Using Energy to Analyze Motion

practice worksheet

**Materials needed:** No extra paper is needed—write on the pages provided.

**Directions:** Each exercise will present a situation—something to be calculated—but unless you’re given actual numbers, you do not need to do the calculation itself. But in every case, do a work-energy analysis, as if preparing for the calculation, as follows:

1. **Choose (and indicate) the initial and final positions of the object** for the motion you wish to analyze.

2. **Choose (and indicate) where the gravitational potential energy \((U_g)\) is zero.** You can state this in words or show on the diagram where you have assigned \(h = 0\). You will need to label other vertical distances on the diagram relative to this zero point.

3. **Identify the various measurable forms of energy present in the mass at its initial and final positions.** Is the mass moving at its initial or final position—and if so, what kind of motion is it? (Translational? Rotational? Both?) Is its initial position at a different vertical (gravitational) position than its final position? Is its initial position at a different position with respect to a spring than its final position? Write the fundamental Work-Energy Equation, then expand/simplify it into its measurable parts, as follows:

\[
E_f = E_i + W + Q
\]

\[
K_{T_f} + K_{R_f} + U_{G_f} + U_{S_f} + U_{E_f} + U_{B_f} + E_{ch_f} + E_{th_f} = K_{T_i} + K_{R_i} + U_{G_i} + U_{S_i} + U_{E_i} + U_{B_i} + E_{ch_i} + E_{th_i} + W + Q
\]

\[
K_{T_f} + K_{R_f} + U_{G_f} + U_{S_f} = K_{T_i} + K_{R_i} + U_{G_i} + U_{S_i} + W
\]

4. **Identify how various forces do work on the mass during its journey from initial to final position.** When working out an actual numerical problem, you may need a FBD and a full application of Newton’s Laws to solve for the values of individual forces. For any of the set-up-only exercises here, however, just identify which forces are doing work on the mass—and write the expression for that work. For example, the work done by a friction force would be written as \(W = F_s s \cos \theta\). You don’t need to expand \(F_s\) any further, but you do need to indicate \(\theta\) (and if it’s 0° or 90° or 180°, simplify \(\cos \theta\) into 1 or 0 or −1). And you do need to label the displacement, \(s\), clearly on the diagram.

5. **Now expand the Work-Energy Equation, substituting the expressions you know for \(K_T, K_R, U_G, U_S, \) and \(W\) for the situation.** (Formulas for \(I\) for various shapes appear on page 216 of the text.) The next page has an example. ---->
EXAMPLE: At point A, a mass $m$ is already sliding to the right, at a speed $v_i$, on a level, frictionless surface. As it slides for a distance $s_a$, forces $F_1$ and $F_2$ are applied to the mass, at angles $\phi_1$ and $\phi_2$ as shown. Then, for a distance $s_b$, the mass slides over a level portion of surface with a friction coefficient $\mu_k$. Then it slides up a frictionless slope, rising a vertical distance $y$, then horizontally once again, as shown, where it encounters some unknown friction over a distance $s_c$. After all that, the mass is still moving to the right with a speed $v$. Do a work-energy analysis on this mass as if you wish to calculate its speed at point B.

The fundamental Work-Energy equation:

$$E_{mech_f} = E_{mech_i} + W_{ext}$$

$$K_{T_f} + K_{R_f} + U_{G_f} + U_{Z_f} = K_{T_1} + K_{R_1} + U_{G_1} + U_{S_1} + W_{ext}$$

Analysis of the external work being done:

$$W_{F1} = F_1s_a\cos\theta,$$ where $\theta = \phi_1$:  

$$W_{F2} = F_2s_a\cos\theta,$$ where $\theta = 180 - \phi_2$:  

$$W_{Fkb} = F_{kb}s_b\cos\theta,$$ where $\theta = 180$: 

To correctly see the angle $\theta$, always place the force and displacement vectors tail to tail.

Detailed expansion of the Work-Energy equation for this situation:

There is no rotational kinetic energy or spring potential energy in this situation; and $h_1 = 0$. Therefore:

$$K_{T_f} + U_{G_f} = K_{T_1} + W_{ext}$$

$$\frac{1}{2}mv_i^2 + mg(y) = \frac{1}{2}mv_i^2 + W_{F1} + W_{F2} + W_{Fkb}$$

$$\frac{1}{2}mv_f^2 + mgy = \frac{1}{2}mv_i^2 + F_1s_a\cos\phi_1 + F_2s_a\cos(180 - \phi_2) - F_{kb}s_b$$
EXERCISE 1. Starting from rest, a block of mass \( m \) is pushed across a level surface (\( \mu_k \)) by a constant force \( F \), as shown here. Suppose you wish to calculate its velocity after traveling a distance \( d \). Do the work-energy analysis necessary to let you do that.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 2. How much work will you do by pushing horizontally a 160 kg box 10.3 m across a rough floor without acceleration, if the effective coefficient of friction is 0.50?

Analysis of the external work being done:
EXERCISE 3: You begin a hike at an elevation of 1500m and climb to a 2660-m peak. If you have a mass of 66.5-kg.

(a) What is the change in your gravitational potential energy?

(b) What is the minimum work you need to do to achieve this?

(c) Will the actual work done be greater than this? Explain.
EXERCISE 4. Tarzan is running at top speed (5.0 m/s) and grasps a vine hanging 4.0 m vertically from a tall tree in the jungle.

(a) How high can he swing upward?
(b) Does the length of the vine affect the answer?
(c) Does Tarzan’s mass affect the answer?

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 5: A mass $m$ is falling freely (without rotation) straight down, initially at a speed $v_i$. Then, after falling an additional distance $d$, its speed is tripled. Set up the work-energy equation that you would need to solve for $d$.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 6: At a vertical distance $d$ above the ground, a freely-falling skydiver (mass $m$) opens her parachute. At that moment she has a speed (vertically downward) of $v_i$. The constant tension force exerted on the skydiver by the parachute thereafter is $F_T$. Do a work-energy analysis as if you were preparing to calculate her impact speed.

**The fundamental Work-Energy equation:**

**Analysis of the external work being done:**

**Detailed expansion of the Work-Energy equation for this situation:**
EXERCISE 7: Starting from rest, a block of mass $m$ is pushed up an incline ($\theta$) by a constant horizontal force, $F$. The block travels a distance $d$ along the incline surface ($\mu_k$) and rises a vertical distance $h$. Do the work-energy analysis necessary to calculate the speed of the block at the top of the incline.

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 8: A block is sliding along a level, frictionless surface. Its speed at point A is $v_A$. Then it encounters an incline whose total vertical rise is $y_B$. The block travels off the upper end of the incline and becomes a projectile, striking the level surface as shown. Prepare an energy analysis that would let you answer the following question: What is the block's speed at point C (the peak of its trajectory) if it is $y_C$ meters above the level surface at that point?

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 9: A block is sliding along a level, frictionless surface. Then it encounters an incline whose total vertical rise is $y_B$. The block travels off the upper end of the incline and becomes a projectile, striking the level surface as shown. Its speed at point $B$ is $v_B$. Prepare an energy analysis that would let you answer the following question: What is the block’s impact speed?

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 10: A block slides, starting at a speed $v$, down a frictionless incline of height $H$ and through the "loop-the-loop" of radius $R$. After the block leaves the loop, it crosses a level surface with friction ($\mu_k$), coming to rest after a distance $d$. Do the work-energy analysis necessary to calculate its speed at the top of the loop. Disregard any rotational kinetic energy.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 11: A block slides, starting at a speed $v_i$, down a frictionless incline of height $H$ and through the "loop-the-loop" of radius $R$. After the block leaves the loop, it crosses a level surface with friction ($\mu_k$), coming to rest after a distance $d$. Do the work-energy analysis necessary to calculate $d$. Disregard any rotational kinetic energy.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 12: A block starts from rest at the top of a frictionless incline with height $y$. Do the work-energy analysis necessary to calculate the speed of the block at the bottom of the incline.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 13: A solid sphere starts from rest at the top of an incline with height $y$. The sphere rolls without slipping. Do the work-energy analysis necessary to calculate the translational speed of the sphere at the bottom of the incline.

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 14: A solid sphere is rolling without slipping along a level surface with a translational speed $v_t$ and rotational speed $\omega_r$. After climbing a vertical distance $y$, the sphere is again rolling without slipping along a level surface with a translational speed $v_f$ and rotational speed $\omega_f$. Write the detailed work-energy equation that describes this motion.

**The fundamental Work-Energy equation:**

**Analysis of the external work being done:**

**Detailed expansion of the Work-Energy equation for this situation:**
EXERCISE 15: A thin rod is rotating in a vertical circle about a pivot attached to one end. When the free end is at the top of the circle, it has a tangential speed $v_i$ and angular speed $\omega_i$. Do a work-energy analysis as if you wish to calculate the tangential speed of that free end when the rod is in the 3 o’clock position shown here (at right).

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation:
EXERCISE 16: An archer pulls the bowstring back for a distance of 0.470 m before releasing the arrow horizontally. The bow and string act like a spring whose spring constant is 425 N/m.

(a) What is the elastic potential energy of the drawn bow?

(b) The arrow has a mass of 0.0300 kg. How fast is it traveling when it leaves the bow?
EXERCISE 17: A vertical spring with a spring constant of 450 N/m is mounted on the floor. From directly above the spring (which is initially relaxed—unstrained), a 0.300-kg block is dropped from rest. It collides with and sticks to the spring, which is compressed by 2.50 cm in bringing the block to a (momentary) halt. Disregarding air resistance, from what height above the compressed spring’s position was the block dropped?

The fundamental Work-Energy equation:

Analysis of the external work being done:

Detailed expansion of the Work-Energy equation for this situation: