SPRING 1975

This Comprehensive Examination for Spring 1975 (#21) consists of six problems of equal weight (20 points each). Half of the problems are judged to be at intermediate undergraduate level, the other half at graduate level. Work carefully and show all your steps so that partial credit can be given liberally in case you do not complete a problem. Use no scratch paper; do all work in the bluebook, using one bluebook per problem.

If something is omitted from the statement of the problem or you feel there are ambiguities, please get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books, and papers, except pencil and bluebook, on the floor.

Some information you may find useful:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta = \sqrt{\frac{3}{4\pi}}\,\frac{z}{r}$$

$$Y_{1,\pm 1} = \mp\sqrt{\frac{3}{8\pi}}e^{\pm i\phi}\sin\theta = \mp\sqrt{\frac{3}{8\pi}}\,\frac{x\pm iy}{r}$$

$$k = 10^{-4}\,\mathrm{eV}/^{\circ}\mathrm{K}$$

$$\int_0^\infty \frac{z^3}{e^z+1}\,dz = \frac{7}{120}\pi^4 = 5.7$$

$$\int_0^\infty \frac{z^2}{e^z+1}\,dz = \frac{3}{2}\zeta(3) = 1.7$$

$$\int_0^\infty \frac{z^3}{e^z-1}\,dz = \frac{\pi^4}{15} = 6.5$$

$$\int_0^\infty \frac{z^2}{e^z-1}\,dz = 2\zeta(3) = 2.3$$

If you need a table of integrals, ask the proctor.

COMPREHENSIVE EXAMINATION #21 Spring 1975

1. A helium atom is in the 1s2p $^1P_1$ state. The atom is oriented so that measurements of the orbital angular momentum component L_z always yield the value $+\hbar$, with respect to a given set of coordinate axes.

5 pts
a) Calculate the expectation value of L_x.

8 pts
b) Determine ΔL_x, the root-mean-square deviation of L_x.

2 pts
c) Determine the parity of the given state.

2 pts
d) Explain why the 1s2p $^1P_1$ state can or cannot decay to the lower-lying 1s2s $^3S_1$ state by an allowed transition.

3 pts
e) Under what circumstances can a helium atom be in a simultaneous eigenstate of L_x, L_y, and L_z?

2. One recent experimental test of the general theory of relativity consisted of the observation of the change in energy of a photon as it traveled a distance of 20 meters in the earth's gravitational field. The following questions are based on that experiment.

3 pts
a) Consider a photon which falls a vertical distance x in a gravitational field g. The frequency of the photon is then shifted by an amount $\Delta\nu$ given(in first order) by

$\Delta\nu = \nu$ [redacted] α ,

where α is a dimensionless parameter. Show that $\alpha = gx/c^2$. A dimensional argument is sufficient.

4 pts
b) The "detector" for this type of experiment makes use of the Mössbauer effect. Without reference to this particular application, give a brief discussion of what the Mössbauer effect is, why it occurs, etc.

8 pts
c) Show how the Mossbauer effect may be used to measure the gravitational frequency shift, and estimate the source velocity needed for observation of the effect. Assume that the 14 keV gamma ray from ^{57}Fe is used; the mean lifetime is 10^{-7} sec.

3 pts
d) State simply the principle of general relativity upon which this frequency shift is based.

2 pts
e) Aside from the great precision obtainable using the Mössbauer effect, why are terrestrial methods superior to astronomical ones for this type of experiment?

3. When a conducting spherical shell is uncharged, it floats on a dielectric liquid with one tenth of its volume submerged. If the sphere has a weight $mg = 0.1$ newton and the liquid has a dielectric constant $K = 4$, to what potential ϕ must the sphere be charged if it is to float half submerged?

4. 10 pts
   a) From the thermodynamics of magnetic systems, show that $-M = \left(\frac{\partial G}{\partial H}\right)_T$,

   where M is the magnetization, G is the Gibbs free energy, H is the magnetic field, and T is the temperature.

   10 pts
   b) In expressing the behavior of thermodynamic variables in the vicinity of the magnetic phase transition of a ferromagnetic material, the hypothesis of "scaling" has been found to be generally valid. As applied to the Gibbs free energy, the scaling hypothesis takes the form $G(\lambda^t\varepsilon, \lambda^u H) = \lambda G(\varepsilon, H)$, where $\varepsilon = \frac{T - T_c}{T_c}$, in which T_c is the critical temperature at which the phase transition occurs, and where t, u, and λ are real numbers. The "scaling law" holds for any value of λ. Using the scaling law and the result from part (a), show that at temperatures just below T_c and $H \to 0$ the magnetization behaves like

$$M = C\,(-\varepsilon)^\beta \quad ,$$

   where $\beta = \frac{1-u}{t}$ and C is a constant.

5. Consider a quantum mechanical rotor constrained to rotate in a plane. The Hamiltonian for such a system is given by

$$H = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\theta^2} \quad ,$$

   where I is the moment of inertia and θ is the angle of rotation with respect to some reference line in the plane.

   4 pts
   a) Obtain the eigenfunctions and energy eigenvalues for this system and indicate the degeneracy, if any, of each level.

   8 pts
   b) Suppose the system is perturbed by a quadrupole interaction of the form

$$H = \frac{V_o}{2}\,(3\cos^2\theta - 1) \quad ,$$

   where V_o is a small positive constant. Calculate the energy of the ground state to second order in V_o (keep terms up to V_o^2).

   8 pts
   c) Assuming the same perturbation, calculate the energy of the first excited state to first order in V_o.

6. In 1950, Enrico Fermi proposed a purely statistical theory (one that did not involve any nuclear dynamics) for the production of spinless pions (positive, negative and neutral) and nucleon, anti-nucleon pairs from the very high energy collision of two nucleons.

In this problem we shall assume, with Fermi, that the pair of colliding nucleons isotropically release all their energy in a small volume, on the order of

$$V = \frac{4}{3}\pi \left(\frac{h}{m_\pi c}\right)^3 \left(\frac{2Mc^2}{W}\right) \sim (1 \text{ Fermi})^3 \quad \text{for } W \sim 10^4 \text{ MeV} .$$

Here $\frac{h}{m_\pi c}$ is the radial extent of the pion field

m_π is the pion mass

c is the velocity of light

M is the mass of a nucleon

W is the total energy (in the C.M.) of the colliding nucleons.

(The factor $\frac{2Mc^2}{W}$ is a relativistic correction to the interaction volume.)

Furthermore we assume:

(i) Thermodynamic equilibrium is reached quickly;

(ii) The pions, nucleons, and antinucleons that result from the collision are created without limit;

(iii) The particles are extremely relativistic, with energies $\varepsilon = c|\vec{p}|$, where $\vec{p}$ is the particle momentum.

10 pts
a) Find an expression for the temperature of the initial pion-nucleon fireball.

10 pts
b) Find an expression for the number of pions produced.

(1.) Helium atom: $1s2p\ {}^1P_1 \quad m_L = +1$.

The angular part of the wave function is the spherical harmonic $Y_{11}(\theta,\phi)$. $L_\pm \equiv L_x \pm i L_y$; $L_x = \frac{L_+ + L_-}{2}$.

(a) $\langle L_x \rangle = \int_{(4\pi)} Y_{11}^* \left(\frac{L_+ + L_-}{2}\right) Y_{11}\, d\Omega \qquad L_\pm Y_{\ell m} = \sqrt{(\ell\mp m)(\ell\pm m+1)}\,\hbar\, Y_{\ell,m\pm1}$

But $L_+ Y_{11} = 0$ and $L_- Y_{11} = \sqrt{2}\,\hbar\, Y_{10}$, which is orthogonal to $Y_{1\phi}$.

Therefore, $\underline{\underline{\langle L_x \rangle = 0}}$.

(b) $\Delta L_x = \sqrt{\langle L_x^2\rangle - \langle L_x\rangle^2} = \sqrt{\langle L_x^2\rangle}$

$$L_x^2 = \frac{(L_+ + L_-)^2}{4} = \frac{1}{4}\left(L_+^2 + L_-^2 + L_+L_- + L_-L_+\right)$$

$$\langle L_x^2\rangle = \frac{1}{4}\int Y_{11}^* L_+L_- Y_{11}\, d\Omega = \frac{1}{4}\int Y_{11}^* L_+ \sqrt{2}\,\hbar\, Y_{10}\, d\Omega$$

$$= \frac{\sqrt{2}\,\hbar}{4}\int Y_{11}^* \sqrt{2}\,\hbar\, Y_{11}\, d\Omega = \frac{\hbar^2}{2}$$

Therefore, $\underline{\underline{\Delta L_x = \frac{\hbar}{\sqrt{2}}}}$.

This may also be obtained from $L_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$ and expressions for the spherical harmonics in terms of x, y, z.

ALTERNATIVE SOLUTION:

$$\underbrace{\langle L_x^2\rangle + \langle L_y^2\rangle}_{\text{EQUAL, by symmetry.}} + \underbrace{\langle L_z^2\rangle}_{\hbar^2} = \langle L^2\rangle = \ell(\ell+1)\hbar^2 = 2\hbar^2$$

$$\therefore\ 2\langle L_x^2\rangle = \hbar^2 \longrightarrow \Delta L_x = \frac{\hbar}{\sqrt{2}}.$$

1. (CONT'D.) ⓒ $P = (-1)^{\sum_i l_i} = (-1)^{0+1} = -1$ Parity is <u>odd</u>.

ⓓ The spin selection rule for allowed transitions is $\Delta S = 0$ (i.e., the electric dipole operator does not affect spin).
However, $\Delta S = 1$ for a singlet-triplet transition $^1P_1 \to {}^3S_1$.
Therefore the transition is <u>forbidden</u>.

ⓔ Suppose
$$\begin{cases} L_x \Psi = m_x \hbar \Psi \\ L_y \Psi = m_y \hbar \Psi \\ L_z \Psi = m_z \hbar \Psi \end{cases}$$ simultaneously.

Then $L_x(L_y \Psi) = L_y(L_x \Psi)$,

or $[L_x, L_y]\Psi = 0$.

But $[L_x, L_y] = i\hbar L_z$, and therefore $L_z \Psi = 0$.

$\longrightarrow m_z = 0$. Similarly, $m_x = m_y = 0$.

$$L^2 = L_x^2 + L_y^2 + L_z^2$$
$$L^2 \Psi = (m_x^2 + m_y^2 + m_z^2)\hbar^2 \Psi = 0$$

$\longrightarrow \underline{l = 0}$. The total orbital angular momentum must be zero (S-state).
This state has <u>spherical symmetry</u>.

2.

Relativity (Solution)

(a) $E(\text{photon}) = h\nu$

an energy E has a gravitational mass E/c^2, and in falling a distance x, its energy increases by $(h\nu/c^2)gx$ (to first order)

$$E' = E + \frac{h\nu}{c^2} gx$$

$$\text{or} \quad \nu' = \nu\left(1 + \frac{gx}{c^2}\right)$$

(b) Nuclear resonance fluorescence (that is, the excitation of a level in a given nucleus by a γ-ray emitted from that level in a different nucleus) is in general not possible owing to the loss in photon energy associated with the nuclear recoil. If, however, the emitting nucleus is bound in a crystal lattice, the recoil momentum is absorbed by the crystal as a whole, and if the recoil energy is less than the phonon (crystal vibrational mode) energy, the γ-ray is emitted from a recoilless nucleus and can then re-excite the corresponding level.

(c) ~~The velocity necessary to achieve complete overlap between the emitted energy and the level energy difference depends on details of crystal structure.~~ A measure of the velocity necessary to pass through the resonance is given by the following:

$$\Gamma = \text{natural width of resonance line} = \frac{\hbar}{\tau} = 6.6 \times 10^{-9}\,\text{eV}$$

$$\Delta E = \text{doppler shift of photon} \approx E\frac{v}{c}$$

$$\Gamma \sim \Delta E \implies v \sim 0.1\,\text{mm/sec}$$

Change in velocity due to gravitational effects:

2. (CONT'D.)

$$\frac{\Delta \nu}{\nu} = \frac{v}{c} = \frac{g x}{c^2} = 2 \times 10^{-15}$$

$$v = \frac{g x}{c} \sim 10^{-3} \text{ mm/sec} \qquad \simeq 1\% \text{ of resonance width}$$

(d) Principle of Equivalence - an experiment performed in a gravitational field of strength g yields a result identical to one performed in a reference frame accelerated at $-g$.

(e) 1. Lack of detailed knowledge of stellar gravitational fields.
2. Separation of gravitational red shift from Doppler red shift.

(3.) Sphere floating on dielectric liquid:

UNCHARGED:  Let V = volume of sphere $= \frac{4}{3}\pi a^3$.

$\frac{V}{10}$ below surface

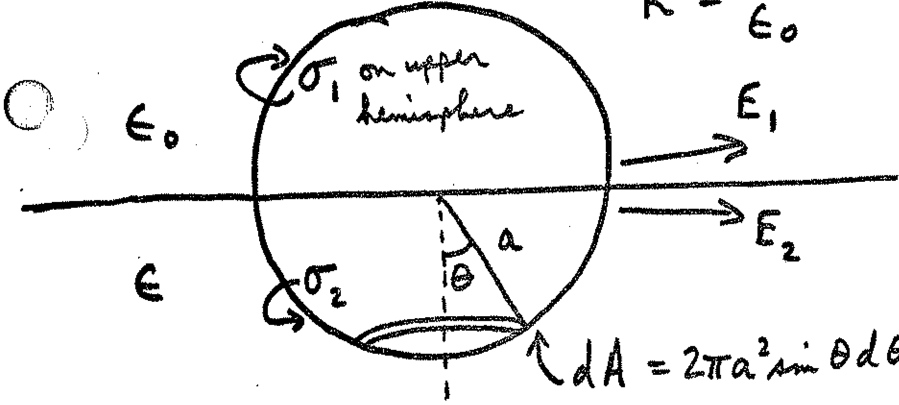
$$mg = \rho \cdot \frac{V}{10} \cdot g \longrightarrow \rho = \frac{10\,m}{V} \quad \text{density of dielectric.}$$

CHARGED: There is a net downward electric force F because the dielectric is polarized.

If sphere is half submerged, $mg + F = \rho \cdot \frac{V}{2} \cdot g$

$$\therefore F = \frac{10\,m}{V} \cdot \frac{V}{2} \cdot g - mg = 5mg - mg = 4mg$$

$$K = \frac{\epsilon}{\epsilon_0}$$



In rationalized MKS units:

$$E_1 = \frac{\sigma_1}{\epsilon_0} \quad ; \quad E_2 = \frac{\sigma_2}{\epsilon}$$

$E_{\text{tangential}}$ is continuous

$$\longrightarrow E_1 = E_2 \quad \text{or} \quad \frac{\sigma_1}{\epsilon_0} = \frac{\sigma_2}{\epsilon}$$

or $\boxed{\sigma_2 = K\sigma_1}$

Net outward pressure on charged surface:

$$\frac{dF_\perp}{dA} = \frac{1}{2}\sigma E = \frac{\sigma^2}{2\epsilon} \qquad F = F_z = \int dF_\perp \cdot \cos\theta$$

$$F = \int \frac{\sigma^2}{\cancel{2}\epsilon} \cos\theta \cdot \cancel{2}\pi a^2 \sin\theta\, d\theta$$

$$= \pi a^2 \left[\frac{\sigma_2^2}{\epsilon} \int_{\theta=0}^{\pi/2} \cos\theta\, d(-\cos\theta) + \frac{\sigma_1^2}{\epsilon_0} \int_{\theta=\pi/2}^{\pi} \cos\theta\, d(-\cos\theta) \right]$$

$$= \frac{\pi a^2}{2} \left[\frac{\sigma_2^2}{\epsilon} - \frac{\sigma_1^2}{\epsilon_0} \right] = \frac{\pi a^2}{2\epsilon_0} (K-1)\, \sigma_1^2$$

In gaussian units $\epsilon_0 \longrightarrow \frac{1}{4\pi}$

$$\phi = \frac{\sigma_1 a}{\epsilon_0} = \sqrt{\frac{2F}{\pi\epsilon_0 (K-1)}} = \underline{\sqrt{\frac{8mg}{\pi\epsilon_0 (K-1)}}} = \sqrt{\frac{8 \times 10^{-1}\ \text{volt}^2}{3 \times 9 \times 10^{-12} \times 3}} = 10^5\ \text{volt}$$

(4.) Solution:

(a) $$T\,dS = dE - H\,dM$$

Thus, we define, as usual

$$G = E - TS - MH$$

so

$$dG = dE - TdS - SdT - MdH - HdM$$

$$= -SdT - MdH$$

Thus $\left(\frac{\partial G}{\partial T}\right)_H = -S$ $\left(\frac{\partial G}{\partial H}\right)_T = -M$

(b) $$G(\lambda^t \epsilon, \lambda^u H) = \lambda G(\epsilon, H)$$

$$\left[\frac{\partial G}{\partial H}(\lambda^t\epsilon, \lambda^u H)\right]_T = \lambda\left[\frac{\partial G(\epsilon,H)}{\partial H}\right]_T$$

$$= -\lambda M(\epsilon, H)$$

$$\left[\frac{\partial G}{\partial H}(\lambda^t\epsilon, \lambda^u H)\right]_T = \frac{\partial G}{\partial H'}(\lambda^t\epsilon, H')\left(\frac{\partial H'}{\partial H}\right) = -\lambda^u M(\lambda^t\epsilon, H')$$

where $H' = \lambda^u H$

For $H \to 0$

$$\lambda^u M(\lambda^t\epsilon, 0) = \lambda M(\epsilon, 0) \quad \text{or} \quad \lambda^{u-1} M(\lambda^t\epsilon, 0) = M(\epsilon, 0)$$

Since $\epsilon = \frac{T - T_c}{T_c}$

Let $\lambda^t = -\frac{1}{\epsilon}$ <u>since scaling holds for any λ</u>

and $\lambda = \left(-\frac{1}{\epsilon}\right)^{\frac{1}{t}} = (-\epsilon)^{-\frac{1}{t}}$

to give

$$(-\epsilon)^{-\left[\frac{u-1}{t}\right]} M(-1, 0) = M(\epsilon, 0)$$

$$C(-\epsilon)^{\beta} = M(\epsilon, 0)$$

(5.) Q.M. (Grad) - Solutions

(a) $-\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\theta^2}\psi = E\psi$

let $\psi_k = Ae^{ik\theta}$, then $\frac{\partial^2\psi}{\partial\theta^2} = -k^2\psi$

$$\boxed{E_k = \frac{\hbar^2 k^2}{2I}} \quad k = 0, \pm 1, \pm 2, \ldots$$

choice of k follows from requirement that ψ be single-valued $\psi(\theta) = \psi(\theta + 2\pi)$

Normalization: $\int_0^{2\pi} \psi^* \psi \, d\theta = 1 \Rightarrow A = \frac{1}{\sqrt{2\pi}}$

$$\boxed{\psi_k = \frac{1}{\sqrt{2\pi}} e^{ik\theta}}$$

Degeneracy = 2 for $k > 0$

(b) $H' = \frac{V_0}{2}(3\cos^2\theta - 1)$

ground state $k = 0$ (non-degenerate)

1st order:

$$\Delta E_1 = \langle 0 | H' | 0 \rangle = \int \psi_0^* H' \psi_0 \, d\theta$$

$$= \frac{1}{2\pi}\frac{V_0}{2}\int_0^{2\pi}(3\cos^2\theta - 1) = \frac{V_0}{4}$$

2nd order $\Delta E_2 = \sum_n \frac{|\langle 0 | H' | n \rangle|^2}{E_0 - E_n}$

$$\langle 0 | H' | n \rangle = \frac{V_0}{2}\frac{1}{2\pi}\int_0^{2\pi}(3\cos^2\theta - 1)e^{in\theta}\, d\theta = \frac{3V_0}{8}\delta_{n2}$$

5. (CONT'D.)

$$\therefore \Delta E_2 = \frac{(3V_0/8)^2}{E_0 - E_2} = \frac{(3V_0/8)^2}{0 - \frac{4\hbar^2}{2I}} = -\frac{9IV_0^2}{128\hbar^2}$$

$$\therefore \boxed{E = 0 + \frac{V_0}{4} - \frac{9IV_0^2}{128\hbar^2}}$$

(c) 1^{st} order shift - note levels are <u>degenerate</u>:

$$\langle n|H'|n'\rangle = \frac{1}{2\pi}\frac{V_0}{2}\int \psi_n^* \, H' \, \psi_{n'} \, d\theta$$

$$= \frac{1}{2\pi}\frac{V_0}{2}\int_0^{2\pi} e^{-in\theta}(3\cos^2\theta - 1)\, e^{in'\theta}\, d\theta$$

$n' = \pm n$

$$\langle n|H'|n\rangle = \frac{1}{2\pi}\frac{V_0}{2}\int_0^{2\pi}(3\cos^2\theta - 1)\, d\theta = \frac{V_0}{4}$$

$$\langle n|H'|-n\rangle = \frac{1}{2\pi}\frac{V_0}{2}\int_0^{2\pi}(3\cos^2\theta - 1)(\cos 2n\theta - i\sin 2n\theta)\, d\theta$$

$$= \frac{3V_0}{8}\,\delta_{n1}$$

$$H' = \begin{pmatrix} V_0/4 & 3V_0/8 \\ 3V_0/8 & V_0/4 \end{pmatrix}$$

diagonalize $\left(V_0/4 - \lambda\right)^2 - \left(\frac{3V_0}{8}\right)^2 = 0 \implies \lambda = -\frac{1}{8}V_0, \ \frac{5}{8}V_0$

$\therefore$ First excited state is split to

$$E = \boxed{\frac{\hbar^2}{2I} - \frac{1}{8}V_0} \text{ and } \boxed{\frac{\hbar^2}{2I} + \frac{5}{8}V_0}$$

6. Solution

Pions are bosons and $\left\{ \begin{matrix} \text{nucleons} \\ \text{anti nucleons} \end{matrix} \right\}$ are fermions

so, the average energy is (in C.M.)

$$W = \frac{V}{(2\pi\hbar)^3}\Big[3 \int d\vec{p} \frac{\epsilon(p)}{e^{\beta\epsilon(p)} - 1}$$

$$+ 8 \int d\vec{p} \frac{\epsilon(p)}{e^{\beta\epsilon(p)} + 1} \Big]$$

$\beta = \frac{1}{k_B T}$ $\quad k_B =$ Boltzmann's constant

$\epsilon(p) = cp$

The Factor 3 arises from the statistical weight of three types of pions $(+, -, 0)$

The Factor 8 arises from statistical weight of 2 types of nucleons [protons + neutrons] each with 2 spin states

and an equal number of ~~anti~~ anti-nucleons, each with 2 spin states.

The angular integrations can be done to give

$$W = \frac{V}{(2\pi\hbar)^3} 4\pi c \Big[3 \int_0^\infty \frac{p^3\,dp}{e^{\beta pc} - 1} + 8 \int_0^\infty \frac{p^3\,dp}{e^{\beta pc} + 1} \Big]$$

6. (CONT'D.)

With a change of variable, we get

$$W = \frac{V}{2\pi^2 \hbar^3 c^3} \left(\frac{1}{\beta}\right)^4 \left[3\int_0^\infty \frac{z^3\,dz}{e^z - 1} + 8\int_0^\infty \frac{z^3\,dz}{e^z + 1} \right]$$

$$\int_0^\infty \frac{z^n}{e^z - 1}\,dz = n!\,\zeta(n+1)$$

$$\int_0^\infty \frac{z^n}{e^z + 1}\,dz = n!\,(1 - 2^{-n})\,\zeta(n+1)$$

where $\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}$

So

$$W = \frac{3!\,V\,\zeta(4)}{2\pi^2 (\hbar c)^3 \beta^4} \left[3 + 8\left(\frac{7}{8}\right)\right] = \frac{(10)(3!)\,V\,\zeta(4)}{2\pi^2 (\hbar c)^3 \beta^4}$$

$$\frac{1}{\beta^4} = \frac{\pi^2 (\hbar c)^3 W}{30\,V\,\zeta(4)}$$

$$T = \frac{1}{k_B} \left[\frac{\pi^2 (\hbar c)^3 W}{30\,V\,\zeta(4)} \right]^{\frac{1}{4}}$$

$$= \frac{1}{k_B} \left[\frac{\pi^3}{10\,\zeta(4)} \right]^{\frac{1}{4}} \left[\left(\frac{m_\pi}{M}\right) (m_\pi c^2)^2 W^2 \right]^{1/4}$$

$$= \frac{1}{k_B} \left[\frac{\pi^3}{10\,\zeta(4)} \right]^{1/4} \left(\frac{m_\pi}{M}\right)^{\frac{1}{4}} \sqrt{(m_\pi c^2) W}$$

6. (CONT'D.) Similarly

$$N_\pi = \frac{V}{2\pi^2\hbar^3}\frac{3}{c^3}\frac{1}{\beta^3}\int_0^\infty \frac{z^2\,dz}{e^z+1}$$

$$= \frac{3V\zeta(3)}{\pi^2\hbar^3c^3\beta^3}$$

$$= \frac{8\zeta(3)}{\pi}\left[\frac{(k_BT)}{m_\pi c^2}\right]^3 \frac{Mc^2}{W}$$

$$= \cancel{\frac{8\zeta(3)}{\pi}\frac{\pi^2(k_BT)^3W}{30}}$$

$$= \frac{8\zeta(3)}{\pi}\left[\frac{\pi}{10\zeta(4)}\right]^{3/4}\left(\frac{m_\pi}{n}\right)^{\frac{3}{4}}\frac{[(m_\pi c^2)W]^{\frac{3}{2}}}{(m_\pi c^2)^3}\frac{Mc^2}{W}$$

$$= \tilde{C}\left(\frac{m_\pi}{n}\right)^{3/4}\frac{(Mc^2)}{(m_\pi c^2)^{\frac{3}{2}}}\sqrt{W}$$

Note: For $W \sim 10^4$ MeV.

$T \sim 10^{13}$ °K

$N_\pi \sim 10^2$