Practice Quizzes
may differ from actual quizzes given in class

Quiz 1: Power series  Write down the first two non-zero terms in the power
expansion of the following functions.

1. \( \frac{1}{1 - x} \)

2. \( \log(1 + x) \)

3. \( \cos(x) \)

4. \( \sin(x) \)

Quiz 2: Evaluating partial derivatives  Evaluate the following partial deriva-
tives.

1. Taking \( k_B \) to be a constant:

   \[ F = k_B T(n \ln n - n) \]

   Find \( \left( \frac{\partial F}{\partial n} \right)_T \)

2.

   \[ U = x^2 + y^2 - 2xy \]

   Find \( \left( \frac{\partial U}{\partial x} \right)_y \)

3. Taking \( k_B \) to be a constant:

   \[ Z = e^{-\frac{x^2 + y^2}{2kBT}} \]

   Find \( \left( \frac{\partial Z}{\partial x} \right)_{y,T} \)

4. Taking \( N, k_B, b \) and \( a \) to be constant, and given

   \[ G = -Nk_B T \ln(V - Nb) + \frac{N^2 k_B T b}{V - Nb} - \frac{2aN^2}{V} \]

   find \( \left( \frac{\partial G}{\partial V} \right)_T \)
Quiz 3: Total differentials  Evaluate the total differential of each the following functions.

1. Taking $k_B$ to be a constant:
   \[ F = k_B T(n \ln n - n) \]

2. \[ U = x^2 + y^2 - 2xy \]

3. Taking $k_B$ to be a constant:
   \[ Z = e^{-\frac{x^2+y^2}{k_B}} \]

4. Taking $N$, $k_B$, $b$ and $a$ to be constant:
   \[ G = -Nk_B T \ln(V - N) + \frac{N^2 k_B T b}{V - N} - \frac{2aN^2}{V} \]

Quiz 4: Changing variables  Given the definitions below, evaluate the requested partial derivative.

1. Taking $k_B$ to be a constant:
   \[ F = k_B T(n \ln n - n) \]
   \[ n = \frac{N}{V} \]
   Find \( \left( \frac{\partial F}{\partial V} \right)_T \)

2. \[ U = z^2 + x^2 + y^2 - 2xy - 2xz \]
   \[ z = \ln(y - x) + xy \]
   Find \( \left( \frac{\partial U}{\partial z} \right)_y \)

3. \[ z = e^{x^2 + y^2} \]
   \[ x = \sin(t + y) \]
   Find \( \left( \frac{\partial z}{\partial x} \right)_t \)
Quiz 5: Limiting cases For each of the following expressions, find the limiting case when $x \ll 1$.

1. $\tan x$

2. $\frac{\sin(x^2)}{\sin x}$

3. $\frac{\sin x}{x}$

4. $\ln(1 + x^2)$

Quiz 6: Finding entropy Given the following expressions for $dQ$ and $T$ for a quasistatic process, solve for the change in entropy from $t = 0$ to $t = t_f$, where $t$ is time. You may take any other variables used to be constant (i.e. independent of time).

1. $dQ = P\,dt$, $T = T_0 + Kt$

2. $dQ = P\,dt$, $T = T_0$

3. $dQ = -Pe^{\frac{t}{T_0}}\,dt$, $T = Ke^{-\frac{1}{TV}}$

Quiz 7: Heat and work For each of the following processes, solve for the heat or work done.

1. A system expands from volume $V_0$ to volume $V_f$. During this process the pressure is given by

$$p = \frac{Nk_BT}{V}$$

where $k_BT$ and $N$ are constant. How much work does the system do on its environment?

2. A system is heated from initial entropy $S_0$ to final entropy $S_f$. During this process the temperature given by

$$T = T_0 + \frac{S - S_0}{C_V}$$

where $T_0$ and $C_V$ are constants. How much energy is transferred into the system by heating during this process?
3. A system expands from volume $V_0$ to volume $V_f$. During this process the pressure is given by

$$p = p_0 \left(\frac{V_0}{V}\right)^\gamma$$

where $p_0$ is the initial pressure. How much work does the system do on its environment?

**Quiz 8: First Law**

1. You heat an insulated piston with a resistor. You run 5 A through the resistor at 10 V for a total of 10 seconds. The pressure is fixed at 1 Pa (which is one N/m²). If the system expands by 0.1 cubic meter, what is the change in internal energy of the system?

2. Consider an insulated cylinder full of an ideal gas, whose internal energy is given by

$$U = \frac{3}{2} N k_B T$$

What happens to the temperature of the gas when I compress the insulated piston? Why?

**Quiz 9: Integrating work** Given the equations of state below, what is the amount of work done when a system isothermally expands from initial volume $V_0$ to final volume $2V_0$?

1. $$p = \frac{N k_B T}{V}$$

2. $$p = p_0 e^{-\frac{p_0 V}{N k_B T}}$$

**Quiz 10: Heat revisited**
The plot above shows two paths from an initial state described by $S_0$ and $T_0$ to a final state given by $S_f$ and $T_f$. Consider the heat added to the system in the first path $Q_A$ and the heat added to the system in the second path $Q_B$. Which of these is greater, or are they equal? Which are positive and which are negative?

Quiz 11: Summation notation  Evaluate the following sums $S$:

1. $$S = \sum_{i=0}^{3} i$$

2. $$S = \sum_{i=1}^{3} i^2$$

3. $$S = \sum_{i=0}^{2} i!$$

4. $$S = \sum_{i=0}^{2} \sum_{j=0}^{2} i^2 j^2$$

Quiz 12: Efficiency

1. The temperature of the surface of the sun is around 6000 K. Room temperature is around 300 K. What is the upper bound on the efficiency of a photovoltaic cell converting light from the sun into electric power, if the cell operates at room temperature? How does this compare with the efficiency of actual solar cells? (very roughly)

2. If an ideal heat engine operates between a hot heat bath at $T_H$ and a cold heat bath at $T_C$, what is the change in entropy of the hot bath when the engine does $W$ work in one cycle?

3. What is the change in entropy of the cold bath?

4. What is the change in entropy of the engine itself?

Quiz 13: Entropy change  For the following processes, in which the system is designated in *italics*:

- Is the change in entropy of the system positive, negative, zero or impossible to determine?
• Is the change of entropy of the surroundings positive, negative, zero, or impossible to determine?

• Is the change in entropy of system plus surroundings positive, negative, zero, or impossible to determine?

For each question, give a brief explanation. Answers without explanation will not receive credit.

1. A hot potato is left on the counter top.
2. A sealed steel piston of air is slowly compressed.

Quiz 14: Internal energy change For the following processes, in which the system is designated in italics, is the change in the internal energy of the system positive, negative, zero or impossible to determine? Give a brief explanation. Answers without explanation will not receive credit.

1. A hot potato is left on the counter top.
2. A sealed steel piston of air is slowly compressed.
3. A sealed, insulated piston of air is slowly compressed.