Practice:

1(a) Consider the vector field $\vec{F} = (x + 2)\hat{i} + (z + 2)\hat{k}$. Calculate the divergence of $\vec{F}$.

(b) In which direction does the vector field $\vec{F}$ point on the plane $z = x$? What is the value of $\vec{F} \cdot \hat{n}$ on this plane where $\hat{n}$ is the unit normal to the plane?

(c) Verify the divergence theorem for this vector field where the volume involved is drawn below.

Required:

2) (Griffiths 1.5):
Use the cross product to find the components of the unit vector $\hat{n}$ perpendicular to the plane shown in the figure below, i.e. the plane joining the points $\{(1,0,0),(0,1,0),(0,0,1)\}$.
3) In Marion and Thornton, Example 5.1, they found the gravitational potential and, a little later, the gravitational field due to a spherical shell of matter (or equivalently, the electric field due to a spherical shell of charge.) In this problem, you will explore the consequences of the divergence theorem for this shell.

(a) Using the value of the gravitational field from that example, find the divergence of the gravitational field everywhere in space. You will need to divide this question up into three parts: \( R < b, \ b < R < a, \) and \( R > a. \)

(b) Discuss the physical meaning of the divergence in this particular example.

(c) For this gravitational field, verify the divergence theorem on a sphere, concentric with the shell, with radius \( Q, \) where \( b < Q < a. \)

(d) Discuss how this example would change if you were discussing the electric field of a uniformly charged spherical shell.

4) (Griffiths 5.5) A current \( I \) flows down a wire of radius \( a. \)

(a) If it is uniformly distributed over the surface, give a formula for the surface current density \( K? \)

(b) If it is distributed in such a way that the volume current density, \( J, \) is inversely proportional to the distance from the axis, give a formula for \( J? \)

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