PH 422: Day 5

1 The Relationship between $\vec{E}$, $V$, and $\rho$

Starting with the electric potential for a point charge, we used the superposition principle to obtain an integral formula for the potential due to any charge distribution, as well as a similar formula for the electric field. Furthermore, the electric field is just (minus) the gradient of the potential, which can be inverted to obtain the potential as an integral of the electric field.

This week, we have seen that the charge density can be recovered as the divergence of the electric field. It remains to show how to recover the charge density from the potential, at which point we are able to compute any of $\vec{E}$, $V$, and $\rho$ from any other.

But we know that

$$\vec{E} = -\vec{\nabla}V$$

$$\frac{\rho}{\epsilon_0} = \vec{\nabla} \cdot \vec{E}$$

from which it is easy to compute

$$\frac{\rho}{\epsilon_0} = -\vec{\nabla} \cdot \vec{\nabla} V$$

which is the desired relation; this is Poisson’s equation. These relationships are nicely summarized in the triangle in Griffiths in Figure 2.35 on page 87.

2 The Laplacian

The second derivative above occurs so often it has its own name and notation. It is called the Laplacian of the function $V$, and is written in any of the forms

$$\Delta V = \nabla^2 V = \vec{\nabla} \cdot \vec{\nabla} V$$

In rectangular coordinates, it is easy to compute

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

The special case of Poisson’s equation with no source, namely

$$\Delta V = 0$$

is called Laplace’s equation. This equation arises in many contexts, not just in electrodynamics.