This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page — find all choices before making your selection. The due time is Central time.

Chapter 8 problems.

001 (part 1 of 2) 5 points

Note: A bungee cord can stretch, but it is never compressed. When the distance between the two ends of the cord is less than its unstretched length \( L_0 \), the cord folds and its tension is zero. For simplicity, neglect the cord’s own weight and inertia as well as the air drag on the ball and the cord.

A bungee cord has length \( L_0 = 30 \text{ m} \) when unstretched; when it’s stretched to \( L > L_0 \), the cord’s tension obeys Hooke’s law with “spring” constant \( 45 \text{ N/m} \). To test the cord’s reliability, one end is tied to a high bridge of height \( 83 \text{ m} \) above the surface of a river) and the other end is tied to a steel ball of mass \( 74 \text{ kg} \). The ball is dropped off the bridge with zero initial speed.

Fortunately, the cord works and the ball stops in the air a few meters before it hits the water — and then the cord pulls it back up.

The acceleration of gravity is \( 9.8 \text{ m/s}^2 \).

Calculate the ball’s height above the water’s surface at this lowest point of its trajectory.

Correct answer: 1.86096 m.

Explanation:

In the absence of air drag and other resistive forces, there are only two forces acting on the ball — the gravity force \( m \vec{g} \) and the cord’s tension \( T \), both conservative. Therefore, the system (the ball plus the cord) has conserved mechanical energy

\[
E_{\text{mech}} = K + U_{\text{grav}} + U_{\text{cord}} = \text{const},
\]

where

\[
K = \frac{m v^2}{2},
\]

\[
U_{\text{grav}} = m g L,
\]

and

\[
U_{\text{cord}} = \frac{k (L - L_0)^2}{2}.
\]

When the ball is dropped off the bridge, it has zero initial speed (hence \( K_i = 0 \)) while the cord is folded rather than stretched (hence \( L = L_0 \)) and therefore \( U_{\text{cord}} = 0 \) as well. Thus the only mechanical energy present at the beginning is the ball’s gravitational energy,

\[
E_{\text{mech}}^{(0)} = U_{\text{grav}}^{(0)} = m g L .
\]

\[
U_{\text{cord}} = \frac{k}{2} (L - L_0)^2 , \quad \text{for } L > L_0 .
\]

At first, this elastic energy gain is less than the gravitational energy release due to the ball going down, the difference goes to the ball’s kinetic energy, and the ball accelerates. But later, the elastic energy outgrows the gravitational energy release, the kinetic energy has to decrease and the ball slows down. At the bottom of the ball’s trajectory, its speed drops all the way to zero, thus \( K = 0 \) and

\[
U_{\text{cord}} = U_{\text{grav}}
\]

\[
\frac{k}{2} (L - L_0)^2 = m g L
\]

\[
L^2 - 2 L_0 L + L_0^2 = \frac{2 m g}{k} L , \quad \text{so}
\]

\[
L^2 - 2 \left( L_0 + \frac{m g}{k} \right) L + L_0^2 = 0 .
\]

Solving this equation yields

\[
L = \left( L_0 + \frac{m g}{k} \right) \pm \sqrt{\left( L_0 + \frac{m g}{k} \right)^2 - L_0^2}
\]

Since

\[
L_0 + \frac{m g}{k} = 30 \text{ m} + \frac{74 \text{ kg}(9.8 \text{ m/s}^2)}{45 \text{ N/m}}
\]

\[
= 46.1156 \text{ m} ,
\]

\[
\sqrt{\left( L_0 + \frac{m g}{k} \right)^2 - L_0^2} = \sqrt{(46.1156 \text{ m})^2 - (30 \text{ m})^2}
\]

\[
= 35.0235 \text{ m} , \quad \text{and}
\]

\[
L = \left( L_0 + \frac{m g}{k} \right) \pm \sqrt{\left( L_0 + \frac{m g}{k} \right)^2 - L_0^2}
\]

\[
= 46.1156 \text{ m} + 35.0235 \text{ m}
\]

\[
= 81.139 \text{ m} .
\]
The height $h$ above the water is
\[ h = H - L = 83 \text{ m} - 81.139 \text{ m} = 1.86096 \text{ m} . \]

002 (part 2 of 2) 5 points
What is the tension of the bungee cord when the ball is at its lowest point? Correct answer: 2301.26 N.

Explanation:
Once we found the ball's position, the force is easy
\[ T = k (H - h - L_0) = k (L - L_0) = (45 \text{ N/m})(81.139 \text{ m} - 30 \text{ m}) = 2301.26 \text{ N}. \]

By the way, the weight of the ball is
\[ W = m g = (74 \text{ kg}) (9.8 \text{ m/s}^2) = 725.2 \text{ N} . \]

003 (part 1 of 1) 0 points
When an object is moved from rest at point $A$ to rest at point $B$ in a gravitational field, the net work done by the field depends on the mass of the object and

1. the path taken between $A$ and $B$ only.

2. the nature of the external force moving the object from $A$ to $B$.

3. both the positions of $A$ and $B$ and the path taken between them.

4. the positions of $A$ and $B$ only. correct

5. the velocity of the object as it moves between $A$ and $B$.

Explanation:
Gravitational force is a conservative force, so in addition to the mass, only the positions are needed.

004 (part 1 of 2) 5 points
A bead slides without friction around a loop-the-loop. The bead is released from a height 16.4 m from the bottom of the loop-the-loop which has a radius 5 m.

The acceleration of gravity is 9.8 m/s$^2$.

What is its speed at point $A$? Correct answer: 11.2 m/s.

Explanation:
Let : \( m = 7 \text{ g} , \ R = 5 \text{ m} , \) and \( h = 16.4 \text{ m} . \)

From the conservation of energy, we have
\[ K_i + U_i = K_f + U_f \]
\[ 0 + m g h = \frac{m v^2}{2} + m g (2 R) \]
\[ v^2 = 2 g (h - 2 R) . \]

Therefore
\[ v = \sqrt{2 g (h - 2 R)} = \sqrt{2 (9.8 \text{ m/s}^2) [16.4 \text{ m} - 2 (5 \text{ m})]} = 11.2 \text{ m/s} . \]

005 (part 2 of 2) 5 points
How large is the normal force on it at point $A$ if its mass is 7 g?
Correct answer: 0.107016 N.

Explanation:
The centripetal force is equal to the normal force due to the track and gravity, \( F_c = N + m g \), so

\[
N = m \left( \frac{v^2}{R} \right) - m g
\]

\[
= (7 \, \text{g})(0.001 \, \text{kg/g})
\times \left[ \frac{(11.2 \, \text{m/s})^2}{5 \, \text{m}} - 9.8 \, \text{m/s}^2 \right]
\]

\[
= 0.107016 \, \text{N}.
\]

Solving for \( y_1 \):

\[
y_1 = \frac{v^2}{2g}
= \frac{(4.40668 \, \text{m/s})^2}{2(9.8 \, \text{m/s}^2)}
= 0.990755 \, \text{m}
\]

Therefore the initial height is:

\[
h = y_1 + y_2
= 0.990755 \, \text{m} + 1.49 \, \text{m}
= 2.48076 \, \text{m}.
\]
008 (part 2 of 2) 5 points
Choosing down the incline as the positive direction, what is its acceleration at its lowest point?
Correct answer: $-5.89779 \text{ m/s}^2$.
**Explanation:**
Only gravity and the spring force act on the block, so

$$-kx + mg \sin \theta = ma$$

Therefore:

$$a = g \sin \theta - \frac{kx}{m}$$

$$= (9.8 \text{ m/s}^2) \sin 37^\circ$$

$$= \frac{(349 \text{ N/m})(0.259232 \text{ m})}{7.67 \text{ kg}}$$

$$= -5.89779 \text{ m/s}^2.$$