Impulse-Momentum Theorem

Definition: Momentum
\[ \vec{P} = m \vec{v} \]

Scaling of velocity with mass
\[ \vec{P} = m \langle v_x, v_y, v_z \rangle \]

What the principle actually says

(i) \[ \sum F_{\text{ext}} = \frac{\Delta \vec{P}}{\Delta t} \]

\[ = \frac{\Delta (m\vec{v})}{\Delta t} \text{ if } m = \text{const. then } \Delta (m\vec{v}) = m\Delta \vec{v} \]

\[ \sum F_{\text{ext}} = m \frac{\Delta \vec{v}}{\Delta t} = m \frac{\Delta \vec{0}}{\Delta t} \]

Insert (i)

\[ \boxed{\Delta \vec{P} = \sum F_{\text{ext}} \Delta t = J} \]

Impulse-momentum theorem

Area under \( F(t) \) curve

* Change in \( \vec{P} \) is from net force over time

Connect to plots?

\[ \sum F_{\text{ext}} \\
\]

area = \( \Delta p_x \)

\[ \begin{array}{c}
\text{ex.} \quad (\Delta D) \\
\text{speeding up} \\
\end{array} \]

\( \vec{V}_i \)

\( \Theta \rightarrow \)

\( \vec{V}_f \)

\( \Theta \rightarrow \)

\( \Delta \vec{P}? \)

\( \vec{P}_f \)

\( \vec{P}_i \)

\( \Delta \vec{P} \)

\* \( \Delta \vec{P}, \sum \vec{F}, \vec{a}, \Delta \vec{v} \) all point in same direction

\[ \text{math} \quad \Delta \vec{P} = \sum \vec{F}_{\text{ext}} \Delta t \]
Example: A loaded tractor-trailer with a total mass of 5000 kg traveling at 3.0 km/hr hits a loading dock and comes to a stop in 0.6 s. What is the average force exerted on the truck by the dock? (Answer: 3 km/hr = 0.833 m/s, \( F = 6944 \) N)

Example: In a slow-pitch softball game, a 0.200-kg softball crosses the plate at 15.0 m/s at an angle of 45.0° below the horizontal. The batter hits the ball toward center field, giving it a velocity of 40.0 m/s at 30.0° above the horizontal. (a) Determine the impulse delivered to the ball. (b) If the force on the ball increases linearly for 4.00 ms, holds constant for 20.0 ms, and then decreases linearly to zero in another 4.00 ms, what is the maximum force on the ball?
(Answers: (a) 10.9 kg·m/s, (b) 455 N)

\[ J^* = \Delta \vec{p} = \vec{p}_f - \vec{p}_i \]

\[ = \left< \vec{p}_x - \vec{p}_i, \vec{p}_y - \vec{p}_i \right> \]

\[ |J^*| = |\Delta \vec{p}| = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = \boxed{10.9 \text{ kg·m/s}} \]

\[ |\Sigma F| = 1 \Delta \vec{p} \]

\[ |\Sigma F_{max}| = 1 \Delta \vec{p} \]

\[ \frac{1}{2} (4 \text{ms}) F_{max} + (20 \text{ms}) F_{max} + \frac{1}{2} (4 \text{ms}) F_{max} = 10.9 \text{ kg·m/s} \]

Solve for \[ F_{max} = 455 \text{ N} \]