PHYSICS DEPARTMENT COMPREHENSIVE EXAMINATION #40

October 3, 1981

General Instructions

This Comprehensive Examination for Fall 1981 (#40) consists of six problems of equal weight (20 points each). Please check that you have all of them.

Work carefully, indicate your reasoning briefly and display your work clearly. Even if you do not complete a problem, it might be possible to obtain partial credit--especially if your understanding is manifest. Use no scratch paper; do all work in the bluebooks, use one bluebook per problem, and be certain that your assigned student letter (but not your name) is on every booklet.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers except pencil (or pen) and bluebook, on the floor. The accompanying booklet contains data and formulas which you may find useful. Please return it at the end of the exam.

1. Two parallel plates of large area A and separation d_1 are connected to a battery of voltage V and internal resistance r and then disconnected. Neglecting all edge effects,

(5) (a) What force is required to hold them apart?

(5) (b) What work is done in separating them a small distance to d_2?

(5) (c) With the separation now d_2, the battery is again connected as in part (a). Will any charge flow and, if so, onto or off from the plates?

(5) (d) How much work is now done with the battery connected in going back to the original separation d_1?

(5) 2. A pi-mesic atom consists of a strongly deformed nucleus and a pion. Since the pion's Bohr radius is small atomic electrons can be ignored. The potential felt by the pion is

$$- \frac{e^2}{r} + V_0(r)\ [1 + \lambda P_2(\cos\theta)].$$

Because of finite size effects the s, p, d, etc., states for a given n are not degenerate but occur at energies $\varepsilon_0(n)$, $\varepsilon_1(n)$, $\varepsilon_2(n)$ . . . when the λ term is ignored.

(5) (a) Obtain an expression for the leading order correction to the ground state energy from the λ term. How does it depend on λ?

(5) (b) Obtain an expression for the quadrupole moment of the (perturbed) ground state. How does it depend on λ? $[\underset{\sim}{Q} = r^2 Y_2^0(\hat{r})]$

(5) (c) For each state of $n = 2$ determine the dependence of the energy shift on λ. What degeneracies will be removed by the perturbation? Which state will be shifted most?

(5) (d) What are the lowest energy states for which degenerate or near-degenerate perturbation theory must be used?

3. Consider light, as a plane wave, to propagate in the z-direction in a medium having a dielectric constant represented by the tensor ε_{ij} whose elements are given by the matrix $\begin{pmatrix} \varepsilon_1 & i\eta & 0 \\ -i\eta & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}$, where ε_1, ε_2 and η are real and $\eta < \varepsilon_1$.

(15) (a) Determine expressions for the two complex propagation constants γ_1 and γ_2 associated with the stable modes that can propagate in this medium (i.e. those polarizations which propagate unchanged), and

(5) (b) indicate the polarizations of these two stable modes. [Remember that $D_i = \sum_j \varepsilon_{ij} E_j$ in a medium such as this.]

4. An artificial satellite orbiting the earth well above the atmosphere travels, in general, in an elliptical path (neglecting many-body effects and considerations of general relativity).

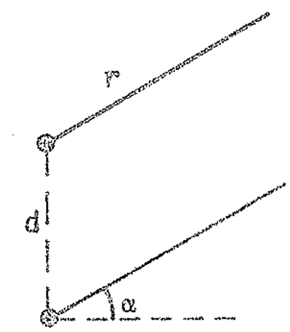
(10) (a) Show that the total energy of the two-body system depends only on the major axis of the ellipse (you may assume the earth to be stationary for purposes of the calculation).

(10) (b) Consider now a satellite in circular orbit around the earth. As it enters the atmosphere, a frictional retarding force $R(v)$ acts on it. Show that the ensuing motion is such that the satellite actually speeds up under these conditions.

5. Two identical vertical dipole antennas are separated by a horizontal distance d and are driven in phase at frequency ω. The radiation field amplitude near each antenna is $E_0 \cos \omega t$ in the mid-plane.

(5) (a) Calculate the radiation-field intensity in the horizontal mid-plane at a distant point $r \gg \lambda$ due to one antenna alone.

(15) (b) Calculate and plot the complete radiation-field intensity pattern due to both antennas everywhere in the horizontal mid-plane, if $d = \lambda$.



(Antennas are perpendicular to paper.)

6. Fifty grams of milk at temperature $T_m = 5^oC$ are added slowly to 250 grams of coffee at temperature $T_c = 90^oC$. Assume the specific heats of both liquids to be the same as that of water.

(5) (a) Calculate the final temperature T_f after equilibrium is reached.

(15) (b) Calculate the total entropy change of the system.

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Undergrad E & M

$E = 4\pi\sigma$ ++++++ ------ $E = \frac{\sigma}{\epsilon_0}$

$dE = V$

$4\pi\sigma = \frac{V}{d}$

$4\pi \frac{q}{A} = \frac{V}{d}$

$q = CV = \left(\frac{A}{4\pi d}\right)V$ $C = \frac{\epsilon_0 A}{d}$

5 (a) $F = qE = \frac{A}{4\pi d_1} V \frac{V}{d_1}$ $F = \frac{A\epsilon V^2}{d^2}$

$= \frac{AV^2}{4\pi d_1^2}$

5 (b) F doesn't change

$W = \frac{AV^2}{4\pi d_1^2}(d_2 - d_1)$

5 (c) charge at d_2 is $\frac{A}{4\pi d_2}V$

lower since d is greater

thus charge flows from plates back to battery

$\Delta q = \frac{AV}{4\pi}\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$

5 (d) $q(x) = \frac{AV}{4\pi x}$

$E = \frac{V}{x}$

$F = qE = \frac{AV^2}{4\pi x^2}$

$W = \int F\,dx = \int_{d_2}^{d_1} \frac{AV^2}{4\pi x^2}\,dx = \frac{AV^2}{4\pi}\left(\frac{1}{d_2} - \frac{1}{d_1}\right) = -\frac{AV^2}{4\pi}\,\frac{d_2 - d_1}{d_1 d_2}$

(2)

(a) $\Delta E_{1s}^{(1)} = \lambda \int \psi_{1s}\ V(r)\ P_2(\cos\theta)\ \psi_{1s}\ d\tau = 0$

$$\Delta E_{1s}^{(2)} = \lambda^2 \sum_n \frac{|\langle \psi_{n20} | V(r) P_2(\cos\theta) | \psi_{1s} \rangle|^2}{E_{10} - E_{n2}}$$

only $l = 2$ states will contribute

(b)

$$\psi^0 + \psi^{(1)} = \psi_{1s} + \lambda \sum_n \frac{\langle \psi_{n20} | V(r) P_2(\cos\theta) | \psi_{1s} \rangle}{E_{10} - E_{n2}} \psi_{n20}$$

$$Q \approx \langle \psi^0 + \psi' | r^2 Y_2^0 | \psi^0 + \psi^{(1)} \rangle$$

$$\approx \sum_n \lambda \frac{\langle \psi_{1s} | r^2 Y_2^0 | \psi_{n20} \rangle \langle \psi_{n20} | V(r)\ P_2(\cos\theta) | \psi_{1s} \rangle}{E_{10} - E_{n2}}$$

$$+ \lambda \frac{\langle \psi_{n20} | r^2 Y_2^0 | \psi_{1s} \rangle \langle \psi_{n20} | V(r)\ P_2(\cos\theta) | \psi_{1s} \rangle^*}{E_1 - E_{n2}}$$

(c) $n = 2$ — 2s — 2p $m=0$ — 2p $m=1$ — 2p $m=2$

since ΔV cons. parity and L_z none of these states are mixed by it. $\Delta E^{(1)} = 0$ for 2s

$$\Delta E_{2pm}^1 = \lambda \langle \psi_{2pm} | V_0(r) P_2(\cos\theta) | \psi_{2pm} \rangle$$

$$= \lambda \langle 1 2 m 0 | 1 m \rangle \langle \psi_{2p} || V_0 P_2 || \psi_{2p} \rangle$$

$$= \lambda \begin{cases} -\sqrt{.4} & m = 0 \\ \sqrt{.1} & m = \pm 1 \end{cases}$$ degen. between $m=0$ + $m=\pm1$ is removed but not between $m = \pm 1$

(d) Quad int mixes 3s and 3d $m=0$ states

these are the lowest

③

Fields $\vec{E}$ and $\vec{H}$ have the general form, $A_\lambda = A^\circ_\lambda e^{-\gamma z}$

where $\gamma = \alpha + i\beta$ and $\lambda = x, y$.

(I) $\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$ yields,

$$\gamma H^\circ_y = \frac{i\omega\varepsilon_1}{c} E^\circ_x - \eta\frac{\omega}{c} E^\circ_y \qquad \text{—— (1)}$$

and $$-\gamma H^\circ_x = \eta\frac{\omega}{c} E^\circ_x + i\frac{\omega\varepsilon_1}{c} E^\circ_y \qquad \text{—— (2)}$$

(II) $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ yields

$$\gamma E^\circ_y = -\frac{i\omega}{c} H^\circ_x \qquad \text{—— (3)}$$

and $$\gamma E^\circ_x = \frac{i\omega}{c} H^\circ_y \qquad \text{—— (4)}$$

Eliminating H°_x and H°_y yields

$$\left.\begin{aligned} \varepsilon_1 E^\circ_x + i\eta E^\circ_y &= -\frac{c^2\gamma^2}{\omega^2} E^\circ_x \\ -i\eta E^\circ_x + \varepsilon_1 E^\circ_y &= -\frac{c^2\gamma^2}{\omega^2} E^\circ_y \end{aligned}\right\} \text{Eigenvalue Statement.}$$

Nontrivial solutions when

$$\begin{vmatrix} \varepsilon_1 - \left(-\frac{\gamma^2 c^2}{\omega^2}\right) & i\eta \\ -i\eta & \varepsilon_1 - \left(-\frac{\gamma^2 c^2}{\omega^2}\right) \end{vmatrix} = 0$$

i.e. $\left(-\frac{\gamma^2 c^2}{\omega^2}\right)^2 - 2\varepsilon_1\left(-\frac{\gamma^2 c^2}{\omega^2}\right) + (\varepsilon_1^2 - \eta^2) = 0$

and so $\gamma^2 = -\frac{\omega^2}{c^2}(\varepsilon_1 \pm \eta)$

<u>Polarizations:</u>

Put γ into eigenvalue statement and get E_y^0 in terms of E_x^0. You get, for $\gamma^2 = -\frac{\omega^2}{c^2}(\varepsilon_1 + \eta)$

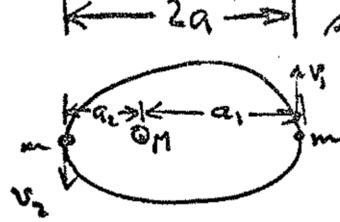
$$\varepsilon_1 E_x^0 + i\eta E_y^0 = (\varepsilon_1 + \eta) E_x^0, \quad \text{or} \quad E_y^0 = -i E_x^0$$

and for $\gamma^2 = -\frac{\omega^2}{c^2}(\varepsilon_1 - \eta)$

$$\varepsilon_1 E_x^0 + i\eta E_y^0 = (\varepsilon_1 - \eta) E_x^0, \quad \text{or} \quad E_y^0 = i E_x^0$$

i.e. $\left.\begin{array}{l} E_1 = E_x^0(1-i) \\ E_2 = E_x^0(1+i) \end{array}\right\}$ <u>CIRCULARLY POLARIZED LIGHT.</u> (stable modes).

(4). An artificial satellite ~~orbiting the earth~~ well above the ~~earth's~~ atmosphere travels, in general ~~about the earth~~ in a elliptical path (neglecting many body effects and general relativistic considerations). Show that the total energy of the two body system depends only on the major axis of the ellipse. (You may assume the earth stationary for purposes of calculation).



$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad \text{—— (1)}$$

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$$\text{(2) —— } \cancel{m}\, a_2 v_2 = \cancel{m}\, a_1 v_1 \qquad \text{but } 2a = a_1 + a_2 \text{ —— (3)}$$

$$E_1 = \underbrace{\tfrac{1}{2}\cancel{m}v_1^2 - \frac{GM\cancel{m}}{a_1}}\; = \;\underbrace{\tfrac{1}{2}\cancel{m}v_2^2 - \frac{GM\cancel{m}}{a_2}} = E_2 \quad \text{—— (4)}$$

$$v_1^2 - v_2^2 = 2GM\left(\frac{1}{a_1} - \frac{1}{a_2}\right) \text{—— (5)}$$

~~$v_1 + v_2$~~

Equations (2) and (5) allow determination of v_1 and v_2 in terms of a_1 and a_2.

i.e. $v_2 = \frac{a_1}{a_2} v_1$ and $v_1^2 - \frac{a_1^2}{a_2^2} v_1^2 = 2GM\left(\frac{1}{a_1} - \frac{1}{a_2}\right)$

$$\therefore \quad v_1^2\left(1 - a_1^2/a_2^2\right) = 2GM\left(\frac{a_2 - a_1}{a_1 a_2}\right)$$

$$v_1^2\left(\frac{\cancel{(a_2 - a_1)}(a_2 + a_1)}{a_2^{\cancel{2}}}\right) = \frac{2GM}{a_1 \cancel{a_2}}\cancel{(a_2 - a_1)}$$

$$\therefore \quad v_1^2 = \frac{2GM\,a_2}{a_1(a_1 + a_2)}.$$

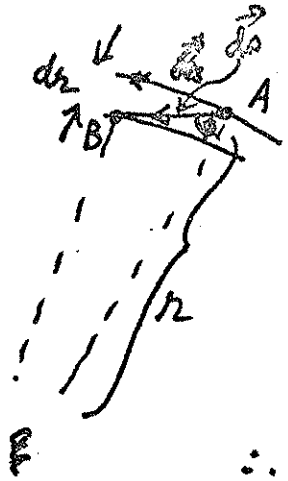
$$\therefore \quad E_1 = \frac{1}{\cancel{2}} m \frac{\cancel{2}GM a_2}{a_1(a_1 + a_2)} - \frac{GMm}{a_1}$$

$$= -\frac{GMm}{a_1}\left[\frac{1}{\cancel{a_1}} - \frac{a_2}{\cancel{a_1}(a_1 + a_2)}\right] = -\frac{GMm}{\cancel{a_1}}\left(\frac{\cancel{a_1} + a_2 - a_2}{a_1 + a_2}\right)$$

$$E_1 = -\frac{GMm}{a_1 + a_2} = -\frac{GMm}{2a},$$

and since E is constant at all pts. in the orbit, the statement has been proven.

"Circular Orbit"



$$\bar{F} = \frac{GMm}{r^2}\cos\phi - R(v) \qquad \text{net force} \quad ——①$$

$$dK = \bar{F}\cdot d\bar{s} = \frac{GMm}{r^2}\underbrace{\cos\phi\, ds}_{-dr} - R(v)\underbrace{ds}_{v\,dt} \quad ——②$$

and $dK = mv\,dv$ ——③

$\therefore$

$$mv\,dv = -\frac{GMm}{r^2}dr - R(v)\,v\,dt \quad ——④$$

⑤ But $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$ $\quad\therefore\quad$ $mv\,dv = -\frac{GMm}{2r^2}dr$

$\therefore$ we can write ④ as $m\cancel{v}\,dv = 2m\cancel{v}\,dv - R(v)\cancel{v}\,dt$

$$m\,dv = R(v)\,dt$$

or $\boxed{\frac{dv}{dt} = +\frac{R(v)}{m}}$ i.e. the artificial satellite actually speeds up upon entering the earth's atmosphere.

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⑥ Consider now an artificial satellite which is initially in circular ~~earth~~ orbit around the earth. As it enters ~~encounters~~ the atmosphere a frictional retarding force $R(v)$ acts on it. Show that the ensuing motion of the satellite is such that it actually speeds up under these conditions.

5) a) At a distant point r, $\theta = \pi/2$, $\quad E(r,t) = \frac{E_0}{r}\cos\omega(t - \frac{r}{c})$

so $I_0 = \varepsilon_0 c \frac{E_0^2}{r^2}\overline{\cos^2\omega(t-\frac{r}{c})}$ $\quad$ but $\overline{\cos^2(\quad)} = 1/2$

so $\underline{\underline{I_0 = \frac{\varepsilon_0 c E_0^2}{2r^2}}}$

b) $E(r,t) = \frac{E_0}{r}\cos\omega(t - \frac{r}{c}) + \frac{E_0}{r}\cos\omega(t - \frac{r}{c} - \frac{d\sin\alpha}{c})$

but $\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$

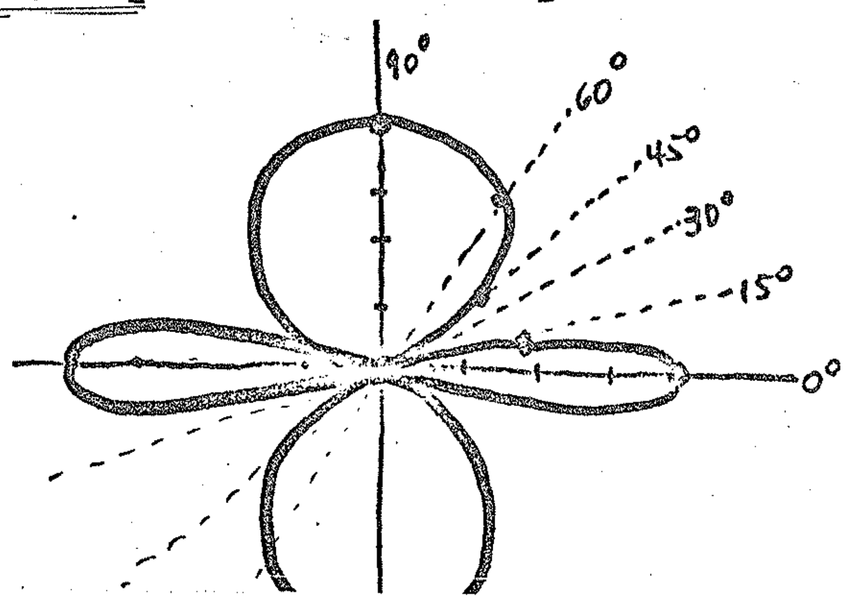
$= \frac{2E_0}{r}\cos\left[\omega(t-\frac{r}{c}) - \frac{\omega d\sin\alpha}{2c}\right]\cos\left(\frac{\omega d\sin\alpha}{2c}\right)$

$\left\{\begin{aligned} \text{calling } \delta &= \frac{\omega}{c}d\sin\alpha \\ &= k\,d\sin\alpha \\ &= \frac{2\pi}{\lambda}d\sin\alpha \end{aligned}\right.$

$= \frac{2E_0}{r}\cos\frac{\delta}{2}\cos\left[\omega(t-\frac{r}{c}) - \frac{\delta}{2}\right]$

so $\underline{\underline{I}} = \varepsilon_0 c\overline{E^2} = \frac{\varepsilon_0 c\, 4E_0^2}{2r^2}\cos^2\frac{\delta}{2} = \underline{\underline{4I_0\cos^2\frac{\delta}{2}}}$ $\quad$ and for $d = \lambda$, $\frac{\delta}{2} = \frac{2\pi}{2\lambda}\lambda\sin\alpha = \underline{\pi\sin\alpha}$

α	$\frac{\delta}{2}$	$\cos^2\frac{\delta}{2}$	$\frac{I(\alpha)}{I_0}$
$0, 180°$	0	1	4
$\pm 30°, 150°$	$\pm\frac{\pi}{2}$	0	0
$\pm 45°, 135°$	$\pm\frac{\pi}{\sqrt{2}}$	0.4	1.4
$\pm 60°, 120°$	$\pm\frac{\sqrt{3}}{2}\pi$	0.8	3.3
$\pm 15°, 165°$	$\pm 0.26\pi$	0.5	1.9
$\pm 90°$	$\pm\pi$	1	4



6

.) $m_m \cancel{c_m} (T_f - T_m) = m_c \cancel{c_c} (T_c - T_f)$

$(m_m + m_c) T_f = m_c T_c + m_m T_m$

$6 \cancel{m_m} T_f = 5 \cancel{m_m} T_c + \cancel{m_m} T_m$

$$\boxed{T_f = \frac{5T_c + T_m}{6}} = \frac{5(90)^\circ c + 5^\circ C}{6} = \underline{\underline{75.8^\circ C}}$$

b) $dQ = dU + dW$

$dQ = T\,dS$

$dW = p\,dV = 0$ at constant volume

$dU = C_v\,dT$

$T_m = 278^\circ K$

$T_c = 363^\circ K$

$C_c = C_m = C_w = 1 \frac{cal}{gm\,^\circ C}$

$$dS = C_v \frac{dT}{T}$$

$$\Delta S = 50 C_m \int_{T_m}^{T_f} \frac{dT}{T} + 250 C_c \int_{T_c}^{T_f} \frac{dT}{T} = 50 C_m \ln\left(\frac{T_f}{T_m}\right) + 250 C_c \ln\left(\frac{T_f}{T_c}\right)$$

$$= 50 \ln \frac{5T_c + T_m}{6T_m} + 250 \ln \frac{5T_c + T_m}{6T_c} \quad \frac{cal}{^\circ C}$$

$$T_f = \frac{5T_c + T_m}{6} = 348.8^\circ K$$

$$= 50 \ln(1.25) + 250 \ln(0.96) \quad \frac{cal}{^\circ K}$$

$$= 50\,(0.227) + 250\,(-0.040) \quad \frac{cal}{^\circ K}$$

$$= 11.35 - 10.00 = \underline{\underline{+1.35 \frac{cal}{^\circ K}}}$$