Math

**Total differentials**

\[ dA = \left( \frac{\partial A}{\partial B} \right)_C dB + \left( \frac{\partial A}{\partial C} \right)_B dC \]

You can:

1. Do algebra
2. Interpret coefficients as partial derivatives
3. Integrate

**Mixed partial derivatives**

\[ \left( \frac{\partial \left( \frac{\partial A}{\partial B} \right)_C}{\partial C} \right)_B = \left( \frac{\partial \left( \frac{\partial A}{\partial C} \right)_B}{\partial B} \right)_C \]

**Chain rules**

\[ \left( \frac{\partial A}{\partial B} \right)_C = \frac{1}{\left( \frac{\partial B}{\partial A} \right)_C} \]
\[ \left( \frac{\partial A}{\partial B} \right)_D = \left( \frac{\partial A}{\partial C} \right)_D \left( \frac{\partial C}{\partial B} \right)_D \]
\[ \left( \frac{\partial A}{\partial B} \right)_C = \left( \frac{\partial A}{\partial C} \right)_B \left( \frac{\partial B}{\partial C} \right)_A \]

**Thermodynamics**

**Entropy**

\[ \Delta S = \int \frac{dQ_{\text{quasistatic}}}{T} \]
\[ dQ =TdS \]
\[ C_\alpha = T \left( \frac{\partial S}{\partial T} \right)_\alpha \]

**First Law**

\[ \Delta U = Q + W \]
\[ dU =dQ +dW \]
\[ dU = TdS -pdV \]

**Second Law**

\[ \Delta S_{\text{system}} + \Delta S_{\text{surrondings}} \geq 0 \]

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**Legendre transforms**

You can add or subtract from \( U \) products of conjugate variables to find new thermodynamic potentials that are convenient when \( T \) or \( p \) are held fixed or controlled.

**Maxwell relations**

From any thermodynamic potential you can use the equality of mixed partial derivatives to create a relationship between two different partial derivatives.

**Statistical mechanics**

\[ P_i = \frac{e^{-\beta E_i}}{Z} \]
\[ Z = \sum_{i}^{\text{all states}} e^{-\beta E_i} \]
\[ \beta = \frac{1}{k_B T} \]
\[ F = -k_B T \ln Z \]
\[ U = \sum_{i} P_i E_i \]
\[ S = -k_B \sum_{i} P_i \ln P_i \]