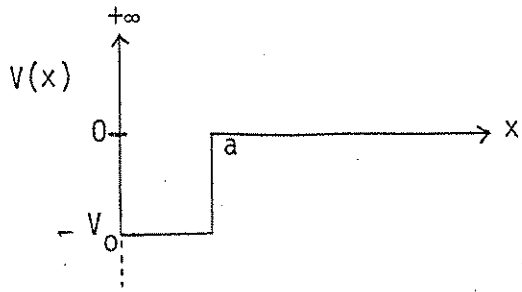
This Comprehensive Examination for Spring 1983 (#45) consists of six problems of equal weight (20 points each). Please check that you have all of them.

Work carefully, indicate your reasoning briefly, and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you may find useful. Please return it at the end of the exam.

(20) 1. A quantum mechanical system of reduced mass m is subject to the one-dimensional rectangular potential well

$$V(x) = \begin{cases} +\infty & x \leqq 0 \\ -V_o & 0<x<a \\ 0 & x \geqq a \end{cases}$$



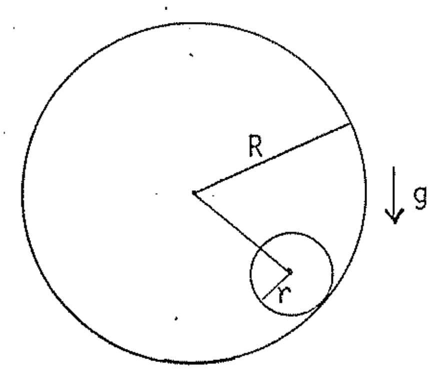
a) Calculate W_o, the smallest value of V_o which allows the system to have three (but not more) bound states.

(Hint: Consider the solutions of the Schrödinger equations inside and outside a. Solve graphically the conditions for "good behavior".)

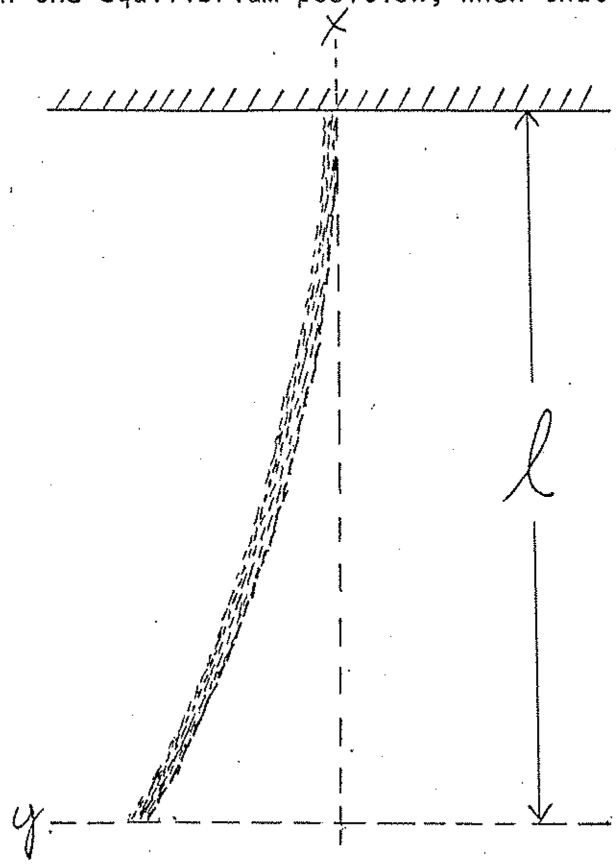
b) Estimate E_{ground}, the energy of the lowest of the three bound states, if $V_o = W_o$.

c) Estimate the kinetic energy of the system when it is in the ground state.

(20) 2. A disk of mass m and radius r rolls without slipping on the inside of a cylinder of radius R in a plane perpendicular to the axis of the cylinder as shown. If it has just enough energy to roll to the top of the cylinder, what will the normal force on it be at the bottom?



3. A thick, uniform, flexible rope of mass M and length ℓ hangs from a fixed point of support (at $y = 0$, $x = \ell$) with its lower end free. It can swing in a plane about its equilibrium position with small-amplitude harmonic motion at frequency ω, assuming the tension in any part of the rope remains the same at it was in the equilibrium position, when that part is displaced horizontally.



(15) a) Analyze the horizontal forces on an element dx of the rope for small displacements ($\sin\theta \approx \tan\theta$, $\cos\theta \approx 1$) from the equilibrium position, write the equation of motion of the element's horizontal ~~harmonic~~ harmonic motion (not the angular motion) and show that the equation of motion can be written

$$\frac{d}{dx}\left(\frac{dL}{dy'}\right) - \frac{dL}{dy} = 0 \quad , \quad \text{provided that} \begin{cases} y' = \dfrac{dy}{dx} \quad \text{and} \\ L = \dfrac{1}{2}my'^2 + \dfrac{\omega^2}{g}myy' \quad \text{where } m \text{ is} \\ \text{the mass of the rope } \underline{\text{below}} \text{ the} \\ \text{element.} \end{cases}$$

(5) b) Show that $\delta \int_0^{\ell} L\,dx$ vanishes with a variation of the shape $y(x)$ by the nature of the end-point conditions.

4. Positronium is a bound state of an electron and a positron. Its 1 s state can be described by a perturbing Hamiltonian

$$H' = A\,\vec{s}_1 \cdot \vec{s}_2 + \frac{eB}{mc}(s_{1z} - s_{2z}),$$

where the electron spin is $\vec{s}_1$ and the positron spin is $\vec{s}_2$. The first term in H' gives the fine structure (or hyperfine structure; in positronium they are the same), and the second term describes the Zeeman effect due to a static external magnetic field $\vec{B}$ along the z-axis.

(02) a) Identify the good (or nearly good) quantum numbers in the limits of very low and very high magnetic fields.

(12) b) Calculate the energies of the four eigenstates of H', in terms of A, B, and fundamental constants.

(04) c) Make a rough plot of the energies of the four states, as a function of magnetic field. It is known that A > 0. Label each state with its quantum numbers in the low-field and high-field limits.

(02) d) Positronium decays by annihilation. Briefly describe the dominant decay modes of the singlet and triplet states of 1 s positronium (at zero magnetic field).

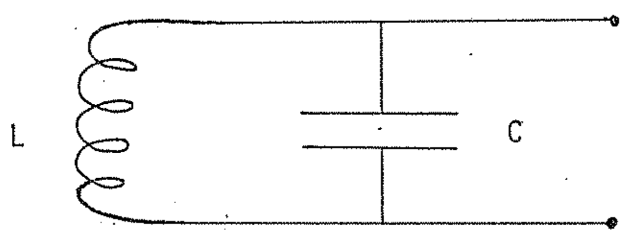
(20) 5. A long grounded conducting cylindrical shell of radius a is oriented along the z axis. It contains: a dielectric with permitivity ε_1 for $z > 0$, ε_2 for $z < 0$, and a point charge at ρ', $z' > 0, \theta$ within the cylinder. Obtain an expression for the electrostatic potential V everywhere.
[The electrostatic Green's function for the inside of a grounded cylinder of radius a is

$$\sum_{mn} \frac{2}{k_{mn}\, J_{m+1}^2 (x_{mn})\, a^2}\; e^{k_{mn}(z_< - z_>)}$$

$$J_m (k_{mn}\rho')\, J_m (k_{mn}\rho)\; e^{im(\theta-\theta')}$$

where $k_{mn} = \frac{X_{mn}}{a}$ and $J_m (X_{mn}) = 0$ (X_{mn} are the roots of the Bessel Functions)]

6. The sensitivity of physical measurements is limited by noise due to thermal fluctuations. Consider an L C circuit in thermal equilibrium at absolute temperature T.



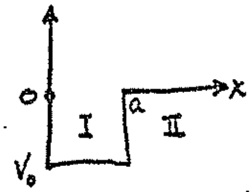
The circuit behaves as a harmonically oscillating system. If the zero-point energy is neglected, the energy eigenvalues are given by $E_n = n\hbar\omega$, where n = 0, 1, 2, . . . and ω is the classical frequency.

(12) a) Evaluate the partition function and from it obtain the average noise energy $\langle E \rangle$ stored in the system. Then calculate the rms noise voltage V across the terminals at a general temperature T.

(08) b) Find the rms noise voltage in the classical limit $kT \gg \hbar\omega$. Obtain an order of magnitude for this voltage if L = 1 μh and C = 1 pf, with T = 300° K.

Solution

(a)



The 1-dimensional S-eq. is $\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V\psi=E\psi}$

Inside, $0<x<a$, $\quad \frac{d^2\psi_1}{dx^2}+\frac{2m}{\hbar^2}(E-V)\psi_1=0 \longrightarrow \underbrace{\frac{d^2\psi_1}{dx^2}+k_1^2\psi_1=0}_{\text{looks like SHM}}$

[$V=-V_0$ and E is negative for a bound state.

$k_1=\frac{\sqrt{2m(|E|+V_0)}}{\hbar}$]

"good behavior"] $\psi_1=0$ at $x=0$, so choose $\boxed{\psi_1=A\sin k_1x}$

Outside, $x\geq a$, $\quad \frac{d^2\psi_{II}}{dx^2}+\frac{2m}{\hbar^2}E\,\psi_{II}=0$

[$V=0$ and E is negative for a bound state

$k_2=\frac{\sqrt{-2m|E|}}{\hbar}$]

Should absolute value bars be here?

$$\rightarrow \underbrace{\frac{d^2\psi_{II}}{dx^2}-k_2^2\psi_{II}=0}$$

has soln's $e^{\pm k_2x}$

"good behavior →0 at $x\to\infty$"] so choose $\boxed{\psi_{II}=Be^{-k_2x}}$

"good behavior" at $x=a$] solutions must join smoothly : $\boxed{\frac{1}{\psi_1}\frac{d\psi_1}{dx}\bigg|_{x=a}=\frac{1}{\psi_2}\frac{d\psi_2}{dx}\bigg|_{x=a}}$

$$\frac{k_1\cos k_1a}{\sin k_1a}=\frac{-k_2e^{-k_2a}}{e^{-k_2a}}$$

or $\boxed{k_1\cot k_1a=-k_2}$

let $z=k_1a$

$y=\cot z$

$d=\frac{\sqrt{2mV_0}}{\hbar}\,a$

then $k_2=\frac{\sqrt{-2m|E|}}{\hbar}=\sqrt{\frac{2mV_0}{\hbar^2}\left(\frac{a}{a}\right)^2-\frac{2m(|E|+V_0)}{\hbar^2}\left(\frac{a^2}{a^2}\right)}$

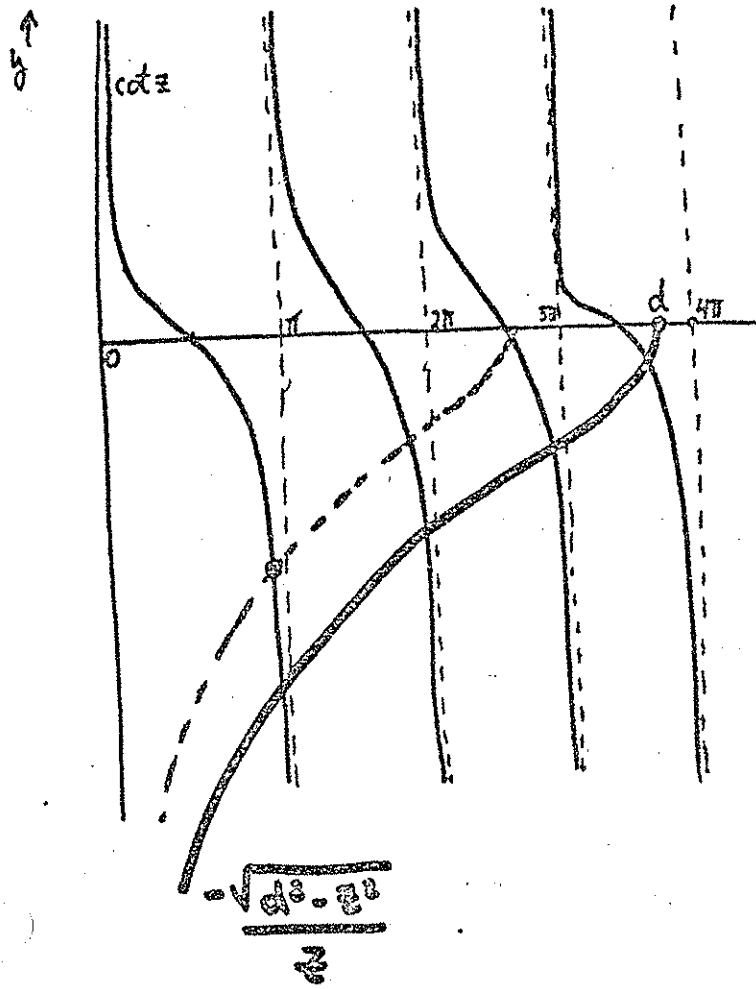
$=\pm\frac{1}{a}\sqrt{d^2-z^2}$

$\left(\frac{z}{a}\right)y=-\frac{1}{a}\sqrt{d^2-z^2}$

or $\boxed{y=-\frac{\sqrt{d^2-z^2}}{z}}$

Let us now plot y against z $(y = \cot z)$

$$-\frac{\sqrt{d^2 - z^2}}{z} \rightarrow \begin{cases} 0 & \text{when } d = z \\ -\infty & \text{when } z \to 0 \end{cases}$$



For this value (arbitrary) of d, there are 4 points of intersection; 4 points which satisfy the "good behavior" condition at $x = a$.

It is now clear that the least value of d which will give three, but no more, intersections is $\boxed{d = \frac{5\pi}{2}}$

ie, $\frac{\sqrt{2mW_0}}{\hbar} a = \frac{5\pi}{2}$

$$\frac{2mW_0}{\hbar^2} a^2 = \frac{25\pi^2}{4}$$

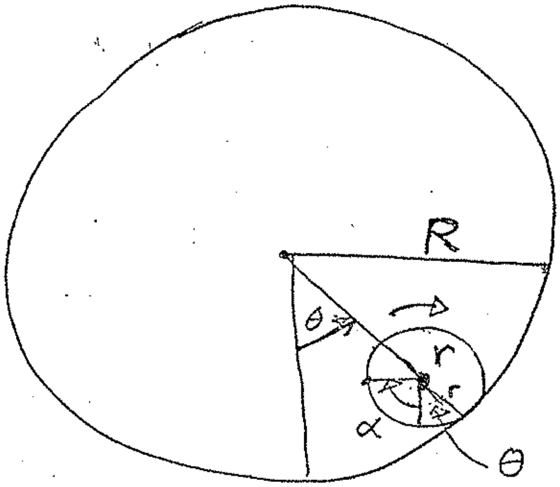
$$\underline{W_0 = \frac{25\pi^2}{8} \frac{\hbar^2}{ma^2}}$$

(b) Under these conditions, the value of E_{ground} is given by the intersection of lowest z. That is, $\underline{z \cong \pi}$. Then $k_2 = \frac{\sqrt{-2m|E|}}{\hbar} = \frac{1}{a}\sqrt{d^2 - z^2} = \frac{1}{a}\sqrt{\frac{25}{4}\pi^2 - \frac{4}{4}\pi^2} = \frac{\pi}{a}\sqrt{\frac{21}{4}}$

$$\text{or} \quad \underline{|E|} = \frac{\pi^2 \hbar^2}{2ma^2}\frac{21}{4} = \underline{\frac{21}{8}\pi^2 \frac{\hbar^2}{ma^2}}$$

(c) and $\underline{T = E - W_0} = \left[-\frac{21}{8} - \left(-\frac{25}{8}\right)\right] \frac{\pi^2\hbar^2}{ma^2} = \underline{\frac{1}{2}\frac{\pi^2\hbar^2}{ma^2}}$

2. Solution



$$I = \int_0^r \rho r'^2 dA' = \frac{m \int_0^r r'^2 \, 2\pi r' dr'}{\int 2\pi r' dr'} = \frac{m \, r^4/4}{r^2/2} = \frac{mr^2}{2}$$

condition of no slipping: $R\theta = r(\theta+\alpha)$

$(R-r)\theta = r\alpha$

condition for just barely rolling to the top

$$\underset{\text{centripetal}}{F_c} = \overbrace{\cancel{m}\dot{\theta}^2(R-r)}^{v^2 = r\omega^2} = \cancel{m}g \text{ (no normal force at top)}$$

$$E = \overbrace{\tfrac{1}{2}m(R-r)^2\dot{\theta}^2}^{\frac{1}{2}mv^2} + \tfrac{1}{2}I\dot{\alpha}^2 - mg(R-r)\cos\theta$$

at top $E = \frac{1}{2}m(R-r)^2\dot{\theta}^2 + \frac{1}{2}I\frac{(R-r)^2\dot{\theta}^2}{r^2} + mg(R-r)$

$= \frac{1}{2}m(R-r)^2\frac{g}{(R-r)} + \frac{1}{2}I\frac{(R-r)^2}{r^2}\frac{g}{(R-r)} + mg(R-r)$

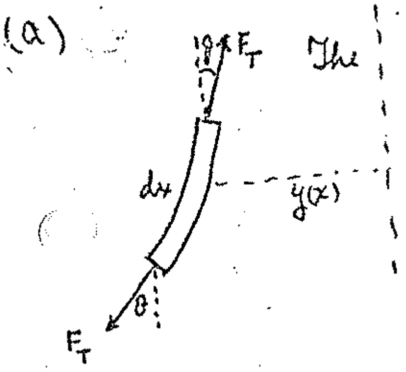
$= mg(R-r)\left[\frac{1}{2} + \frac{1}{2}\frac{I}{mr^2} + 1\right] = \frac{7}{4}mg(R-r)$

at bottom $E = \frac{7}{4}mg(R-r) = \frac{1}{2}m(R-r)^2\dot{\theta}^2 + \frac{1}{4}m(R-r)^2\dot{\theta}^2 - mg(R-r)$

$N - mg = F_c$, $N = mg + m\underbrace{(R-r)\dot{\theta}^2}_{\to \frac{v^2}{R'}}$

$= mg + \frac{(11/\cancel{4})\,mg\,\cancel{(R-r)}}{(3/\cancel{4})\,\cancel{(R-r)}} = \frac{14}{3}mg$

(a)



The forces acting on an element of the rope, of length dx are shown,

If the tension in the rope is F_T, the net horizontal force on the element is

$$\left[\left(F_T \sin\theta\right)_{x+dx} - \left(F_T \sin\theta\right)_x\right] \approx \left[\left(F_T \tan\theta\right)_{x+dx} - \left(F_T \tan\theta\right)_x\right] = \left[\left(F_T \frac{dy}{dx}\right)_{x+dx} - \left(F_T \frac{dy}{dx}\right)_x\right]$$

$$= \frac{d}{dx}\left[F_T \frac{dy}{dx}\right] dx$$

and this is the force that provides the acceleration $\frac{d^2y}{dt^2} = -\omega^2 y$ for horizontal harmonic motion

If $M(x)$ is the mass of the rope which lies below the element, then

1) the vertical component of the tension is $F_T \cos\theta \approx F_T = M(x)\, g$

2) the element has mass $dm = \frac{d(M(x))}{dx}\, dx$

Thus, the equation of horizontal harmonic motion is

$$\frac{d}{dx}\left[F_T \frac{dy}{dx}\right] = (dm)(-\omega^2 y)$$

$$\frac{d}{dx}\left[M(x)\, g \frac{dy}{dx}\right] = \left(\frac{d}{dx}(M(x))\right)(-\omega^2 y) \qquad \text{and, dividing by } g,$$

$$\frac{d}{dx}\left[m \frac{dy}{dx}\right] + \frac{\omega^2}{g} y \frac{dm}{dx} = 0 \qquad \text{but } y\frac{dm}{dx} = \frac{d}{dx}(ym) - m\frac{dy}{dx}$$

or $$\frac{d}{dx}\left(m \frac{dy}{dx}\right) + \frac{\omega^2}{g}\frac{d}{dx}(my) - \frac{\omega^2}{g} m \frac{dy}{dx} = 0$$

finally, $$\frac{d}{dx}\left(m \frac{dy}{dx} + \frac{\omega^2}{g} m y\right) - \frac{\omega^2}{g} m \frac{dy}{dx} = 0 \qquad \text{which has the form}$$

$$\frac{d}{dx}\left(\frac{dL}{dy'}\right) - \frac{dL}{dy} = 0 \qquad \text{provided } \left[\begin{array}{l} y' = \frac{dy}{dx} \\ L(y,y') = \frac{1}{2} m \left(\frac{dy}{dx}\right)^2 + \frac{\omega^2}{g} m y \frac{dy}{dx} \end{array}\right.$$

(The equation of motion is a modified Bessel's Equation which admits various types of solutions)

b) $L(y, y') = \frac{1}{2} m \left(\frac{dy}{dx}\right)^2 + \frac{\omega^2}{g} m y \frac{dy}{dx}$

then $\delta \int_0^l L \, dx = \left[\left(\frac{dy}{dx} + \frac{\omega^2 y}{g}\right)(m \, \delta y)\right]_0^l$

now $\underline{\text{at } x = l}$, $\underline{\delta y = 0}$ (the point of support is $\underline{\text{fixed}}$)

$\underline{\text{at } x = 0}$ $\quad \cancel{m\,dx}\, g \frac{dy}{dx} = (\cancel{m\,dx})(-\omega^2 y)$ ie, the horizontal force causes the horizontal acceleration.

or $\quad \underline{\frac{dy}{dx} + \frac{\omega^2}{g} y = 0}$

so

$$\underline{\delta \int_0^l L \, dx = 0}$$

Solution

(a) Low field: S, m $\qquad H' = A\vec{S}_1\cdot\vec{S}_2 + \frac{eB}{mc}(S_{1z} - S_{2z})$

High field: m_1, m_2

(b) In $|m_1 m_2\rangle$ basis, write $|\psi\rangle = \sum_{m_1 m_2} a_{m_1 m_2} |m_1 m_2\rangle$.

$$H'|\psi\rangle = E|\psi\rangle$$

$$\sum_{m_1' m_2'} \langle m_1' m_2' | H' | m_1 m_2\rangle a_{m_1' m_2'} = E_{m_1 m_2} a_{m_1 m_2}$$ Schrödinger eq. in matrix form

$$H' = A\left[\frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+}) + S_{1z}S_{2z}\right] + \frac{eB}{mc}(S_{1z} - S_{2z})$$

$$H'|\tfrac{1}{2},\tfrac{1}{2}\rangle = A\cdot\frac{\hbar}{2}\cdot\frac{\hbar}{2}|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$H'|-\tfrac{1}{2},-\tfrac{1}{2}\rangle = A\cdot\left(-\frac{\hbar}{2}\right)\cdot\left(-\frac{\hbar}{2}\right)|-\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$|\frac{1}{2},\frac{1}{2}\rangle$ and $|-\frac{1}{2},-\frac{1}{2}\rangle$ are degenerate triplet states with energy $E_{S=1} = \frac{\hbar^2}{4}A$ and no Zeeman shift.

The $m=0$ states are coupled:

$$H'|\tfrac{1}{2},-\tfrac{1}{2}\rangle = A\cdot\tfrac{1}{2}\hbar^2|-\tfrac{1}{2},\tfrac{1}{2}\rangle + \left[-A\frac{\hbar^2}{4} + \frac{e\hbar B}{mc}\right]|\tfrac{1}{2},-\tfrac{1}{2}\rangle = E$$

$$H'|-\tfrac{1}{2},\tfrac{1}{2}\rangle = A\cdot\tfrac{1}{2}\hbar^2|\tfrac{1}{2},-\tfrac{1}{2}\rangle + \left[-A\frac{\hbar^2}{4} + \left(-\frac{e\hbar B}{mc}\right)\right]|-\tfrac{1}{2},\tfrac{1}{2}\rangle = E$$

$$\begin{array}{cc} |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle & |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \end{array}$$
$$\begin{pmatrix} -A\frac{\hbar^2}{4} + \frac{e\hbar B}{mc} - E & A\frac{\hbar^2}{2} \\ A\frac{\hbar^2}{2} & -A\frac{\hbar^2}{4} - \frac{e\hbar B}{mc} - E \end{pmatrix}\begin{pmatrix} a_{\frac{1}{2},-\frac{1}{2}} \\ a_{-\frac{1}{2},\frac{1}{2}} \end{pmatrix} = 0$$
$$\begin{array}{cc} |-\tfrac{1}{2}\,\tfrac{1}{2}\rangle & |-\tfrac{1}{2}\,\tfrac{1}{2}\rangle \end{array}$$

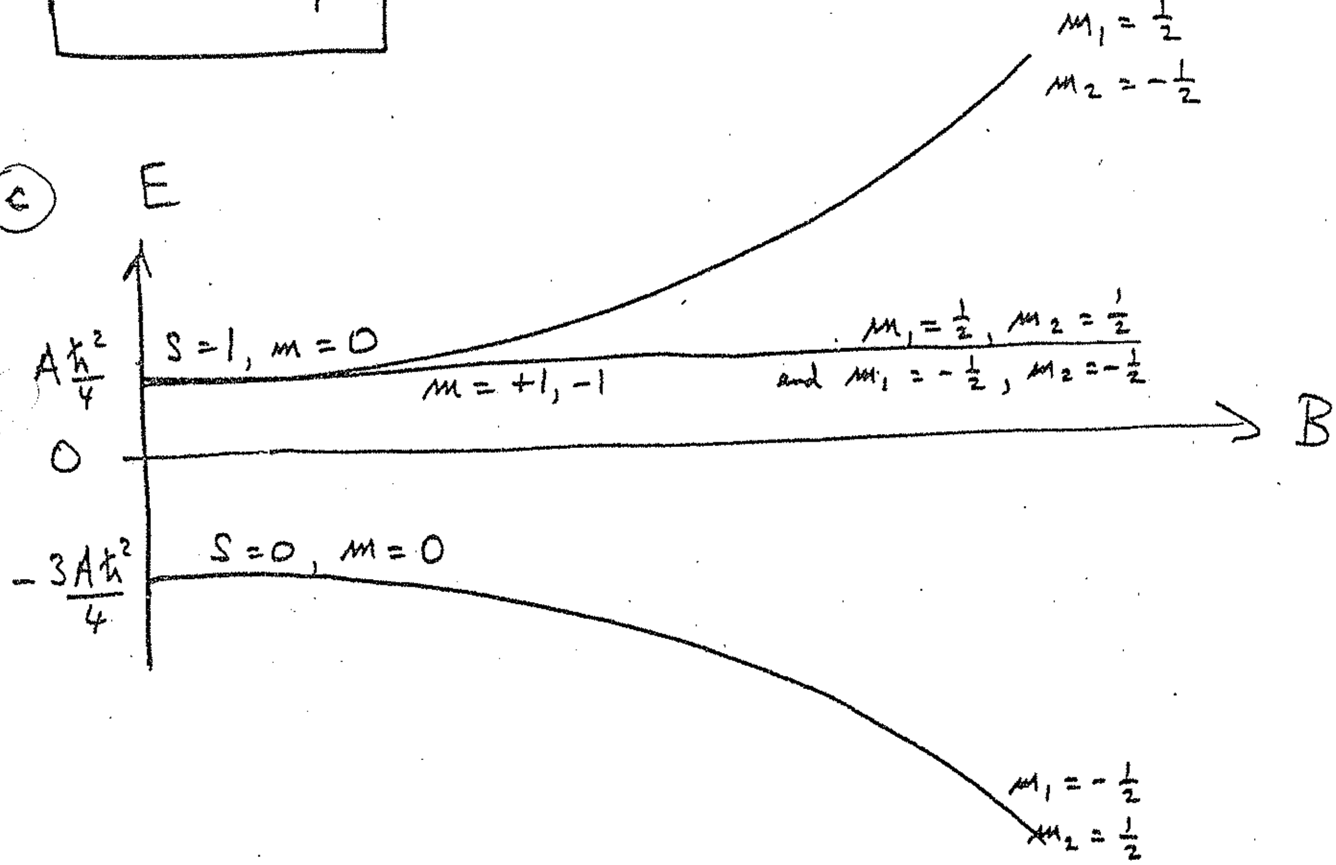
$$\text{Det}(\ ) = 0 : \quad \left(-A\frac{\hbar^2}{4} - E\right)^2 - \left(\frac{e\hbar B}{mc}\right)^2 - A^2\frac{\hbar^4}{4} = 0$$

$$E + A\frac{\hbar^2}{4} = \pm\sqrt{A^2\frac{\hbar^4}{4} + \left(\frac{e\hbar B}{mc}\right)^2}$$

$$\boxed{E = -A\frac{\hbar^2}{4} \pm A\frac{\hbar^2}{2}\sqrt{1+\left(\frac{2eB}{\hbar mcA}\right)^2}}$$ for $m=0$ states

$$\boxed{E = A\frac{\hbar^2}{4}}$$ for $m = \pm 1$ states

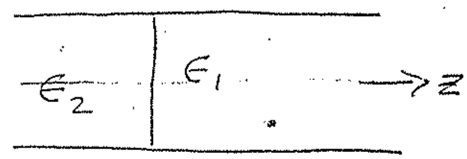
(c)



(d) The e^+e^- system annihilates into photons.
Single-γ decay violates momentum conservation.
Two-γ decay is allowed if the e^+e^- system has $S=0$ (singlet).
The triplet state has a three-γ dominant decay mode (and a longer natural lifetime).

Solution

5. In infinite homogeneous medium a point charge has a solution $\Phi = \frac{q}{\epsilon} \frac{1}{|\vec{r} - \vec{r}'|}$ thus the singularity in the cylinderical geometry can be represented by $\frac{q}{\epsilon_1} G(r, r')$. but to satisfy the boundary conditions we add solutions



of Laplace's eqs. which are not ∞ at $z = \pm\infty$:

$$1: \quad \Phi = \frac{q}{\epsilon_1}\left[G(r, r') + \sum_{mn} A_{mn} e^{im\theta} e^{-k_{mn} z} J_m(\right.$$

$$2: \quad \Phi = \frac{q}{\epsilon_1} \sum B_{mn} e^{im\theta} e^{k_{mn} z} J_m(k_{mn}\rho)$$

where A_{mn} + B_{mn} are unknown constants

cont. of Φ at $z = 0 \Rightarrow$

$$\frac{2}{k_{mn} J_{m+1}^2 (X_{mn})\, a^2} e^{-k_{mn} z'} J_m(\rho') e^{-im\theta'}$$

$$+ A_{mn} = B_{mn}$$

5 (cont.)

Cont. of D at $Z=0$ gives $\epsilon_2 E = \epsilon_1 E_1$

$$\epsilon_1 \left[\frac{2}{k_{mn} J_{m+1}^2 (X_{mn}) a^2} \cancel{k_{mn}}\, e^{-ik_{mn} z'} J_{mn}(\rho') e^{-im\cdot} - \cancel{k_{mn}}\, A_{mn} \right] = \epsilon_2 \cancel{k_{mn}}\, B_{mn}$$

Dividing out $\epsilon_1 k_{mn}$ and adding gives

$$B_{mn} = \frac{2}{1+\frac{\epsilon_2}{\epsilon_1}} \frac{2}{k_{mn} J_{m+1}^2 (X_{mn}) a^2} e^{-ik_{mn} z'} J_m(\rho') \times e^{-im\theta'}$$

or dividing out $\epsilon_2 k_{mn}$ and subtracting gives

$$A_{mn} = -\frac{1-\frac{\epsilon_1}{\epsilon_2}}{1+\frac{\epsilon_1}{\epsilon_2}} \frac{2}{k_{mn} J_{m+1}^2 (X_{mn}) a^2} e^{-ik_{mn} z'} J_m(\rho') e^{-im}$$

thus in region 1 $\quad \Phi = \frac{8}{\epsilon_1}\left[G(r, r') + \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} G(\rho\theta z, \rho'\theta' - \right.$

$\rho, \theta, z; \rho' \theta', z'$

pt charge

image charge

and in 2 $\quad \Phi = \frac{28}{\epsilon_1 + \epsilon_2} G(r, r')$

pt charge soln reduced by $\frac{2\epsilon_1}{\epsilon_1 + \epsilon_2}$

(6 pts.)

(a) Partition function: $Z = \sum_n e^{-\beta E_n}$

where $\beta = \frac{1}{kT}$ and $E_n = n\hbar\omega$. $\omega = \frac{1}{\sqrt{LC}}$ classical frequency.

$$Z = \sum_{n=0}^{\infty} \left(e^{-\beta\hbar\omega}\right)^n = \frac{1}{1-e^{-\beta\hbar\omega}}$$

Average energy: $\langle E \rangle = \sum_n E_n e^{-\beta E_n} \cdot \frac{1}{Z} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$

$$\frac{\partial Z}{\partial \beta} = \frac{\hbar\omega e^{-\beta\hbar\omega}}{\left(1-e^{-\beta\hbar\omega}\right)^2}$$

$$\therefore \langle E \rangle = -\frac{\hbar\omega e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}} = \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$$

The average energy stored in the circuit is also given by

$$\langle E \rangle = \langle \tfrac{1}{2}CV^2 \rangle + \langle \tfrac{1}{2}LI^2 \rangle = \langle CV^2 \rangle$$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{\langle E \rangle}{C}}$$

$$\boxed{V_{rms} = \sqrt{\frac{\hbar\omega/C}{e^{\hbar\omega/kT}-1}}}$$ where $\omega = \frac{1}{\sqrt{LC}}$

(b) If $\frac{\hbar\omega}{kT} << 1$: $e^{\frac{\hbar\omega}{kT}} - 1 \cong \frac{\hbar\omega}{kT}$

$$V_{rms} = \sqrt{\frac{\hbar\omega}{C} \cdot \frac{kT}{\hbar\omega}}$$

$$\boxed{V_{rms} = \sqrt{\frac{kT}{C}}}$$

This result follows also from the equipartition theorem:

$$\left\langle \tfrac{1}{2} C V^2 \right\rangle = \tfrac{1}{2} kT \quad \longrightarrow \quad \langle V^2 \rangle = \frac{kT}{C}$$

Order of magnitude: $kT \cong \frac{1}{40}$ eV

$C = 10^{-12}$ farad

$$V_{rms} \cong \sqrt{\frac{\frac{1}{40} \times 1.6 \times 10^{-19} \text{ coul.volt}}{10^{-12} \text{ coul/volt}}}$$

$$\cong \underline{\underline{10^{-4} \text{ volt.}}}$$