

Currents in Normal-Metal Rings Exhibit Aharonov–Bohm Effect

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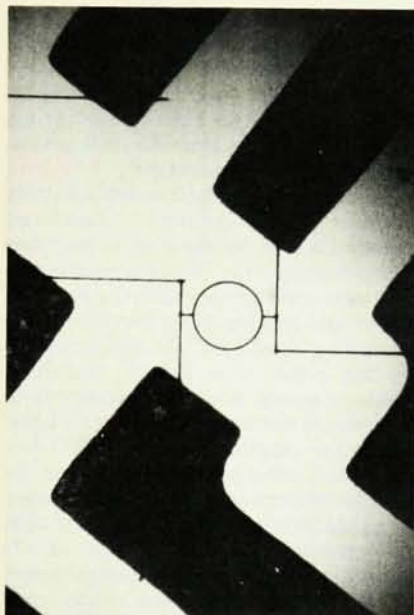
Currents in normal-metal rings exhibit Aharonov-Bohm effect

Ever since Yakir Aharonov and David Bohm pointed out, in 1959, that electrons propagating around a magnetic field through a field-free vacuum should exhibit a quite surprising interference effect, its experimental and theoretical investigation has been contributing to our understanding of the fundamental character of the quantum theory. The recent appearance of this Aharonov-Bohm effect in a rather different context may eventually have a profound effect on our view of the solid state on a "mesoscopic" scale—between the microscopic and the truly macroscopic.

The recent observation, by Richard Webb and his IBM colleagues,¹ of clear Aharonov-Bohm oscillations in very small, nonsuperconducting metallic rings indicates that electrons can maintain quantum-mechanical phase coherence in quite ordinary materials at low temperature over distances corresponding to thousands of atomic diameters. Quantum interference phenomena in condensed matter that had previously seemed the exclusive province of superconductors are entering the realm of disordered normal metals and semiconductors, when the devices are small enough (10 000 Å) so that the scattering of electrons is almost entirely elastic. On this mesoscopic scale—made available by the advance of microfabrication technology—we are also beginning to see the breakdown of the ensemble-averaging description of lattice imperfections appropriate to solid-state systems with very large numbers of atoms; sample-dependent idiosyncrasies are showing up as "fingerprints" of the impurity distribution unique to an individual tiny device.

Soon after the IBM discovery, the same effect was seen by a group at Yale,² and a Purdue group has now seen Aharonov-Bohm oscillations in a tiny semiconductor heterostructure.³

Aharonov and Bohm, in their classic 1959 paper, pointed out a straightforward but nonetheless counterintuitive consequence of the appearance of the vector potential in the Hamiltonian in the standard quantum-mechanical treatment of electromagnetic interactions. They noted that the vector potential will affect the phase of an electron



A gold ring used by the IBM group to observe Aharonov-Bohm oscillations in normal metal. The microlithographic polycrystalline ring has a diameter of 1 micron and a linewidth of 0.03 microns. The gold leads introduce a current that splits to traverse the ring by separate semicircular routes. Quantum interference is observed by measuring the effective resistance of this parallel circuit as a function of the magnetic flux threading the ring.

wavefunction, with observable consequences, even when the electron is restricted to regions of space where the electric and magnetic field intensities vanish. They suggested the following experiment: Let an electron beam (in vacuum) be split coherently so that it travels from a common source to a common detector by two different paths. Depending on the details of the two paths, the reunited coherent beams will exhibit interference at the detector. The Aharonov-Bohm configuration provides no electric or magnetic fields anywhere along either path. The only magnetic field is confined to a long solenoid that threads between the two paths in a region excluded to the electrons.

The surprising but inescapable prediction—if standard quantum mechanics is right—is that the interference

pattern between the reunited beams depends on the magnetic field strength inside the solenoid, even though neither this field nor any electric field is experienced directly by the electrons. Specifically, a total magnetic flux Φ in the solenoid should shift the phase of the interference (relative to the case $\Phi = 0$) by precisely $\Phi e / \hbar$.

Because the wavelengths in question are very small, the observation of this effect would require an exceedingly tiny solenoid. But the Aharonov-Bohm effect was soon verified with an ultrathin magnetized iron "whisker" lying between two slits in a vacuum electron-beam apparatus. Since the early 1960s the effect has been seen often enough in vacuum so that only a few diehard skeptics still doubt its reality. Just as with the various experimental tests of Bell's theorem (see PHYSICS TODAY, April, page 38), quantum mechanics always seems to triumph over challenges by "common sense."

Interference in condensed matter. The issue in the recent experiments and their antecedents in the Soviet Union is a different one. The question is whether one can indeed observe such quantum interference effects in ordinary condensed matter between electrons that have experienced many collisions along their way. Long-range coherence is of course the essence of superconductivity and related phenomena—Josephson oscillation, flux quantization and the like. But what can one expect to see in disordered normal metals or semiconductors?

The recent story begins with a surprising prediction by theorists Boris Altshuler, Arkady Aronov and Boris Spivak at the Leningrad Nuclear Physics Institute in 1981. Considering an ultrathin normal-metal cylindrical shell of moderate length but tiny transverse dimensions at low temperature, they asked how the "magnetoresistance" (electrical resistance as a function of magnetic field) of such a structure would depend on the intensity of a magnetic flux axially threading the cylinder. Their curious conclusion was that the magnetoresistance should be an oscillating function of the total flux, with a period of $h/2e$. But $h/2e$ is just the flux quantum one associates with

superconductivity. The factor 2 multiplying the electron charge indicates that supercurrents are composed of Cooper pairs of electrons. Why should one be seeing manifestations of the superconducting flux quantum $h/2e$ in normal metals? The analogous "flux quantum" in the Aharonov-Bohm effect should be h/e , without the factor 2. That is to say, in the Aharonov-Bohm effect the displacement of the interference pattern depends on the flux *modulo* h/e ; increasing the flux by integral multiples of h/e changes the interference phase by multiples of 2π . The theory of Altshuler and company predicted the $h/2e$ oscillations as a manifestation of "coherent backscattering," a phenomenon related to the ordinary Aharonov-Bohm effect, but different enough in detail so that the former, but not the latter, was predicted to appear in the cylindrical geometry.

The experimental verification was furnished later that year by Yu. V. Sharvin (Institute of Solid State Physics, Moscow) and his son D. Yu. Sharvin (Institute of Physical Problems, Moscow). With a metal layer a few hundred angstroms thick deposited on a 10 000-Å-thick quartz fiber several millimeters long, they did indeed see a few cycles of magnetoresistance oscillation with a flux period of $h/2e$. But there was no evidence of the longer-period h/e oscillations corresponding to the ordinary Aharonov-Bohm effect. Some skepticism was generated by the failure of the first attempts to reproduce this extraordinary result in the US and Western Europe. But in the last two years the Sharvin phenomenon has been firmly established in a number of laboratories with a variety of related geometries.

The Sharvin experiment, as well as the subsequent IBM experiments that ultimately found the h/e oscillation, differ from the original Aharonov-Bohm *Gedankenexperiment* in a way that makes them seem, at first glance, less counterintuitive. But this complication, forced by experimental necessity, is a quite secondary issue. In all of these latter-day experiments with electrons traveling through normal metals rather than in vacuum, the magnetic field does not, in fact, vanish along the electron paths. The magnetic flux is produced by a large solenoid surrounding the entire device. But, all the theoretical analyses conclude, the essential issue is the magnetic flux surrounded by the electric currents. The preponderance of this flux is in the hole encompassed by the cylinder or ring in question. The magnetic field in the metal conductor itself is presumed to produce only secondary effects.

The quest for h/e . In common with the other geometries that yielded $h/2e$ oscillations but no sign of the h/e period required by Aharonov-Bohm, the Shar-

vin device was very small in two dimensions and much larger in the third. This turned out ultimately to be the crucial factor. What constitutes "small" in this context? IBM theorist Rolf Landauer has long advocated a position in which electrical resistance is calculated from reflection and transmission probabilities. He argues that true dissipation—"normal-sign resistance"—occurs only when electrons experience inelastic collisions, principally in scattering off lattice phonons. Elastic scattering off impurities or lattice imperfections, on the other hand, provides no mechanism for energy dissipation. Furthermore, while inelastic collisions destroy quantum-mechanical phase coherence, elastic scattering, though it will in general shift phase, preserves "phase memory".

Can one avail oneself experimentally of this phase memory? Landauer points out that as one goes to very low temperatures, the inelastic-scattering mechanisms diminish with the weakening of lattice vibrations, leaving elastic scattering dominant. At temperatures on the order of a kelvin, therefore, phase memory would be preserved in disordered normal metals over phase-coherence lengths of about 10 000 Å—the mean diffusion length between inelastic-scattering events. Landauer suggests that one sees apparently normal resistance in a cold sample of this size only because of the thermal reservoirs represented by the thick leads usually present in resistance measurements. Inducing a current by magnetic induction in a sufficiently small loop of normal metal at low temperature without such leads, he speculated in an unpublished 1966 IBM memorandum, might produce all sorts of phenomena reminiscent of superconductivity—perhaps even persistent currents.

In the 1960s, suggestions that one might see Aharonov-Bohm h/e oscillations in a small normal-metal device at low temperatures were made by Nina Byers and C. N. Yang at the Institute for Advanced Study, Felix Bloch at Stanford, and Leon Gunther and Yoseph Imry at Tufts. These early papers did not treat the scattering problem in any detail. In 1983, Imry (then at Tel Aviv University) joined forces with Landauer and Markus Büttiker at IBM. Asking how a current would be induced in a small, cold ring of normal metal (without leads) by a time-varying magnetic flux threading the ring, they predicted⁴ that one would see something very much like the ac Josephson effect in superconductors, except for the factor of 2 that characterizes Cooper pairs. A voltage V across a Josephson junction induces a current oscillation given by $\omega = 2eV/\hbar$. Büttiker and his collaborators predicted a current oscillation in the normal loop with a fre-

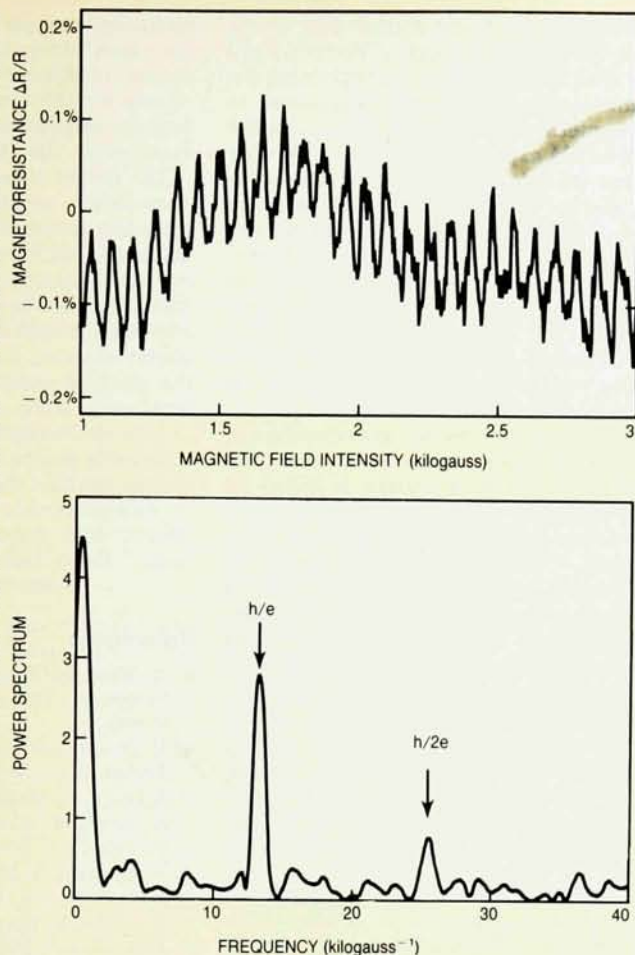
quency $\omega = eV/\hbar$, V in this case being the emf induced in the loop by the time-varying magnetic flux. Note that the flux quantum—the change in flux required to get back to a given current—is h/e , the period characteristic of the Aharonov-Bohm effect.

Perhaps the most striking prediction of this paper is that the current will persist after one stops changing the flux. "If you can ignore inelastic scattering, you can store energy, but you can't dissipate it," Landauer explains. MIT theorist Patrick Lee points out that this prediction is highly speculative, and its experimental test is still some years off.

The actual experiment carried out by Webb and his colleagues was helped along by a series of more experimentally relevant theoretical calculations. Instead of a leadless ring with magnetically induced current, Imry and his Tel Aviv colleagues Yuval Gefen and Mark Azbel considered⁵ a ring with leads for running an external current through the ring and measuring its resistance. If one runs a current into such a ring through one lead and takes it out through another lead at the other side, one in effect splits the current temporarily into two parallel paths. If these paths are shorter than the quantum-mechanical phase-coherence length—the diffusion length for inelastic scattering—the two parallel current branches will preserve their phase memory until they reunite. Because there will be many elastic-scattering events and many possible microscopic paths for an electron traversing either finite-thickness path, one cannot specify the phase changes any coherent pair of electrons will undergo on their separated journeys. There will be an anarchic mix of constructive and destructive interferences.

If one now threads a static magnetic flux through the loop, all these phases will change *à la* Aharonov-Bohm. One cannot say *a priori* whether the new interferences will decrease or increase the current throughput that measures the magnetoresistance of the ring. The only thing that the Aharonov-Bohm equation and the Landauer resistance formalism tell us is that things will revert to the *status quo ante* whenever the total flux change is an integral multiple of h/e . Thus, the theorists predicted, one will see the magnetoresistance oscillate with increasing magnetic field with a flux periodicity of h/e . But one cannot predict the initial phase of this Aharonov-Bohm oscillation at $B = 0$. That will be a sample-dependent signature characteristic of the imperfections of each individual ring. Gefen pointed out that this initial-phase randomness yields the ultimate explanation for why the h/e oscillation cannot be seen in long, thin cylinders and other

Aharonov-Bohm oscillations of magnetoresistance (top) measured across a tiny gold ring at IBM as a function of the magnetic field intensity threading the ring. The principal period of oscillation is 76 gauss, corresponding to a flux change of h/e through the ring. The Fourier transform (bottom) also shows a second-harmonic peak at $1/38$ gauss $^{-1}$, corresponding to a flux period of $h/2e$. Both peaks persist out to field intensities of 80 kilogauss. The enhancement in the Fourier transform near 0 gauss $^{-1}$ corresponds to longer-scale aperiodic fluctuation of the magnetoresistance due to magnetic flux through the metal itself.



geometries related to the Sharvin experiment. In a subsequent paper,⁶ Büttiker, Imry, Landauer and Shlomo Pinhas (Tel Aviv) generalized these ideas to take account of the finite thickness of the ring.

Webb and his colleagues Sean Washburn, Corwin Umbach and Robert Laibowitz put all of this theorizing to the test in the spring of 1984. Their first results were disappointingly ambiguous, but they produced intriguing "reproducible noise." Using microlithographic techniques developed at IBM, the group produced gold rings with diameters ranging from 0.2 to 0.8 microns and metal widths ranging from 0.04 to 0.13 microns. Though the dimensions are impressive, the gold was quite ordinary polycrystalline material, not particularly free of impurities. The experiment is performed by placing such a ring in the magnetic field of a much larger solenoidal coil at a temperature below 1 K. The solenoid produces an effectively uniform field transverse to the ring plane, with a field intensity that can be varied continuously from 0 to 80 kilogauss. With two thin leads attached to opposite sides of the ring, one runs a current through the ring at a given magnetic field intensity and measures the magnetoresistance of

the ring at that field intensity. As the magnetic field was increased in the early experiments, the group saw lots of intriguing structure in the plot of magnetoresistance against field intensity, but the picture was unclear. There were hints of h/e periodicity and also $h/2e$ periodicity, but the overall curve appeared disappointingly noisy and ambiguous. One hopeful sign was that the apparent noise was reproducible; running up the field a second time with the same ring always reproduced the same random, aperiodic pattern.

Encouraged by the reproducibility of the "noise," Imry and IBM theorist Douglas Stone set out to suggest an explanation and a remedy. If the metal ring were infinitesimally thin, they argued, one would expect a cycle of magnetoresistance oscillation to be completed whenever the magnetic field intensity has increased by h/e divided by the area enclosed by the ring. But if the area of the metal itself is non-negligible, a second, aperiodic pattern of longer characteristic fluctuation scale should be superposed on the first—namely h/e divided by the area of the metal, corresponding to a secondary Aharonov-Bohm effect due to paths encircling magnetic flux inside the metal rather than the hole.

The obvious remedy is to reduce the area of the metal relative to the hole as much as possible, so that the secondary fluctuation will have a much longer scale of variation, clearly distinguishable from the principal Aharonov-Bohm oscillation corresponding to the magnetic flux through the hole. Last March, employing a ring of 0.8 microns diameter and 0.04 microns metal width with new, finer leads, the group finally saw very well-defined h/e oscillations persisting throughout the entire magnetic-field range from 0 to 80 kilogauss. Given the area enclosed by the ring, a single h/e oscillation corresponds to a field change of about 76 gauss. Thus the group observed more than a thousand Aharonov-Bohm oscillations over the magnetic range of the solenoid, with no sign of signal weakening at high fields. The $h/2e$ oscillations observed in cylinders, by contrast, wash out after only a few oscillations as the magnetic field is increased.

The Fourier transform (with respect to reciprocal field intensity) of the group's data shows clearly that h/e is the principal flux period of oscillation. A second spectral enhancement shows up at much longer periods, corresponding to the aperiodic effect explained by Stone and Imry. Finally, a significantly weaker second-harmonic peak appears at $h/2e$. Because this $h/2e$ peak persists out to the highest magnetic field intensities, the group does not attribute it to the effect predicted by Altshuler and company and measured by the Sharvins, but rather to a second harmonic of the Aharonov-Bohm effect.

The $h/2e$ oscillations first seen by the Sharvins are attributed by the Altshuler theory to a somewhat different mechanism—coherent backscattering. Imagine the cylinder sliced up into many thin rings. Some trajectories will bring an electron all the way around such a ring and back to its starting point. To every such trajectory there corresponds a precise time-reversed image, experiencing the same sequence of scatterings in reverse order. When no magnetic flux threads the cylinder, such a clockwise-counterclockwise pair of equivalent trajectories will always reinforce one another, being perfectly in phase. Therefore these equivalent pairs will weigh heavily in ensemble averages relative to more general pairs of round-trip trajectories, which will interfere with all sorts of phases. Turning on the magnetic flux in the hole will, in general, scramble all the phases, *à la* Aharonov-Bohm. But one returns to the original state of affairs whenever half an Aharonov-Bohm flux quantum $h/2e$ has been added to the field. It requires only half a flux quantum because each of the perfectly paired electrons makes a complete circuit around the flux in the backscattering

case. Thus one needs only a phase shift of π in each trajectory to regain coherence. The resemblance to the superconducting flux quantum is, as they say, coincidental.

In most light metals, when these backscattering coherences are undisturbed, the magnetoresistance is a maximum—essentially because coherent backscattering is an impediment to diffusion. Thus, as one raises the flux through a cylinder, one observes oscillatory variation of its magnetoresistance with a flux period of $h/2e$. But why is this $h/2e$ effect dominant in the long-cylinder geometry, while the Aharonov–Bohm h/e effect dominates in thin rings? The essential difference, implicit in the Altshuler–Aronov–Spivak theory, is the starting phase. For a thin ring, the starting phase of the h/e Aharonov–Bohm oscillation at zero flux is sample dependent in a random way. The $h/2e$ coherent-backscattering effect, by contrast, always yields maximum resistivity at zero flux.

In the Sharvin-type experiments, the cylinder is much longer than the quantum-mechanical phase-coherence length; thus one can think of such a cylinder as a series of independent thin rings separated by inelastic-scattering events. In such a series of rings the Aharonov–Bohm h/e signal is completely washed out because each little ring has its own random starting phase, which depends on the distribution of imperfections in that particular slice. The coherent-backscattering $h/2e$ oscillation, on the other hand, will persist because it has the same starting phase all along the cylinder.

The IBM group, joined by Christian Van Haesendonk, have in fact recently demonstrated this effect with particular clarity. They repeated their h/e experiment, but this time measuring the summed magnetoresistance of an array of rings in series. As they increased the number of rings in the circuit, they saw the h/e signal gradually replaced by $h/2e$ oscillations. The h/e signal decreases as the reciprocal square root of the number of rings in series, just as one would expect for an averaging over random starting phases.

The h/e effect is really much more spectacular to look at than is its $h/2e$ rival, because it persists through thousands of oscillations out to very high magnetic field intensities. Stone and Imry have recently explained⁷ why the $h/2e$ coherent-backscattering oscillation dies out so quickly with increasing field. The $h/2e$ effect is a delicate interference involving a sum over relatively few coherent terms. This precise balance, they pointed out, is easily upset by any significant magnetic field in the metal itself. The h/e oscillation, by contrast, is a much more robust sum over a large number of random terms—

which don't quite cancel out if the device is small enough. "You can't kill a dead horse," says Imry, explaining the immunity of the h/e oscillation to magnetic fields in the metal. Unlike the IBM experimenters, Daniel Prober and his Yale colleagues have seen² $h/2e$ coherent-backscattering oscillation in their small ring, whose dimensions are similar to those of the IBM ring. Presumably the difference is the presence of fewer magnetic impurities in the silver Yale ring.

The fact that the random h/e terms don't quite cancel when the device is small enough points up the novel character of physics on the mesoscopic scale. In the theory of Altshuler and company, the h/e oscillation is killed by performing the customary ensemble average over all possible impurity locations. This is appropriate for large systems, and also for the relatively long Sharvin cylinder. But on the new, mesoscopic scale, ensemble averaging appears to be too gross a treatment of the relatively small number of scattering centers involved. "Device people have been sensitive to this sort of 'fingerprint' variation of defect signatures from one tiny sample to the next for a long time," Landauer told us. "The solid-state theorists are just catching up."

Intrigued as much by the aperiodic reproducible noise in the IBM experiments as by the h/e periodicity, Lee and Stone (now at Stony Brook) have proposed⁸ a rather general theory of universal conductance fluctuations in metals. They argue that conductance fluctuations on the order of e^2/h with changing magnetic field are a universal feature of

quantum transport in the low-temperature limit, independent of sample size or degree of disorder. Their theory accounts for the amplitudes of both the periodic oscillations and aperiodic fluctuations in the IBM experiments.

The recent observation of h/e Aharonov–Bohm oscillations in a gallium arsenide heterostructure by Supriyo Datta and his Purdue colleagues³ holds out some technological promise. One should be able to see such interference effects in comparatively large semiconductor devices, Datta argues, because the electron wavelengths at the Fermi level are so much longer than in metals. "I believe that optimum semiconductor structures can be fabricated by present technology that will show large magnetic and electrostatic Aharonov–Bohm effects, with potential device applications," Datta told us.

—BERTRAM SCHWARZSCHILD

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Measuring the Hubble constant

Hubble's celebrated constant, which relates the recessional velocities of galaxies to their distances, is perhaps the most important number in extragalactic astronomy. Using it, along with their favorite models, astronomers and cosmologists derive the age of the universe, estimate its size, calculate the luminosity of quasars and much more. Unfortunately, astronomers are far from agreeing on a value for Hubble's constant. Some believe that H_0 is about 50 km/sec Mpc; others think it is closer to 100 km/sec Mpc. Depending on the details of the cosmological model chosen, this can lead to a discrepancy of a factor of 2—ten billion years or so—in the age of the universe.

The controversy over the value of H_0 is fueled by a lack of agreement on the best method for determining the distances to galaxies. In his seminal 1929 publication, Edwin Hubble estimated that H_0 was around 530 km/sec Mpc;

today, even astronomers who cannot agree on a value of H_0 believe that Hubble grossly underestimated the distances to the galaxies, making his value far too large. Techniques for measuring the distances to galaxies have become considerably more sophisticated in the last 50 years, but all depend on a web of inference and uncertainty that makes their ultimate accuracy difficult to estimate. However, an ingenious new technique that combines very-long-baseline interferometry with optical observations of supernovas promises someday to cut through this confusing web.

High-school geometry. Reduced to its conceptual bones, the new technique uses concepts familiar to any student of high-school geometry. Norbert Bartel, Irwin Shapiro, Marc Gorenstein and Carl Gwinn of the Harvard-Smithsonian Center for Astrophysics, Alan Rogers of the Haystack Observatory,