Fall 1973

This Comprehensive Examination for Fall 1973 (#16) consists of six problems of equal weight (20 points each). Half of the problems are judged to be at intermediate undergraduate level, the other half are graduate level. Work carefully and show all your steps so that partial credit can be given liberally in case you do not complete a problem. Use no scratch paper; do all work in the bluebook -- you may get some credit for it!

If something is omitted from the statement of the problem or you feel there are ambiguities, get up and ask your question quietly and privately, so as not to disturb the others. Put all materials, books and papers, except pencil and bluebook, on the floor.

Some information you may find useful follows:

$\pi = 3.14159$

$h \cong 6.63 \times 10^{-34}$ joule-sec.

$(1+\delta)^n \approx 1+n\delta+ \ldots \quad \delta \ll 1$

$\rho e^{i\phi} = x+iy$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

If you need a table of integrals, ask the proctor.

1. The first excited state of a ^{57}Fe nucleus has an energy 14.4 keV above the ground state and a mean lifetime of 10^{-7} seconds.

   a) Calculate an estimate of the width of the emission line, expressed in electron volts.

   b) A free ^{57}Fe atom, initially at rest, makes a transition to the ground state from the first excited state of the nucleus. Calculate the energy of the emitted gamma ray.

   c) Determine whether such a gamma ray can excite another free ^{57}Fe nucleus, initially at rest.

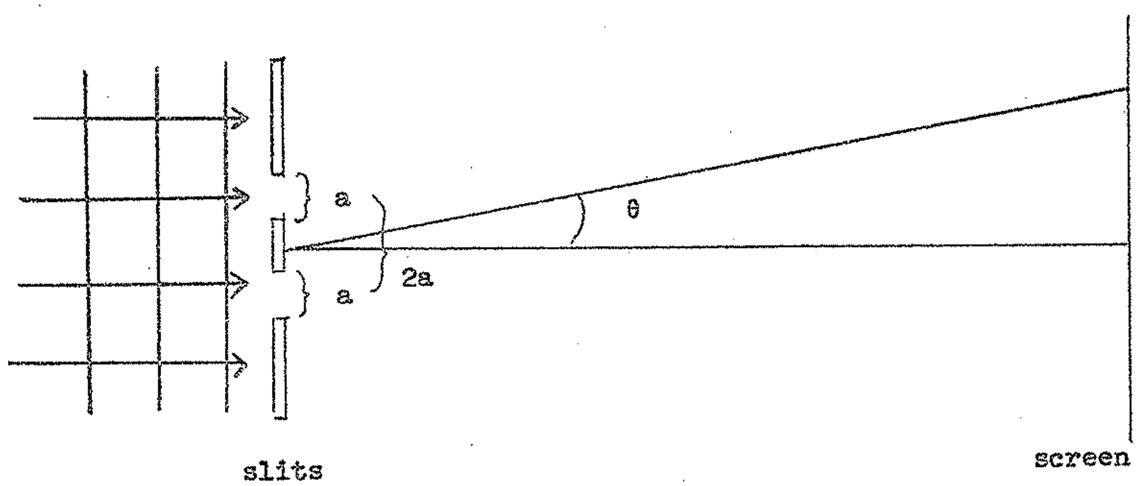
   d) Qualitatively, describe the Mössbauer effect.

2. The Stark effect on the ground state of the hydrogen atom produces (for small applied electric field E) an energy shift proportional to E^2. This shift is usually written as $\Delta\varepsilon = -\frac{1}{2}\,\alpha E^2$ where α is the polarizability of the atom.

   a) Show that the first-order perturbation vanishes, thus proving that the Stark effect is at least quadratic in E.

   b) Evaluate the second-order perturbation in an approximate way, assigning an upper and lower limit to the polarizability.

3. Two slits of width a are separated by a distance 2a. A monochromatic plane wave with wavelength λ impinges normally on the slits from the left. A screen is placed a large distance away to the right of the slits.



   a) Calculate the light intensity distribution $I(\theta)$ on the screen <u>if one of the slits is covered.</u>

   b) With both slits open, how many interference fringes lie within the central diffraction envelope? Sketch the intensity distribution.

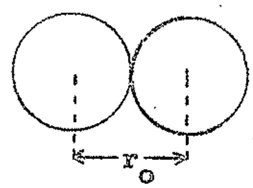
4. In the treatment of an imperfect gas, a generalized form of the equation of state is the so-called virial expansion

$$p = \frac{NkT}{V}\,(1 - b_1\rho + b_2\rho^2 + \ldots .\; b_\ell\rho^\ell + \ldots .).$$

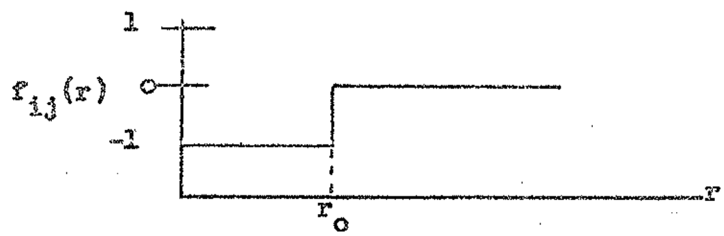
At moderate gas densities, only the leading non-ideal term $-b_1\rho$ is of significance, where ρ is the gas number density N/V. Assume that surface effects are unimportant and rare and that the interaction between any two molecules i and j depends only on their relative separation r. Under these restrictions b_ℓ is given by

$$b_\ell = \frac{1}{(\ell+1)!V} \int_{(V)} f_{ij}(r)\; d\tau_i d\tau_j$$

where $d\tau$ is the element of volume in which the molecule can be found. For a rigid sphere interaction model



the interaction yields the cluster function



Calculate in reasonable approximation the equation of state of a non-ideal gas composed of interacting hard spheres of radius $\frac{1}{2}\, r_o$, and show that it can be written in the same form as that for an ideal gas. Interpret the physical significance of any deviation from the ideal gas equation of state.

5. A charge distribution is present at time t=o in an isolated finite region filled with an isotropic homogeneous medium having conductivity σ, dielectric constant ϵ and permeability μ. No external influences act after t=o.

   a) Starting with the equation of continuity (charge conservation) and Ohm's law (an empirical relation assumed valid for this medium), calculate the time dependence of the charge density $\rho(\vec{r},t)$ at each point $\vec{r}$ within the medium.

   b) Describe the charge distribution (qualitatively) at a time t >> the relaxation time.

6. Calculate the average energy associated with the one-dimensional wave packet

$$f(x,t) = \frac{1}{\sqrt{2}}\psi_1(x,t) + \frac{1}{\sqrt{2}}\psi_2(x,t)$$

where ψ_1 and ψ_2 are the normalized eigenfunctions of the two lowest energy states of the one-dimensional infinite square well of width L.