PH 320: Day 9

1 Curvilinear coordinates

The standard physics conventions for cylindrical and spherical coordinates differ from the standard (American) math conventions; make sure you understand the physics conventions. It is useful to be comfortable both with the coordinates, and with the associated unit vectors \( \hat{r} \), etc.

In the basic coordinate systems, we have

\[
\begin{align*}
\textrm{d}\vec{r} &= dx \hat{i} + dy \hat{j} + dz \hat{k} \\
&= dr \hat{r} + r d\phi \hat{\phi} + dz \hat{z} \\
&= dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}
\end{align*}
\]

It is easy to use (the components of) \( d\vec{r} \) to determine the area of surfaces adapted to these coordinates. For instance, on a \((z = \text{constant})\) plane, one has

\[
dA = dx \, dy = r \, dr \, d\phi
\]

and on the surface of a cylinder one has

\[
dA = r \, d\phi \, dz
\]

and on the surface of a sphere one has

\[
dA = r \, d\theta \, d\phi
\]

Similarly, the volume elements in rectangular, cylindrical, and spherical coordinates are given by

\[
d\tau = dx \, dy \, dz = r \, dr \, d\phi \, dz = r \, dr \, d\theta \, d\phi
\]

where we use \( d\tau \) rather than \( dV \) to avoid confusion with the electric potential.