Definitions for Analytic Functions

A function $w(z)$ is **analytic** (or **regular** or **holomorphic** or **monogenic**) in a region of the complex plane if has a derivative at every point in the region. (Analytic at a point, or on a line, means in an open region around that point or line.)

A **regular point** of $w(z)$ is a point at which $w(z)$ is analytic.

A **singular point** of $w(z)$ is a point at which $w(z)$ is not analytic.

An **isolated singular point** of $w(z)$ is a point at which $w(z)$ is not analytic but for which $w(z)$ is analytic in a neighborhood of that point.

In the limit that $z \to z_0$, if a function $w(z) \to \infty$ and $(z - z_0)^n w(z) \to \infty$ for $n \leq m$, but $(z - z_0)^m w(z)$ is finite, then $w(z)$ is said to have a **pole of order** $m$ at $z_0$.

A **branch point** is a singular point in a multiple-valued function such that the function is discontinuous when going around an arbitrarily small circle around this point in the domain.