PRACTICE:
Do more examples from the in-class worksheets.

REQUIRED:

1) A positively charged dielectric spherical shell of inner radius $a$ and outer radius $b$ has a spherically symmetric internal charge density

$$\rho = \alpha \frac{\sin(kr)}{kr}$$

where $\alpha$ and $k$ are constants with appropriate units.

(a) Write the volume charge density everywhere in space as a single function, sketch the charge density, and find the total charge.
Assume $b$ is large enough (what does this mean?) that there are charges of BOTH signs of charge in your shell.

(b) Use Gauss’s Law and symmetry arguments to find the electric field in each of the regions given below:
   (i) $r < a$
   (ii) $a < r < b$
   (iii) $r > b$

(c) Pick sensible values of $\alpha$ and $k$ (what values do you choose and what units?) and sketch the $r$-component of the electric field as a function of $r$.

2) Referring to the charge distribution in the previous problem, take the limit as $a \to b$ so that the shell becomes infinitely thin, but keeping the total charge $Q$ constant.

(a) Give a formula for the charge density everywhere in space.
   Be careful: Integrate your charge density to get the total charge as a check.

(b) Use Gauss’s Law and symmetry arguments to find the electric field at each region given below:
   (i) $r < b$
   (ii) $r > b$

(c) Using your previous values of $\alpha$ and $k$, sketch the $r$-component of the electric field as a function of $r$.

CHALLENGE:

3) Take the limits of the shell in the previous problem as $a \to b$ and then $b \to 0$, so that the shell becomes a point charge, but keeping the total charge $Q$ constant.

(a) Give a formula for the charge density everywhere in space.
   Be careful: Integrate your charge density to get the total charge as a check.

(b) Use Gauss’s Law and symmetry arguments to find the electric field for $r > 0$. 
REQUIRED:

4) We know that the electric field everywhere in space due to an infinite plane of charge with charge density located in the \( xy \)-plane at \( z = 0 \) is

\[
\vec{E}(z) = \begin{cases} 
  \frac{\sigma}{2\epsilon_0} \hat{\imath} & z > 0 \\
  -\frac{\sigma}{2\epsilon_0} \hat{\imath} & z < 0
\end{cases}
\]

(Mentally check that this is true for both positive and negative values of \( \sigma \).)

(a) Sketch the \( z \)-component of the electric field as a function of \( z \).

(b) Draw a similar picture, and write a function that expresses the electric field everywhere in space, for an infinite conducting slab in the \( xy \)-plane, of thickness \( d \) in the \( z \)-direction, that has a charge density \( +|\sigma| \) on each surface.

(c) Repeat for a charge density \( -|\sigma| \) on each surface.

(d) Now imagine two conductors, one each of the two types described above, separated by a distance \( L \). Use the principle of superposition to find the electric field everywhere. Discuss whether your answer is reasonable. Does it agree with the known fact that the electric field inside a conductor is zero? Has superposition been correctly applied? Is Gauss’ Law correct? Try to resolve any inconsistencies.