Physics 201
Final Exam
12/6/2012

Collaboration is not allowed. Allowed on your desk are: up to ten 8.5 x 11 inch doubled sided sheets of notes that are bound together, non-communicating/graphing scientific calculator, 1 page of scratch paper, writing utensils, and the exam. You will have 110 minutes to complete this exam.

1. (6 points) A crane of height \( h \) rests on the ground and is not fixed to the ground in any way. The supports of the crane have a width \( d \). The crane can lift objects and move them up, down, left, or right. (a) What is the range for the x-component of the center of mass of the crane+object system if the crane is to not tip over? Explain. (b) Increasing the height of the crane increases the y-component of the center of mass of the crane+object system. Does increasing the height of the crane increase, decrease, or have no change to the x-component of the center of mass of the crane+object system? (c) Does the height of the crane effect the stability of the system if there is no wind and the crane remains perfectly rigid? Explain.

a.) For stability the center of mass must be within the furthest Normal force so,
\[ -\frac{d}{2} \leq X_{com} \leq \frac{d}{2} \]

b.) **No change** on \( X_{com} \) but a change on \( Y_{com} \)

c.) If the crane remains rigid and there is no wind (or earthquakes) it can theoretically be as tall as you like, as long as the condition in part (a) is met.

No effect on stability
For questions 2 through 11 circle all correct answers (a given problem may have more than one correct answer). Each correctly circled answer will receive 2 points and each incorrectly circled answer will lose 1 point - no partial credit is available for these problems. The minimum score you will receive on any particular problem is zero. There are a total of 13 correct answers in this section.

2. A stationary bomb explodes in gravity-free space breaking into a number of small fragments. Which of the following statements concerning this event are true?
   F (a) Kinetic energy is conserved in this process.
   F (b) The fragments must have equal kinetic energies.
   F (c) The sum of the kinetic energies after the explosion is zero.
   T (d) The vector sum of the linear momentum of the fragments must be zero.
   F (e) The velocity of any one fragment must be equal to the velocity of any other fragment.

3. An object falls near Earth at terminal velocity through a distance d. Which of the following statements concerning this situation are true?
   F (a) The change in the kinetic energy is equal to negative the change in potential energy.
   F (b) The magnitude of the change in the momentum is greater than zero.
   T (c) The work done by non-conservative forces is equal to the change in the gravitational potential energy.
   T (d) The change in potential energy during the first half of the fall (d/2) is equal to the change in potential energy during the second half of the fall.
   F (e) The gravitational force is greater than the force from air resistance.

4. An object not near the surface of the Earth is falling freely towards the Earth. Neglect air resistance and gravitational forces from other bodies in space. Which of the following statements are true regarding this situation?
   F (a) As the object falls the net work done by all of the forces acting on the block is zero.
   F (b) The kinetic energy of the object increases by equal amounts over equal distances.
   T (c) The kinetic energy of the object increases by increased amounts over equal distances.
   F (d) The momentum of the object increases by equal amounts in equal times.
   T (e) The momentum of the object increases by increased amounts over equal times.
   F (f) The center of mass of the Earth+object system is accelerating.

5. Two cars are accelerating along a drag strip. It is observed that the distance between the two cars is increasing. Which one of following statements concerning this situation are necessarily true?
   ? (a) The velocity of each car is increasing.
   ? (b) At least one of the cars has a non-zero acceleration.
   ? (c) The leading car has the greater acceleration.
   ? (d) The trailing car has the smaller acceleration.
   T (e) Both cars could be accelerating at the same rate.
6. Which of the following situations are not possible?
- (a) A body has zero velocity and non-zero acceleration.
- (b) A body travels with a northward velocity and a northward acceleration.
- (c) A body travels with a northward velocity and a southward acceleration.
- (d) A body travels with a constant velocity and a time-varying acceleration.
- (e) A body travels with a constant acceleration and a time-varying velocity.

7. A man stands on a scale in a moving elevator and notices that the scale reading is 20% larger than when he weighs himself in his bathroom. Which of the following statements can not be true?
- (a) The tension in the elevator's supporting cable must exceed the weight of the elevator and its contents.
- (b) The elevator could be moving upward with increasing speed.
- (c) The elevator could be moving downward with decreasing speed.
- (d) The elevator could be moving upward at constant speed.

8. A ball is whirled on the end of a string in a horizontal circle of radius $R$ at constant speed $v$.
   The radial acceleration of the ball can be increased by a factor of 4 by
   - (a) keeping the speed fixed and increasing the radius by a factor of 4.
   - (b) keeping the radius fixed and increasing the speed by a factor of 4.
   - (c) keeping the radius fixed and increasing the period by a factor of 4.
   - (d) keeping the radius fixed and decreasing the period by a factor of 4.
   - (e) keeping the speed fixed and decreasing the radius by a factor of 4.

9. A particle travels along a curved path between two points P and Q as shown. The displacement of the particle does not depend on
- (a) the location of P.
- (b) the location of Q.
- (c) the distance traveled from P to Q.
- (d) the shortest distance between P and Q.
- (e) the direction of Q from P.

10. A gerbil in your hand is being raised up at a constant velocity. Which of the following statements are false regarding this situation?
- (a) The normal force on the gerbil must be larger than the gravitational force.
- (b) The impulse acting on the gerbil is zero.
- (c) The gerbil travels through constant distances over equal time intervals.
- (d) The normal force does positive work on the gerbil.
- (e) The change in kinetic energy increases from one second to the next.

11. If gold was sold by its apparent weight, which of the following statements would make you the most money?
- (a) Buy your gold at the north pole and sell it at the equator.
- (b) Buy your gold at the equator and sell it at the north pole.
- (c) Buy your gold in London and lose it while you travel to Berlin.
12. (3 points) The momentum-versus-time graph is shown for a 500 g object. Draw the corresponding acceleration-versus-time graph. Include an appropriate vertical scale.

\[ V = \frac{p}{m} \quad \text{and} \quad a = \frac{\Delta v}{\Delta t} \sim \text{slope} \quad \Rightarrow \quad a = \frac{1}{m} \frac{\Delta p}{\Delta t} \]

13. (4 points) A puck on a horizontal, frictionless surface is attached to a string that passes through a hole in the surface, as shown in the figure. As the puck rotates about the hole, the string is pulled downward, bringing the puck closer to the hole. During this process what happens to the puck's (a) angular speed, (b) linear speed and (c) angular momentum? Explain.

\[ \Sigma T_{ext} = 0 \quad \Rightarrow \quad \Delta L = 0 \quad \Rightarrow \quad L = I \omega \]

(c) **no change**

(a) \[ I = mr^2 \quad \text{as } \omega \uparrow, \quad I \uparrow + [ \square ] \quad \Rightarrow \quad L = \text{constant} \]

(b) \[ v = \omega r \quad \Rightarrow \quad r \uparrow \Rightarrow \frac{r}{2} \quad \Rightarrow \quad I \rightarrow \frac{I}{4} \quad \Rightarrow \quad \omega \rightarrow 4\omega \]

so \( \omega \) increases at a greater rate than \( r \) decreases.

So \[ V \text{ Increases} \]
14. (2 points) A 10-kg crate is placed in the bed of a truck at rest on horizontal ground. The truck accelerates uniformly and the crate does not slip during this acceleration.
(a) What force does work on the crate during this acceleration?
\[
\mathbf{F} = F_\text{ts} + F_\text{g}
\]
(b) What is the sign of the work done on the crate.
positive

15. (8 points) It is common to see birds of prey rising upward on wind thermals. The paths they take may be spiral-like. You can model the spiral motion as uniform circular motion combined with a constant upward velocity. It is observed that a bird is undergoing this type of spiral motion and completes 4 revolutions in a 24 s time period. The radius of the circle the bird is moving around is 8 m and the bird rises 120 m in this 24 s time period. During this motion what is (a) the magnitude of the acceleration and (b) the speed of the bird?

\[\begin{align*}
\mathbf{v}_t &\leftarrow \mathbf{v}_y \\
(\overrightarrow{\Delta y}) &\leftarrow h \\
\end{align*}\]

(a) \[\begin{align*}
\mathbf{a}_y &= 0, \quad \mathbf{a}_t = 0, \quad \mathbf{a}_r = \frac{\mathbf{v}_c^2}{r} = \omega^2 r = 8.77 \text{ m/s}^2 \\
\omega &= \frac{\Delta \theta}{\Delta t} = \frac{4(2\pi)\text{rad}}{24 \text{s}} \\
\vert \overrightarrow{a} \vert &= 8.77 \text{ m/s}^2
\end{align*}\]

(b) \[\begin{align*}
\Delta y &= v_y \Delta t + \frac{1}{2} a_y \Delta t^2 \\
V_y &= \frac{\Delta y}{\Delta t} = 5 \text{ m/s} \\
V_r &= 0, \quad \mathbf{v}_e = \omega r = 8.37 \text{ m/s}
\end{align*}\]
16. (10 points) To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 600 gram falcon flying at 20.0 m/s ran into a 1.5 kg raven flying at 9.0 m/s. The falcon hit the raven at right angles to its original path and bounced directly back with a speed of 5.0 m/s. (a) By what angle did the falcon change the raven's direction of motion? (b) This is an example of an inelastic collision, a collision in which kinetic energy in the system is changed. In this case kinetic energy is lost to work from deformation during the collision and ultimately ends up as heat. What is the change in the kinetic energy of the system during the collision?

\[ \Sigma P_i = \Sigma P_f \]

\[ M_1 V_{1x} + M_2 V_{2x} = M_1 V_{1x} + M_2 V_{2x} \]

\[ V_{f2x} = \frac{M_1 V_{f1x} - M_1 V_{f1x}}{M_2} = 10 \text{ m/s} \]

\[ M_1 V_{f1y} + M_2 V_{f2y} = M_1 V_{f1y} + M_2 V_{f2y} \]

\[ V_{2y} = V_{2y} = 9.0 \text{ m/s} \]

\[ \tan \theta_{f2} = \frac{V_{f2x}}{V_{f2y}} \]

\[ \theta_{f2} = 48.0^\circ \]

\[ \Sigma \Delta K = \Sigma K_f - \Sigma K_i \]

\[ = \frac{1}{2} \left[ M_1 (V_{f1x}^2 - V_{f1x}^2) + M_2 (V_{f2x}^2 - V_{f2x}^2) \right] \]

\[ = -37.5 \text{ J} \]
17. (12 points) A uniform beam 4.0 m long and weighing 2500 N carries a 3500 N weight 1.50 m from the far end, as shown in the figure. It is supported by a frictionless hinge at the wall and a metal wire at the far end. (a) How strong does the wire have to be? That is, what is the maximum tension it must be able to support without breaking? (b) What are the horizontal and vertical components of the force that the hinge exerts on the beam? (c) If the wire was to break, what would be the initial angular acceleration of the beam? The moment of inertia for a beam of length l and mass m, about an axis through the end of the beam, is 1/3·m·l^2.

(a) \[ \Sigma \tau = \begin{vmatrix} \bar{r}_1 & \bar{r}_2 \end{vmatrix} \begin{bmatrix} \sin \Theta_f \cdot \tau_f \end{bmatrix} - \begin{bmatrix} m_1 g \sin \Theta_f \cdot \bar{r}_1 + m_2 g \bar{r}_2 \end{bmatrix} = \mathbf{I} \begin{bmatrix} \gamma \end{bmatrix} \]

\[ F_f = \frac{m_1 g \bar{r}_1 + m_2 g \bar{r}_2}{\sin \Theta_f} = 6875 \text{N} \]

(b) \[ \Sigma F_x = F_{u} - F_f \cos 30 = m_1 g \gamma \sin 30 \]

\[ F_u = 5954 \text{N} \]

\[ \Sigma F_y = F_{u} \sin 30 - m_1 g - F\gamma = m_2 g \gamma \]

\[ F_u = 2563 \text{N} \]

(c) \[ \Sigma \tau = -m_1 g \bar{r}_1 - m_2 g \bar{r}_2 = \frac{1}{3} m_1 l^2 \alpha \]

\[ \alpha = \frac{-3g(m_1 r_1 + m_2 r_2)}{m_1 l^2} = 10.1 \text{ rad/s}^2 \]
18. (10 points) A grasshopper leaps into the air from the edge of a vertical cliff, as shown in the figure. Use information from the figure to find (a) the initial speed of the grasshopper and (b) the height of the cliff. Neglect air resistance.

a) \[
\begin{align*}
\text{Rise} & \\
K & \\
\Delta x &= 0 \\
\frac{\Delta x}{\Delta t} &= \frac{v_k}{2} \\
v_{ix} &= \sqrt{v_k^2 - v_{ix}^2} \\
v_{ix} &= \sqrt{1.164^2 - 0.944^2} \\
v_{ix} &= 0.944 \\
\Delta x &= 1.06 \text{ m} \\
\end{align*}
\]

\[
\begin{align*}
\theta_0 &= 30^\circ \\
\Delta y &= 6.74 \text{ cm} \\
v_{iy} &= v_i \sin 30^\circ \\
v_{iy} &= 1.149 \text{ m/s} \\
v_{iy} &= 1.149 \text{ m/s} \\
v_{iy} &= \frac{v_{iy}}{\sin 30^\circ} = 1.499 \text{ m/s} \\
\end{align*}
\]

b) \[
\begin{align*}
\text{Entire Flight} & \\
\Delta y &= \frac{1}{2} g \Delta t^2 + v_y \Delta t \\
\Delta y &= 4.66 \text{ m} \\
\end{align*}
\]
19. (10 points) An object of mass $m$, slides a distance $d$, down a frictionless incline that makes an angle $\theta$ with respect to the horizontal. Then the object slides across a horizontal surface that is also frictionless except for a patch of length $l$ that has a coefficient of kinetic friction between the surface and the object of $\mu_k$. After the horizontal surface the object is redirected up a frictionless vertical section a distance $h$ to a relaxed massless spring of constant $k$. The object then continues higher, compressing the spring by an amount $\Delta X_{sf}$. Find an expression for the compression of the spring in terms of the variables $m$, $d$, $\theta$, $l$, $\mu_k$, $h$, $k$, and the gravitational constant $g$.

\[ \sum E_i + W_{nc} = \sum E_f \]

\[ \frac{1}{2}mv_i^2 + mgd\sin\theta + \frac{1}{2}kX_{sf}^2 + \frac{1}{2}k\Delta X_{sf}^2 \]

\[ mgds\sin\theta - \mu_kmg \cdot l = mg(h + \Delta X_{sf}) + \frac{1}{2}k\Delta X_{sf}^2 \]

Quadratic eq. for $\Delta X_{sf}$

Rearrange

\[ \frac{1}{2}k \Delta X_{sf}^2 + mg \Delta X_{sf} + (mgh + \mu_kmg \cdot l - mgds\sin\theta) = 0 \]

Form of $Ax^2 + Bx + C = 0$

\[ \Delta X_{sf} = \frac{-mg \pm \sqrt{(mg)^2 - 4 \left( \frac{k}{2} \right) (mgh + \mu_kmg \cdot l - mgds\sin\theta)}}{2 \left( \frac{k}{2} \right)} \]