Definitions for Normed Vector Spaces

A set of objects (vectors) \{\vec{u}, \vec{v}, \vec{w}, \ldots\} is said to form a \textit{linear vector space} over the field of scalars \{\lambda, \mu, \ldots\} (e.g. real numbers or complex numbers) if:

1. the set is closed, commutative, and associative under (vector) addition;
2. the set is closed, associative, and distributive under multiplication by a scalar;
3. there exists a \textit{null vector} \vec{0};
4. multiplication by the scalar identity 1 leaves the vector unchanged;
5. all vectors have a corresponding \textit{negative vector};

An \textit{inner product} \langle \vec{u}|\vec{v}\rangle is a generalization of the dot product with the following properties:

\begin{align*}
\langle \vec{u}|\vec{v}\rangle &= \langle \vec{v}|\vec{u}\rangle^* \\
\langle \vec{u}|\lambda \vec{v} + \mu \vec{w}\rangle &= \lambda \langle \vec{u}|\vec{v}\rangle + \mu \langle \vec{u}|\vec{w}\rangle
\end{align*}