26.51 An electron gun shoots electrons \((q = -e, m = 9.1 \times 10^{-31} \text{ kg})\) at a metal plate that is 4 mm away in vacuum. The plate is 5.0 V lower in potential than the gun. How fast must the electrons be moving as they leave the gun if they are to reach the plate?

\(v \geq \left[ \frac{16.0 \times 10^{-19} \text{ J}}{9.1 \times 10^{-31}} \right]^{1/2} \text{ m/s} = 1330 \text{ km/s}.

26.52 Suppose that two protons are released when \(2.0 \times 10^{-14} \text{ m}\) apart. Find their speeds when they are \(5.0 \times 10^{-14} \text{ m}\) apart. (Mass of proton = \(1.67 \times 10^{-27} \text{ kg}\).)

From the law of conservation of energy,

\[
\left(U_e\right)_{\text{start}} - \left(U_e\right)_{\text{end}} = K_{\text{end}} - K_{\text{start}}
\]

Because \(K\) at the start is zero, and because the protons have identical kinetic energies,

\[
\frac{e^2}{4\pi \varepsilon_0 \left(\frac{1}{2.0 \times 10^{-14} \text{ m}} - \frac{1}{5.0 \times 10^{-14} \text{ m}}\right)} = 2(\frac{1}{2}mv^2) - 0
\]

Substituting known values for \(e\) and \(m\) yields \(v = 2034 \text{ km/s}\).

26.53 The electron beam in a television tube consists of electrons accelerated from rest through a potential difference of about 20 kV. How large an energy do the electrons have? What is their speed? Ignore relativistic effects for this approximate calculation. \((m_e = 1.9 \times 10^{-31} \text{ kg})\)

\(v = \sqrt{2E_e/\frac{m_e}{2}}\)

26.54 A proton \((q = e, m = 1.67 \times 10^{-27} \text{ kg})\) is accelerated from rest through a potential difference of 1 MV. What is its final speed?

\(v = \sqrt{2E_p/\frac{m_p}{2}}\)

26.55 Two metal plates are attached to the two terminals of a 1.50-V battery. How much work is required to carry a +5-\(\mu\)C charge \((a)\) from the negative to the positive plate, \((b)\) from the positive to the negative plate?

26.56 The plates described in Prob. 26.55 are in vacuum. An electron \((q = -e, m = 9.1 \times 10^{-31} \text{ kg})\) is released at the negative plate and falls freely to the positive plate. How fast is it going just before it strikes the plate?

\(v = \sqrt{2E_e/\frac{m_e}{2}}\)

26.57 A lead pellet (mass. 2 g) fired from an air rifle has a speed of 150 ft/s. Through what difference in potential would this pellet have to fall to acquire the same speed, assuming it carries a charge of 1 \(\mu\)C?

\(v = \sqrt{\frac{1}{2}mv^2}\)

26.58 A proton \((q = e)\) is released from a point \(P\) which is 10 \(^{-14}\) m from a heavy nucleus which has a charge of 80e. 
ELECTRIC POTENTIAL AND CAPACITANCE

The nucleus of a mercury atom. How large will the kinetic energy of the proton be when it gets far away from the nucleus? What will be its speed?

The proton will be repelled and caused to move radially toward infinity by the positive nucleus. In effect, it falls through the potential difference between point \( P \) and infinity. This potential difference is just the absolute potential at point \( P \),

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{80 \times 1.6 \times 10^{-10}}{10^{-14}} \right) = 12 \times 10^6 \text{ V}
\]

In falling through this potential difference, the proton will thus acquire a kinetic energy of \( K = 12 \text{ MeV} \). In writing this, we assume the nucleus to remain nearly at rest (cf. Prob. 26.45). We then have

\[
\frac{1}{2}mv^2 = (12 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})
\]

Using the proton mass \( 1.67 \times 10^{-27} \text{ kg} \) and solving for the proton's speed, we find that \( v = 4.8 \times 10^7 \text{ m/s} \).

26.59 The nucleus of the radium atom has a charge of \( +88e \) and a radius of about 0.007 pm. With what speed must a proton be shot at the atom if it is to reach a radius of 0.01 pm? The inner radius of the electron cloud, \( r_a \), is about 50 pm.

The potential due to the electrons alone is approximately constant and equal to \((-88e)/r_a\), within the inner radius. The potential due to the nucleus is \((+88e)/r \) with \( r << r_a \).

Therefore the approximate potential difference between infinity and \( r = 0.01 \text{ pm} \) is simply the absolute potential at this radius from a point charge with \( q = 88e \):

\[
V = \frac{88}{1.6 \times 10^{-19}} \left( \frac{9 \times 10^9}{10^{-14}} \right) = 12.7 \text{ MV}
\]

For the proton,

kinetic energy at infinity = potential energy at \( r \) \[ \frac{1}{2}mv^2 = qV \]

Using \( m = 1.67 \times 10^{-27} \text{ kg} \), \( q = 1.60 \times 10^{-19} \text{ C} \), and \( V = 12.7 \times 10^6 \text{ V} \) in this equation gives \( v = 4.9 \times 10^7 \text{ m/s} \).

26.60 The potential difference between the two plates in Fig. 26-19 is 100 V. If the system is in vacuum, what will be the speed of a proton released from plate \( B \) just before it hits plate \( A \)?

\[
\begin{array}{c}
\text{B} \\
+ \\
+ \\
+ \\
100 \text{ V} \\
+ \\
\end{array}
\quad
\begin{array}{c}
\text{A} \\
+ \\
\bigcirc \rightarrow \text{Eq} \\
- \\
- \\
\end{array}
\]

Fig. 26-19

The mass and charge of a proton are \( 1.67 \times 10^{-27} \text{ kg} \) and \( 1.60 \times 10^{-19} \text{ C} \), respectively. When the proton is moved from plate \( B \) to plate \( A \), it loses a potential energy \( q(V_B - V_A) \), where \( V_B - V_A = 100 \text{ V} \) in this case. This appears as kinetic energy of the proton at plate \( A \). The law of conservation of energy therefore tells us that

loss in potential energy = gain in kinetic energy \[ q(V_B - V_A) = \frac{1}{2}mv^2 \]

Placing in the values and solving for \( v \), we find \( 140 \text{ km/s} \).

26.61 The potential difference applied to the filament of a bulb of a 5-cell flashlight is 7.5 V. How much work is done by the flashlight cells (i.e., how much chemical energy is lost) in transferring 60 C of charge through the filament?

\[ W = Vq = 7.5(60) = 450 \text{ J} \]

26.62 A proton is accelerated from rest in a Van de Graaff accelerator by a potential difference of 0.9 MV. What is the kinetic energy of the proton after acceleration?

The kinetic energy of the proton is equal to the work done on it by the potential difference through which it moved. \( W = Vq = (9 \times 10^6)(1.60 \times 10^{-19}) = 1.44 \times 10^{-13} \text{ J} \).
26.63 In the Bohr model of the hydrogen atom, the electron was pictured to rotate in a circle of radius $0.053 \text{ nm}$ about the nucleus. (a) How fast should the electron be moving in this orbit? (b) How much energy is needed to tear the electron loose from the nucleus of the atom?

In order for the electron to travel in the circular orbit, a centripetal force must be furnished to it. This is provided by the coulomb attraction between it and the nucleus. We therefore have (after assuming that the nucleus remains stationary—a good assumption, since it is 1840 times more massive than the electron)

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{ee}{r^2}$$

In this expression $m$ is the mass of the electron $(9.1 \times 10^{-31} \text{ kg})$, and $e$ is the magnitude of the charge on the electron as well as on the nucleus, $1.6 \times 10^{-19} \text{ C}$. After placing in the values and solving for $v$, we find that $v = 2.2 \times 10^8 \text{ m/s}$. The energy needed to tear the electron loose (the ionization energy $\Phi$) is given by energy conservation as

$$\Phi = \Delta PE + \Delta KE = [0 - (-e)V] + \left[ 0 - \frac{1}{2}mv^2 \right] = e \frac{e}{4\pi\varepsilon_0} - \frac{1}{2} \left( \frac{1}{4\pi\varepsilon_0} \right) e^2 \frac{e}{8\pi\varepsilon_0} = 2.15 \times 10^{-18} \text{ J}$$

26.4 CAPACITANCE AND FIELD ENERGY

26.64 What is a capacitor and how does one measure capacitance?

A capacitor, or condenser, consists of two conductors with equal and opposite charges separated by an insulator or dielectric. The capacitance of a capacitor is defined by

$$\text{capacitance } C = \frac{\text{magnitude of charge } q \text{ on either conductor}}{\text{magnitude of potential difference } V \text{ between conductors}}$$

For $q$ in coulombs and $V$ in volts, $C$ will be in farads (F). Convenient submultiples of the farad are:

$$1 \mu F = 1 \text{ microfarad} = 10^{-6} \text{ F} \quad 1 \text{ pF} = 1 \text{ picofarad} = 10^{-12} \text{ F}$$

26.65 Find an expression for the capacitance of a parallel-plate capacitor made up of two parallel conducting plates, each of area $A$, and separated by a distance $d$. Assume that $d$ is much smaller than the dimensions of the plates.

Assume a charge $q$ on one plate and $-q$ on the other. The electric field between the plates is constant and perpendicular to the plates. Then, using Prob. 25.71, $V = Ed = (\varepsilon_0 d A) = [d/(\varepsilon_0 A)]q$, and so $C = q/V = (\varepsilon_0 A)/d$. In this derivation we have ignored deviations in $E$ at the edges of the plates.

26.66 Find the energy stored in a capacitor with charge $q$.

We start with neutral plates and bring charge across in increments $dq'$. The energy $U$ is then the total work done in bringing $q$ units of charge across:

$$U = \int_0^V V(q')dq' = \frac{1}{C} \int_0^V q' dq' = \frac{q^2}{2C}$$

26.67 A plane-parallel capacitor has circular plates of radius $r = 10.0 \text{ cm}$, separated by a distance $d = 1.00 \text{ mm}$. How much charge is stored on each plate when their electric potential difference has the value $V = 100 \text{ V}$? Discuss the accuracy of the calculation.

From Prob. 26.65

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \times (3.14 \times 10^{-2} \text{ m})^2}{1.00 \times 10^{-3} \text{ m}} = 2.8 \times 10^{-10} \text{ F} = 280 \text{ pF}$$

It is not appropriate to quote the capacitance to more than two significant figures because (1) ignores the effects of the edges of the capacitor plates. Figure 26-20 shows that these effects occur in a region whose radial extent $\Delta r$ is comparable to the separation $d$ of the plates. Hence the ratio $\Delta r/r = d/r = 10^{-3} \text{ m} / 10^{-1} \text{ m} = 1$ percent gives a measure of the accuracy to be expected from the equation.

The magnitude $|q|$ of the charge stored on either plate of the capacitor is $|q| = CV = (2.8 \times 10^{-10} \text{ F})(1.00 \times 10^2 \text{ V}) = 28 \text{ nC}$.  

A spherical capacitor is to be constructed by using a metal sphere of radius \( b \) as one plate and a concentric spherical metal shell as the other plate. The inner radius of the shell is \( a > b \); show that the capacitance of the device is

\[
C = \frac{4\pi \varepsilon_0 ab}{a - b}
\]

We use the spherical symmetry and associated properties of \( E \). Assume a charge \( Q \) on the inner plate. Then \( E = \frac{Q}{(4\pi \varepsilon_0 r^2)} \) and \( \Delta V = \frac{Q}{(4\pi \varepsilon_0)} \left[ \frac{1}{b} - \frac{1}{a} \right] \). By definition \( C \) is \( \frac{Q}{\Delta V} = \frac{(4\pi \varepsilon_0) / (1/b - 1/a)}{(a - b)} \).

Refer to Prob. 26.68. Show that if the separation of the spheres is very small in comparison with their radii, the capacitance is given by the parallel-plate relation \( C = (\varepsilon_0 A)/d \).

Write \( 4\pi a^2 = 4\pi ab \) for the area of either sphere. Then \( C = (\varepsilon_0 A)/(a - b) \). But \( a - b = d \), and so the desired result is obtained.

How much charge is stored in a capacitor consisting of two concentric spheres of radii 30 and 31 cm if the potential difference is 500 V? Assume \( K = 1 \) for air.

For concentric spheres the capacitance is

\[
C = \frac{\kappa ab}{(9 \times 10^9)(a - b)} = \frac{(0.30)(0.31)}{(9 \times 10^9)(0.31 - 0.30)} = 1.033 \times 10^{-9} \text{ F} = 1.033 \text{ nF}
\]

\[
Q = CV = (1.033 \times 10^{-9})(500) = 517 \text{ nC}
\]

A metal sphere mounted on an insulating rod carries a charge of 6 nC when its potential is 200 V higher than its surroundings. What is the capacitance of the capacitor formed by the sphere and its surroundings?

\[
C = \frac{Q}{V} = \frac{6 \times 10^{-9}}{200 \text{ V}} = 30 \text{ pF}
\]

A capacitor is charged with 9.6 nC and has a 120-V potential difference between its terminals. Compute its capacitance and the energy stored in it.

\[
C = \frac{Q}{V} = \frac{(9.6 \times 10^{-9} \text{ C})/(120 \text{ V})}{8 \times 10^{-11} \text{ F} = 80 \text{ pF}}
\]

For the energy we can use any of three equivalent forms. \( E = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \left( \frac{Q}{C} \right)^2 \). Using the middle form, \( E = \frac{1}{2}(9.6 \times 10^{-9} \text{ C})(120 \text{ V}) = 5.76 \times 10^{-7} \text{ J} = 576 \text{ nJ} \).
26.73 A charge of 600 μC is placed on a 20-μF capacitor. Find the potential difference between the terminals of the capacitor.

\[ Q = CV \quad 600 \times 10^{-6} = 20 \times 10^{-6} V \quad V = 30 V. \]

26.74 What is the charge on a 300 pF capacitor when it is charged to a voltage of 1 kV?

\[ q = CV = (300 \times 10^{-12} F)(1000 V) = 3 \times 10^{-7} C = 0.3 \mu C. \]

26.75 A charge of 50 μC is placed on a 2-μF capacitor. What is the stored energy?

\[ W = \frac{Q^2}{2C} = \frac{(50 \times 10^{-6})^2}{2(2 \times 10^{-6})} = 625 \mu J. \]

26.76 Compute the energy stored in a 60-pF capacitor (a) when charged to a potential difference of 2 kV, (b) when the charge on each plate is 30 nC.

(a) \[ E = \frac{1}{2}CV^2 = \frac{1}{2}(60 \times 10^{-12} F)(2000 V)^2 = 1.2 \times 10^{-6} J \]

(b) \[ E = \frac{1}{2}Q^2 = \frac{1}{2}(30 \times 10^{-6} C)^2 = 7.5 \times 10^{-12} J. \]

26.77 Find the energy stored in a 5-μF capacitor charged to a potential difference of 500 V.

\[ W = \frac{1}{2}CV^2 = \frac{1}{2}(5 \times 10^{-6} )(500^2) = 0.625 J. \]

26.78 If a 4-μF capacitor has a potential difference of 1000 V, what is its stored energy?

\[ W = \frac{1}{2}CV^2 = \frac{1}{2}(4 \times 10^{-6} )(1000)^2 = 2 J. \]

26.79 A 1.2 μF capacitor is charged to 3 kV. Compute the energy stored in the capacitor.

\[ \frac{1}{2}CV^2 = \frac{1}{2}(1.2 \times 10^{-6} F)(3000 V)^2 = 5.4 J. \]

26.80 A capacitor of arbitrary shape with air (K = 1) between its plates has capacitance C. What is the capacitance when wax of dielectric constant K is between the plates?

Let \( Q \) be the charge placed on one conductor and \( -Q \) on the other. This will not change by placing dielectric between the conductors. What will change is the electric field in the region between the conductors. Because of induced charges on the interface between dielectric and conductor, the effective charge giving rise to the electric field is reduced by the dielectric constant factor, \( K \). Since the field is reduced everywhere by the same factor, the potential difference will be reduced by the same factor. Thus if \( V \) is the potential difference without dielectric and \( V' \) with dielectric, \( V' = V/K \). Then \( C' = Q/V' = KQ/V = KC. \)

26.81 A capacitor with air between its plates has a capacitance of 8 μF. Determine its capacitance when a dielectric with dielectric constant 6.0 is between its plates.

\[ C \quad \text{with dielectric} = K (C \quad \text{with air}) = (6.0)(8 \mu F) = 48 \mu F \]

26.82 A certain parallel-plate capacitor consists of two plates, each with area 200 cm², separated by a 0.4-cm air gap. (a) Compute its capacitance. (b) If the capacitor is connected across a 500-V source, what are the charge on it, the energy stored in it, and the value of \( E \) between the plates? (c) If a liquid with \( K = 2.60 \) is poured between the plates so as to fill the air gap, how much additional charge will flow onto the capacitor from the 500-V source?

(a) For a parallel-plate capacitor, \( C = (\varepsilon_0 A)/d = [(8.85 \times 10^{-12})(200)]/(0.004) = 44 \mathrm{pF}. \]

\[ q = CV = (4.4 \times 10^{-11} F)(500 V) = 22 \mu C \quad \text{Energy} = \frac{1}{2}qV = \frac{1}{2}(2.2 \times 10^{-6} C)(500 V) = 5.5 \mu J. \]

\[ E = \frac{V}{d} = \frac{500 V}{4 \times 10^{-3} m} = 125 \text{ kV/m}. \]

(c) The capacitor will now have a capacitance 2.60 times larger than before. Therefore, \( q' = (2.60)(22) = 57 \text{ nC}, \) and so \( q' - q = 35 \text{ nC} \) must flow onto it.
26.83 Two parallel conducting plates of area 100 cm² and 5 mm apart are given equal and opposite charges of 0.20 μC. The region between the plates is filled with a dielectric of \( K = 5 \). Compute (a) the capacitance of the system and (b) the voltage difference between the plates.

(a) \[
C = \frac{KA}{k4\pi d} = \frac{5(0.01)}{(9 \times 10^9)(4\pi)(0.005)} = 8.8 \times 10^{-12} \text{ F} = 88 \text{ pF}.
\]

(b) \[
Q = CV = 0.2 \times 10^{-6} = 88 \times 10^{-12} \text{ V} \quad V = 2270 \text{ V}.
\]

26.84 A 5-μF capacitor with air between the metal plates is connected to a 30-V battery. The battery is then removed, leaving the capacitor charged. (a) Calculate the charge on the capacitor. (b) The air between the plates is replaced by oil with \( K = 2.1 \). Find the new value of the capacitance and the new potential difference between the plates.

(a) \[
Q = CV = 5 \times 10^{-9}(30) = 150 \mu\text{C}.
\]

(b) The charge on the plates remains the same when the oil replaces the air. The capacitance increases by a factor \( K \).

\[
C' = KC = 10.5 \mu\text{F} \quad V' = \frac{V}{K} = 14.3 \text{ V}
\]

26.85 Consider a parallel-plate capacitor with plate area \( A \) and charge \( Q \). (a) Find the force on one plate because of the charge on the other. (b) Compute the work done in separating the plates from essentially zero separation to a separation \( d \). (c) Compare this work with the energy stored in the capacitor as given in Prob. 26.66.

Assume that \( Q \) remains unchanged.

(a) Since each plate contributes equally to the field between the plates the field due to one plate is \( \sigma/2\epsilon_0 \) and this exerts a force \( (\sigma/2\epsilon_0)\sigma A \) on the other, so \( F = (\sigma^2 A)/(2\epsilon_0) \).

(b) The work done is given by \( Fd = (\sigma^2 A)/(2\epsilon_0) d \).

(c) Note that \( Q^2 = (\sigma A)^2 \) and the capacitance is given by \( (\epsilon_0 A)/d \); thus \( Fd = Q^2/(2C) \), the usual expression for the stored energy.

26.86 Show that the electrical energy of a parallel-plate capacitor may be thought of as residing in the electric field, with an energy density of \( \rho_e = (\epsilon_0 E^2)/2 \) at any field point.

The situation is depicted in Fig. 26-21. By Prob. 26.66,

\[
U = \frac{1}{2} qV \quad V = Ed \quad E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} \quad \text{Thus} \quad U = \frac{1}{2}(\epsilon_0 AE)(Ed) = \frac{\epsilon_0 E^2 \tau}{2}
\]

where \( \tau = Ad \) is the volume between the plates. Then \( \rho_e = U/\tau = (\epsilon_0 E^2)/2 \).

While this derivation is for the simple case of the constant field between two parallel plates, the result is true for the electrostatic energy of an arbitrary charge distribution.

26.87 Calculate the electric field, the electric-field energy density, and the energy stored in the plane-parallel capacitor of Prob. 26.67.

\[
E = \frac{V}{d} = \frac{100 \text{ V}}{1.00 \times 10^{-3} \text{ m}} = 100 \text{ kV/m} \quad \rho_e = \frac{\epsilon_0 E^2}{2} = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(10^4 \text{ N/C})^2 = 0.0441 \text{ J/m}^3
\]

\[
U = \rho_e \pi r^2 d = (4.4 \times 10^{-2} \text{ J/m}^3)\pi(0.1 \text{ m})^2(0.001 \text{ m}) = 1.4 \mu\text{J}
\]
26.88 Obtain expressions for the electric-field energy density $\rho_\varepsilon$ and energy content $U$ for the spherical capacitor shown in Fig. 26-22, when the inner and outer spheres hold charges $+|q|$ and $-|q|$, respectively.

1. By Gauss' law,

$$E = \frac{|q|}{4\pi\varepsilon_0 r^2} \quad (r_1 \leq r \leq r_2)$$

Giving

$$\rho_\varepsilon = \frac{\varepsilon_0 E^2}{2} = -\frac{q^2}{32\pi^2\varepsilon_0 r^4}$$

Using the volume element indicated in Fig. 26-22,

$$U = \iiint \rho_\varepsilon \, dv = \int_{r_1}^{r_2} \rho_\varepsilon (4\pi r^2 \, dr) = \frac{q^2}{8\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{q^2}{8\pi\varepsilon_0} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

As a check, we write $U = q^2/(2C)$, or

$$C = \frac{1}{2} \frac{q^2}{U} = \frac{1}{2} \frac{8\pi\varepsilon_0}{\left( \frac{1}{r_1} - \frac{1}{r_2} \right)} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}$$

in agreement with Prob. 26.68. Also, as $r_2 \to \infty$, $U \to q^2/(8\pi\varepsilon_0 r_1)$ (in agreement with Prob. 26.48) and $C \to 4\pi\varepsilon_0 r_1$, the capacitance of an isolated sphere of radius $r_1$.

26.89 The unit of electrostatic energy density is the $J/m^3$, which is the same as the $Pa$, the unit of stress or pressure. Is this merely accidental?

1. No: in fact, by Prob. 26.85, the energy density in a parallel-plate capacitor is

$$\rho_\varepsilon = \frac{Fd}{Ad} = \frac{F}{A}$$

pressure on either plate.

26.5 CAPACITORS IN COMBINATION

26.90 Show that capacitances add linearly when connected in parallel and reciprocally when connected in series.

1. Figure 26-23(a) shows three capacitors connected in parallel. We wish to find the capacitance of a single

![Fig. 26-23](image)
capacitor that will behave equivalently to the combination: \(C_{eq}\). If a voltage \(V\) is placed across the terminals we have \(q_1 = C_1 V\); \(q_2 = C_2 V\); \(q_3 = C_3 V\). The total charge stored is thus \(q = q_1 + q_2 + q_3\). Hence \(q - C_{eq}V = C_1 V + C_2 V + C_3 V\), or dividing out \(V\), \(C_{eq} = C_1 + C_2 + C_3\).

For the capacitors in series, as depicted in Fig. 26-23(b) we must have \(q_1 = q_2 = q_3\). The voltages are \(V_1 = q_1/C_1\), \(V_2 = q_2/C_2\), and \(V_3 = q_3/C_3\). The voltage across the equivalent capacitor is \(V = V_1 + V_2 + V_3 = q/C_{eq}\), with \(q = q_1 = q_2 = q_3\). Thus \(q/C_{eq} = q/C_1 + q/C_2 + q/C_3\), and dividing out by \(q\), \(1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3\).

26.91 The parallel capacitor combination shown in Fig. 26-24 is connected across a 120-V source. Determine the equivalent capacitance \(C_{eq}\) and the charge on each capacitor.

![Fig. 26-24](image)

For a parallel combination,

\[
C_{eq} = C_1 + C_2 = 6 \text{ pF} + 2 \text{ pF} = 8 \text{ pF}
\]

Each capacitor has a 120-V potential difference impressed on it. Therefore,

\[
q_1 = C_1 V_1 = (2 \text{ pF})(120 \text{ V}) = 240 \text{ pC} \quad q_2 = C_2 V_2 = (6 \text{ pF})(120 \text{ V}) = 720 \text{ pC}
\]

The charge on the combination is \(q_1 + q_2 = 960 \text{ pC}\). Or we could write

\[
q = C_{eq} V = (8 \text{ pF})(120 \text{ V}) = 960 \text{ pC}.
\]

26.92 Determine the capacitance of a parallel combination of one 12-\(\mu\)F and two 6-\(\mu\)F capacitors.

\[
C = C_1 + C_2 + C_3 = 12 + 6 + 6 = 24 \mu\text{F}
\]

26.93 The series combination of two capacitors shown in Fig. 26-25 is connected across 1000 V. Compute

![Fig. 26-25](image)

(a) the equivalent capacitance \(C_{eq}\) of the combination, (b) the magnitudes of the charges on the capacitors, (c) the potential differences across the capacitors, (d) the energy stored in the capacitors.

\[
(a) \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3 \mu\text{F}} + \frac{1}{6 \mu\text{F}} = \frac{1}{2 \mu\text{F}}
\]

from which \(C = 2 \mu\text{F}\).

(b) In a series combination, each capacitor carries the same charge, which is the charge on the combination. Thus, using the result of (a),

\[
q_1 = q_2 = q = C_{eq} V = (2 \times 10^{-12} \text{ F})(1000 \text{ V}) = 2 \text{nC}
\]
(c) \[ V_1 = \frac{q_1}{C_1} = \frac{2 \times 10^{-9} \text{C}}{3 \times 10^{-12} \text{F}} = 667 \text{V} \]
\[ V_2 = \frac{q_2}{C_2} = \frac{2 \times 10^{-9} \text{C}}{6 \times 10^{-12} \text{F}} = 333 \text{V} \]

(d) energy in \( C_1 \) is \( \frac{1}{2}q_1V_1 = \frac{1}{2}(2 \times 10^{-9} \text{C})(667 \text{V}) = 6.7 \times 10^{-6} \text{J} \)

energy in \( C_2 \) is \( \frac{1}{2}q_2V_2 = \frac{1}{2}(2 \times 10^{-9} \text{C})(333 \text{V}) = 3.3 \times 10^{-6} \text{J} \)

energy in combination is \((6.7 + 3.3) \times 10^{-6} \text{J} = 10 \times 10^{-6} \text{J} \)

The last result is also directly given by \( \frac{1}{2}qV \) or \( \frac{1}{2}C_\text{eq}V^2 \).

26.94 If a 6-\( \mu \)F capacitor and a 12-\( \mu \)F capacitor are connected in series, what is the capacitance of the combination?

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12} \]
Thus \[ C = 4 \mu \text{F} \]

26.95 Find the equivalent capacitance of a 1-\( \mu \), a 2-\( \mu \), and a 6-\( \mu \)F capacitor connected in series.

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{3}{6} + \frac{1}{6} = \frac{10}{6} \]
and \[ C = 0.6 \mu \text{F} \]

26.96 If you need a capacitor with \( C = 0.25 \mu \text{F} \), but the only ones in the storeroom have \( C = 1.00 \mu \text{F} \), must you delay finishing your experiment?

I No. You can connect four of the available capacitors in series.

26.97 Three capacitors (2, 3, and 4 \( \mu \)F) are connected in series with a 6-V battery. When the current stops, what is the charge on the 3-\( \mu \)F capacitor? What is the potential difference between the two ends of the 4-\( \mu \)F capacitor?

I The equivalent capacitance of the three in series is \( \frac{1}{3} \) = 0.92 \( \mu \)F. Capacitors in series each carry the same charge, which is the same as the charge on the equivalent capacitor: \( Q_\text{eq} = C_\text{eq}V = 0.92(6) = 5.5 \mu \text{C} \). The \( V \) across 4 \( \mu \)F is \( V = Q_\text{eq}/C = 5.5/4 = 1.38 \text{V} \).

26.98 Three capacitors are connected as shown in Fig. 26-26. If a 12-V potential difference is applied to the terminals, what will the total capacitance be?

![Diagram of capacitors connected in series](image)

I For the two capacitors in series,

\[ \frac{1}{C_A} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} \]

and thus the capacitance of the upper branch is \( C_A = 2 \mu \text{F} \). For the two parallel branches
\[ C = C_A + C_B = 2 + 4 = 6 \mu \text{F} \]
(capacitance of the system)

26.99 Refer to Prob. 26.98 and find the charge on each capacitor.

I Use the general formula \( Q = CV \) successively for \( C \) and for \( C_B \).

\[ Q = CV = 6 \times 10^{-6}(12) = 72 \mu \text{C} \] (charge on the system)
\[ Q_B = C_BV = (4 \times 10^{-6})(12) = 48 \mu \text{C} \] (charge on the 4-\( \mu \)F capacitor)
The total charge on the system is the sum of the charges on the upper branch and on the lower branch; thus, \( Q_u = Q - Q_n = 24 \ \mu\text{C} \). Both the 3- and the 6-\( \mu \)F capacitors carry this same charge since they are in series.

26.100 Refer to Probs. 26.98 and 26.99 and find the voltage on each capacitor.

I The 4-\( \mu \)F capacitor has a potential difference of \( \frac{12}{5} \), the applied voltage. Use \( Q = CV \) for the 3-\( \mu \)F capacitor.

\[
Q_u = (3 \times 10^{-6}) V_5 \quad \text{or} \quad 2.4 \times 10^{-5} = (3 \times 10^{-6}) V_5 \quad \text{so} \quad V_5 = 8 \ \text{V}
\]

Use \( Q = CV \) for the 6-\( \mu \)F capacitor.

\[
Q_u = (6 \times 10^{-6}) V_6 \quad \text{or} \quad 2.4 \times 10^{-5} = (6 \times 10^{-6}) V_6 \quad \text{and} \quad V_6 = 4 \ \text{V}
\]

Their sum must of course equal the terminal voltage, and indeed \( 8 \ \text{V} + 4 \ \text{V} = 12 \ \text{V} \).

26.101 In the circuit of Fig. 26-27, find the total capacitance.

First find the capacitance \( C_r \) of the parallel section.

\[
C_r = C_2 + C_3 = 6 + 4 = 10 \ \mu\text{F}
\]

\( C_r \) is in series with \( C_1 \), so

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_r} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10} \quad \text{and} \quad C = 3.33 \ \mu\text{F}
\]

26.102 Refer to Prob. 26.101 and find the potential difference across each capacitor.

I Find the total charge \( Q \) on the system first.

\[
Q = CV = 3.33 \times 10^{-3}(1000) = 3.33 \times 10^{-3} \ \text{C}
\]

Then

\[
V_1 = \frac{Q}{C_1} = \frac{3.33 \times 10^{-3}}{5 \times 10^{-5}} = 0.667 \times 10^3 = 667 \ \text{V} \quad \text{and} \quad V_2 = V_3 = V - V_1 = 1000 - 667 = 333 \ \text{V}.
\]

As a check we can use the equivalent of the two parallel capacitors, \( C_r \).

\[
V_2 = V_3 = \frac{Q}{C_r} = \frac{3.33 \times 10^{-3}}{10 \times 10^{-5}} = 333 \ \text{V}.
\]

26.103 Find the equivalent capacitance of the combination shown in Fig. 26-28.

As a check we can use the equivalent of the two parallel capacitors, \( C_r \).

\[
V_2 = V_3 = \frac{Q}{C_r} = \frac{3.33 \times 10^{-3}}{10 \times 10^{-5}} = 333 \ \text{V}.
\]
26.104 Two capacitors in parallel, 2 and 4 μF, are connected, as a unit, in series with a 3-μF capacitor. The combination is connected across a 12-V battery. Find the equivalent capacitance of the combination and the potential difference across the 2-μF capacitor.

Reduce the system as shown in Fig. 26-29, to find \( C_{eq} = 2.0 \) μF. The charge on the equivalent capacitor \( Q = CV = 2(12) = 24 \) μC. This is also the charge on the equivalent 6-μF capacitor. So V across it (and the 2 μF) is \( Q/C = 24/6 = 4.0 \) V.

26.105 Find the equivalent capacitance of the combination shown in Fig. 26-30. Also find the charge on the 4-μF capacitor.

Reduce the circuit as shown in Fig. 26-31, to find \( C_{eq} = 5.4 \) μF. The charge on the 4 μF is the same as on the 2.4 μF. The value of \( V \) across the 2.4 is 10 V, so \( Q = CV = 2.4(10) = 24 \) μC.

26.106 Two capacitors, 3 μF and 4 μF, are individually charged across a 6-V battery. After being disconnected from the battery, they are connected together with the negative plate of one attached to the positive plate of the other. What is the final charge on each capacitor?

\[ q_1 = 18 \, \mu C \]

\[ q_2 = 24 \, \mu C \]

(a) Before

(b) After

The situation is shown in Fig. 26-32. Before being connected, their charges are

\[ q_3 = CV = (3 \times 10^{-6} \, \text{F})(6 \, \text{V}) = 18 \, \mu C \quad q_4 = CV = (4 \times 10^{-6} \, \text{F})(6 \, \text{V}) = 24 \, \mu C \]

As seen in the figure, the charges will partly cancel when the capacitors are connected together. Their final charges are given by \( q_3 + q_4 = q_3 - q_1 = 6 \, \mu C \). Also, the potential across each is now the same, so that
\[ V = \frac{q}{C} \text{ gives} \]

\[ \frac{q_3}{3 \times 10^{-6} \text{ F}} = \frac{q_5}{4 \times 10^{-6} \text{ F}} \quad \text{or} \quad q_5 = 0.75q_3 \]

Substitution of this in the previous equation gives

\[ 0.75q_3 + q_4 = 6 \mu\text{C} \quad \text{or} \quad q_4 = 3.43 \mu\text{C} \]

Then \( q_5 = 0.75q_3 = 2.57 \mu\text{C} \).

26.107 Two capacitors, \( C_1 = 3 \mu\text{F} \) and \( C_2 = 6 \mu\text{F} \), are connected in series and charged by connecting a battery of voltage \( V = 10 \text{ V} \) in series with them. They are then disconnected from the battery, and the loose wires are connected together. What is the final charge on each?

\[ \text{The capacitors are charged in series so they originally have equal charges. When the loose wires are reconnected, they neutralize each other giving zero final charge.} \]

\[ \text{Before} \quad \text{After} \quad \text{Fig. 26-33} \]

\[ \text{C}_1 \quad \text{C}_2 \quad \text{C}_1 \quad \text{C}_2 \]

\[ V \]

\[ \text{Before} \quad \text{After} \quad \text{Fig. 26-34} \]

\[ \text{C}_1 \quad \text{C}_2 \]

26.108 Repeat Prob. 26.107 if after being disconnected from the battery, the capacitors are disconnected from each other. They are now reconnected as shown in Fig. 26-33. What is the final charge on each?

\[ \text{The original charge on each is } Q = C_{eq}V = 20 \mu\text{C}. \text{ After being connected as shown, } Q_1 + Q_2 = 2Q = 40 \mu\text{C}. \text{ Also } V_1 = V_2 \text{ and so } Q_1/C_1 = Q_2/C_2. \text{ Solving these two equations simultaneously gives for the two charges } 26.7 \mu\text{C} \text{ on the } 6 \mu\text{F} \text{ and } 13.3 \mu\text{C} \text{ on the } 3 \mu\text{F}. \]

26.109 If two capacitors \( C_1 = 4 \mu\text{F} \) and \( C_2 = 6 \mu\text{F} \) are originally connected to a battery \( V = 12 \text{ V} \), as shown in Fig. 26-34, and then disconnected and reconnected as shown, what is the final charge on each capacitor?

\[ \text{Originally, } Q_1 = 48 \mu\text{C}, Q_2 = 72 \mu\text{C}; \text{ hence, } Q_1 + Q_2 = 72 - 48 = 24 \mu\text{C}. \text{ Also, } V_1 = V_2 \text{ gives } Q_1/C_1 = Q_2/C_2. \text{ Solving simultaneously gives } 9.6 \text{ and } 14.4 \mu\text{C}. \]