Symmetries Homework 6  
Due 10/16/15 @ 4:30 pm

QUIZ:

1. Give the expression for $d\vec{r}$ in rectangular, cylindrical, and spherical coordinates.

PRACTICE:

2. Imagine you’re standing on a landscape with a local topology described by the function $f(x, y) = kx^2y$, where $k = 20 \text{m}^3/\text{km}^3$ is a constant, $x$ and $y$ are east and north coordinates, respectively, with units of kilometers. You’re standing at the spot $(3 \text{ km}, 2 \text{ km})$ and there is a cottage located at $(1 \text{ km}, 2 \text{ km})$. At the spot you’re standing, what is the slope of the ground in the direction of the cottage? Plot the function $f(x, y)$ in Mathematica. Does your result makes sense from the graph?

REQUIRED:

3. Consider the fields at a point $\vec{r}$ due to a point charge located at $\vec{r}'$.
   
   (a) Write down an expression for the electrostatic potential $V(\vec{r})$ at a point $\vec{r}$ due to a point charge located at $\vec{r}'$. (There is nothing to calculate here.)
   
   (b) Write down an expression for the electric field $\vec{E}(\vec{r})$ at a point $\vec{r}$ due to a point charge located at $\vec{r}'$. (There is nothing to calculate here.)
   
   (c) Working in rectangular coordinates, compute the gradient of $V$.
   
   (d) Write several sentences comparing your answers to the last two questions.

4. (a) Find the electric field around a finite, uniformly charged, straight wire, at a point a distance $r$ straight out from the midpoint, starting from Coulomb’s Law. 
   
   (b) Find the electric field around an infinite, uniformly charged, straight wire, starting from Coulomb’s Law.

5. Find the electric field around an infinite, uniformly charged, straight wire, starting from the following expression for the electrostatic potential:

$$V(\vec{r}) = \frac{2\lambda}{4\pi\epsilon_0} \ln \frac{s_0}{s}$$

6. Three charges are situated at the corners of a square (side $s$). Two have charge $-q$ and are located on opposite corners. The third has charge $+q$ and is opposite an empty corner.

   (a) How much work does it take to bring in another charge, $+q$, from far away and place it at the fourth corner?
   
   (b) How much work does it take to assemble the whole configuration of four charges?