REQUIRED:

1. Consider a spin-1/2 particle with a magnetic moment. At time $t = 0$, the state of the particle is $|\psi(t = 0)\rangle = |+\rangle$.
   (a) If the observable $S_x$ is measured at time $t = 0$, what are the possible results and the probabilities of those results?
   (b) Instead of performing the above measurement, the system is allowed to evolve in a uniform magnetic field $\vec{B} = B_0 \hat{y}$. Calculate the state of the system after a time $t$ using the $S_z$ basis.
   (c) At time $t$, the observable $S_x$ is measured. What is the probability that a value $\hbar/2$ will be found?
   (d) Draw a schematic diagram of this experiment, similar to Fig. 3.2.

2. Consider a two-state quantum system with a Hamiltonian
   \[ H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \]  
   Another physical observable $A$ is described by the operator
   \[ A = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \]  
   where $a$ is real and positive. Let the initial state of the system be $|\psi(0)\rangle = |a_1\rangle$, where $|a_1\rangle$ is the eigenstate corresponding to the larger of the two possible eigenvalues of $A$. What is the frequency of oscillation of the expectation value of $A$? Compare this frequency to the Bohr frequency.

3. A quantum mechanical system starts out in the state:
   \[ |\psi(0)\rangle = C (3|a_1\rangle + 4|a_2\rangle) \]  
   where $|a_i\rangle$ are the normalized eigenstates of the operator $A$ corresponding to the eigenvalues $a_i$. In this $|a_i\rangle$ basis, the Hamiltonian of this system is represented by the matrix:
   \[ H = E_0 \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \]  
   (a) If you measure the energy of this system, what values are possible, and what are the probabilities of measuring those values?
(b) Find the state from the previous part as a function of time.

(c) Calculate the expectation value \( \langle A \rangle \) of the observable \( A \) as a function of time.
   (This part of the problem is a Challenge. It is NOT required. If you can do this, you can do anything!)