Graphical Understanding of Work

\[ W = \int \vec{F} \cdot d\vec{s} = \int_{x_1}^{x_2} F_x \, dx + \int_{y_1}^{y_2} F_y \, dy + F_z \, dz \]

- Area under \( F_x(y) \) curve
- Area under \( F_y(y) \) curve

\[ W_{ac} = m \cdot g \cdot \Delta y \]

\[ W_{bc} = m \cdot g \cdot \Delta y \]

\[ W_{ab} = m \cdot g \cdot \Delta y \]

Work by Gravity

\[ \Delta y = \tan \theta \cdot \Delta x \]

\[ W_{ac} = m \cdot g \cdot \Delta x \cdot \cos \theta \]

\[ W_{bc} = m \cdot g \cdot \Delta y \]

\[ W_{ab} = m \cdot g \cdot \Delta y \]

\[ W_{bc} = 0 \]

So, \( W_{ac} = W_{ab} + W_{bc} \)

Work from gravity is independent of path taken.

It only depends on change in height (\( \Delta y \)).

So, \( W^2 \) is a function of \( \Delta y \) but path taken.

Let's define function of \( y \) and call it Potential Energy (PE).

\[ W^2 = -\Delta U^2 = m \cdot g \cdot \Delta y \]

Forces that create path independent work are called conservative forces, we use PE function for them.
Example: A 2-kg mass is compressed 1 m against a spring whose spring constant (k) is 20 N/m. Once released the mass slides down a frictionless ramp then back up to another spring with the same spring constant. After compressing the second spring it comes to rest and is held in place. (a) Find the kinetic energy, spring and gravitational potential energy and total energy throughout the mass’s journey. (b) Plot these as a function of position. (c) What is the final compression of the second spring?
Example: A 2 kg block is pushed upward from underneath by a constant 50-N vertical force. The block is sliding on the bottom side of an inclined plane that makes an angle of 70º with the vertical. The coefficient of kinetic friction between the block and the plane is 0.2. If after traveling a total distance of 4 m the block is traveling at a speed of 4.443 m/s up the incline, what was the initial velocity of the block? (Answer: 1 m/s, up the incline)

Example: A 1.80-kg block slides on a rough, horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11 cm before coming to rest. The coefficient of kinetic friction between the block and the surface is 0.56, what is the force constant of the spring? (Answer: 415 N/m)

Example: A 60-kg skier starts from at the top of a 6-m-high frictionless hill. After sliding to the bottom of the hill the skier slides over a horizontal patch of rough snow that has a coefficient of kinetic friction equal to 0.2. How far will they slide before coming to rest? (Answer: 30 m)

Example: An object of mass m, starting from rest slides a distance d, down a frictionless incline that makes an angle θ with respect to the horizontal. Then the object slides across a horizontal surface that is also frictionless except for a patch of length l that has a coefficient of kinetic friction between the
distance $d$, down a frictionless incline that makes an angle $\theta$ with respect to the horizontal. Then the object slides across a horizontal surface that is also frictionless except for a patch of length $l$ that has a coefficient of kinetic friction between the surface and the object of $\mu_k$. After the horizontal surface the object is redirected up a frictionless vertical section a distance $h$ to a relaxed massless spring of constant $k$. The object then continues higher, compressing the spring by an amount $\Delta X_{s,f}$. Find an expression for the compression of the spring in terms of the variables $m$, $d$, $\theta$, $l$, $\mu_k$, $h$, $k$, and the gravitational constant $g$. 