PH 422: Day 6

1 Dot Products and Components

The previous activity used the concepts of the parallel and perpendicular components of \( \vec{E} \). What are components?

Start in rectangular coordinates, where

\[
\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}
\]

Similarly, in cylindrical coordinates,

\[
\vec{E} = E_r \hat{r} + E_\phi \hat{\phi} + E_z \hat{z}
\]

What is the \( x \)-component of \( \vec{E} \)? Surely just \( E_x \). The \( r \)-component? \( E_r \).

How could you find these components, given \( \vec{E} \)? The (scalar) component of a vector field in a given direction is just the projection in that direction. Projections are dot products! Thus,

\[
E_x = \vec{E} \cdot \hat{i}
\]

and

\[
E_r = \vec{E} \cdot \hat{r}
\]

Given a surface, it makes sense to ask what the component \( E_\perp \) of the electric field is, perpendicular to the surface. By the same reasoning, we have

\[
E_\perp = \vec{E} \cdot \hat{n}
\]

where \( \hat{n} \) is the unit normal to the surface. The parallel component \( E_\parallel \) is more subtle, since there are several directions parallel to the surface. One often speaks of vector components, for instance

\[
\vec{E}_\perp = E_\perp \hat{n}
\]

and we can now define the parallel (vector) component of \( \vec{E} \) as

\[
\vec{E}_\parallel = \vec{E} - \vec{E}_\perp
\]

from which the magnitude \( E_\parallel = |\vec{E}_\parallel| \) can be computed using the Pythagorean Theorem if desired.
2 Currents

How do we measure current? By counting the charges which go past. So use a stopwatch, and count. What are the units? Coulombs per second. What answer do we get? That depends on the charge density $\lambda$ and the velocity $\vec{v}$. At any point in a wire, the current is given by

$$\vec{I} = \lambda \vec{v}$$

and the number of charges which go past per unit time is clearly just the magnitude, $I = |\vec{I}|$. Since the direction of the current is obvious in a wire, both $I$ and $\vec{I}$ are called the current.

Similarly, the surface current density $\vec{K}$ and the (volume) current density $\vec{J}$, given respectively by

$$\vec{K} = \sigma \vec{v} \quad (1)$$

$$\vec{J} = \rho \vec{v} \quad (2)$$

Be very careful with the dimensions here, current densities do not have the dimensions of current per unit area or volume, respectively, but rather the dimensions of charge density times speed.

But what is the total current in these cases? The key idea is that one must insert a (1- or 2-dimensional) “gate”, and count the charges which cross the gate per unit time. But only the motion of the charges perpendicular to the gate is relevant. This is (2- and 3-dimensional) flux; the total current is given respectively by

$$I = \int \vec{K} \cdot \hat{n} \, ds \quad (3)$$

$$I = \int \vec{J} \cdot \hat{n} \, dA = \int \vec{J} \cdot d\vec{A} \quad (4)$$
