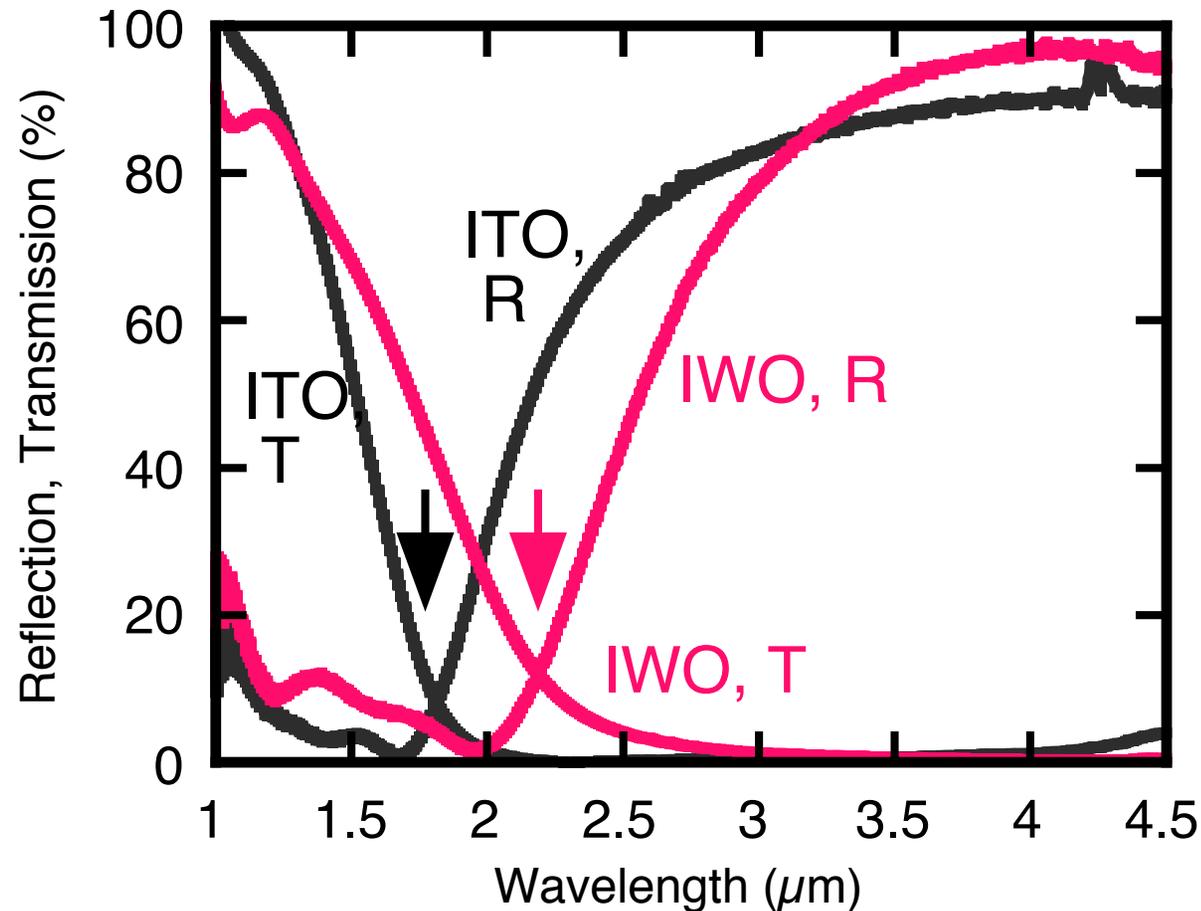
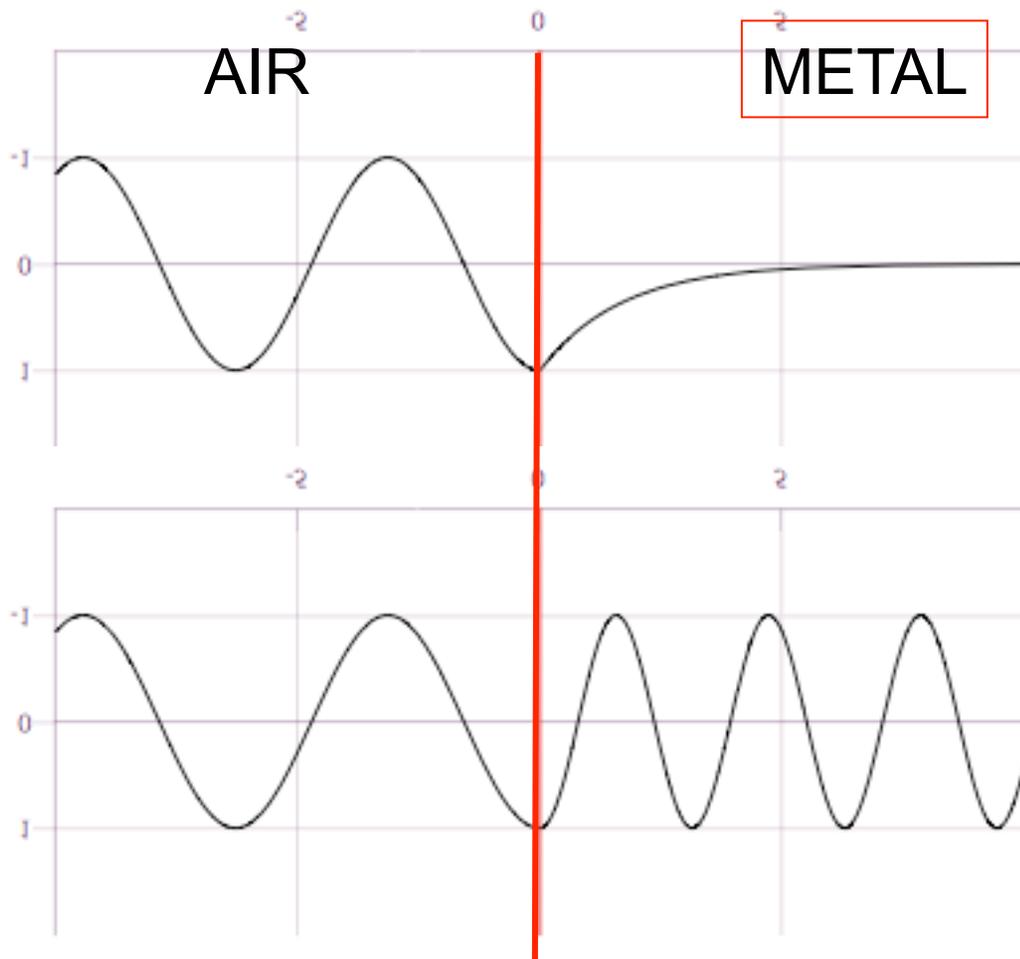


# PH575 Spring 2019

## Lecture #17/18

Free electron metals: Optical response, Kittel Ch.  
14 pp. 152-156; 395-403

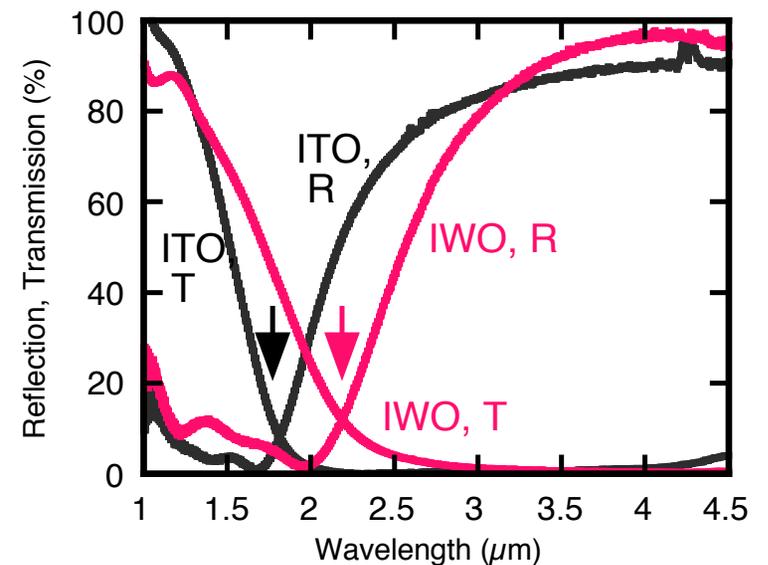


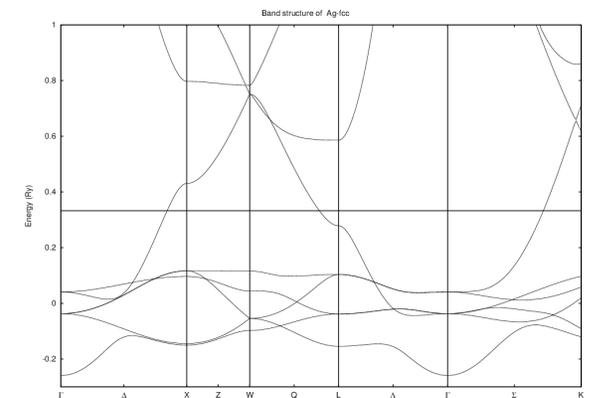
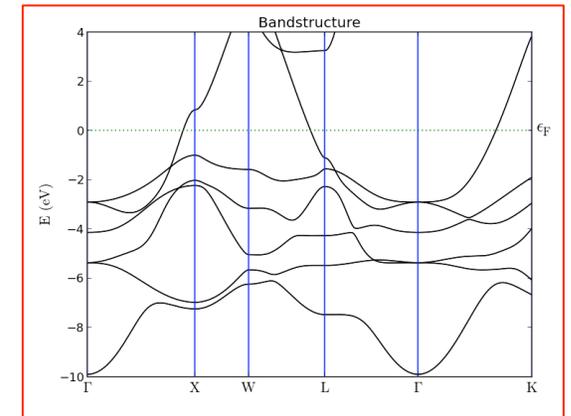
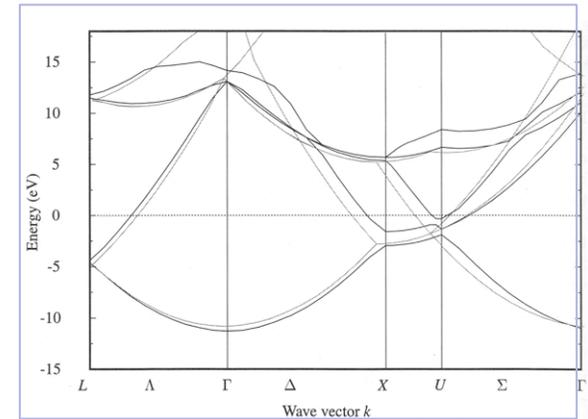
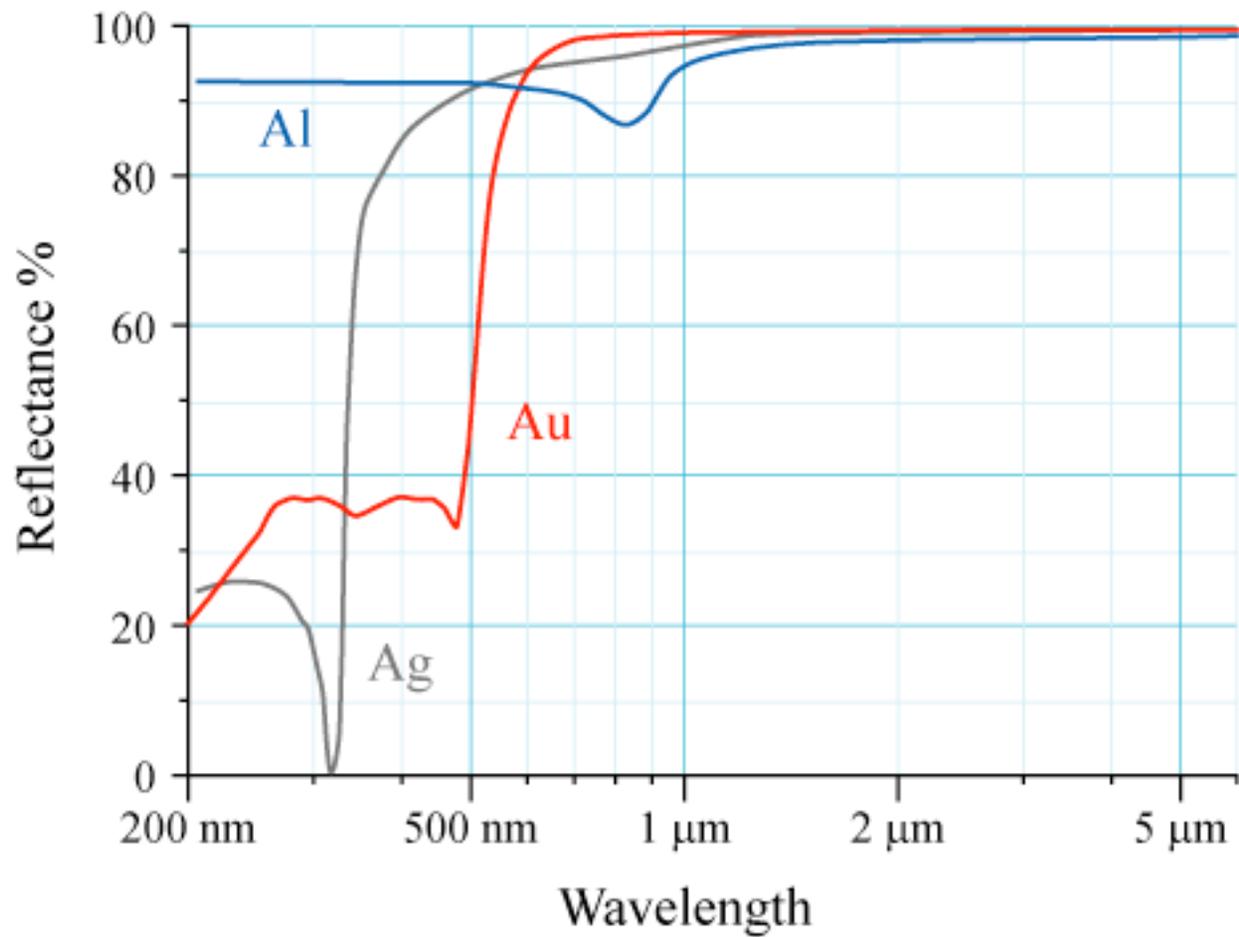


$\omega < \omega_p$  No propagation;  
reflection

$\omega > \omega_p$  Transparent

PLASMA frequency separates  
reflection (low frequency)  
from transparency (high frequency)



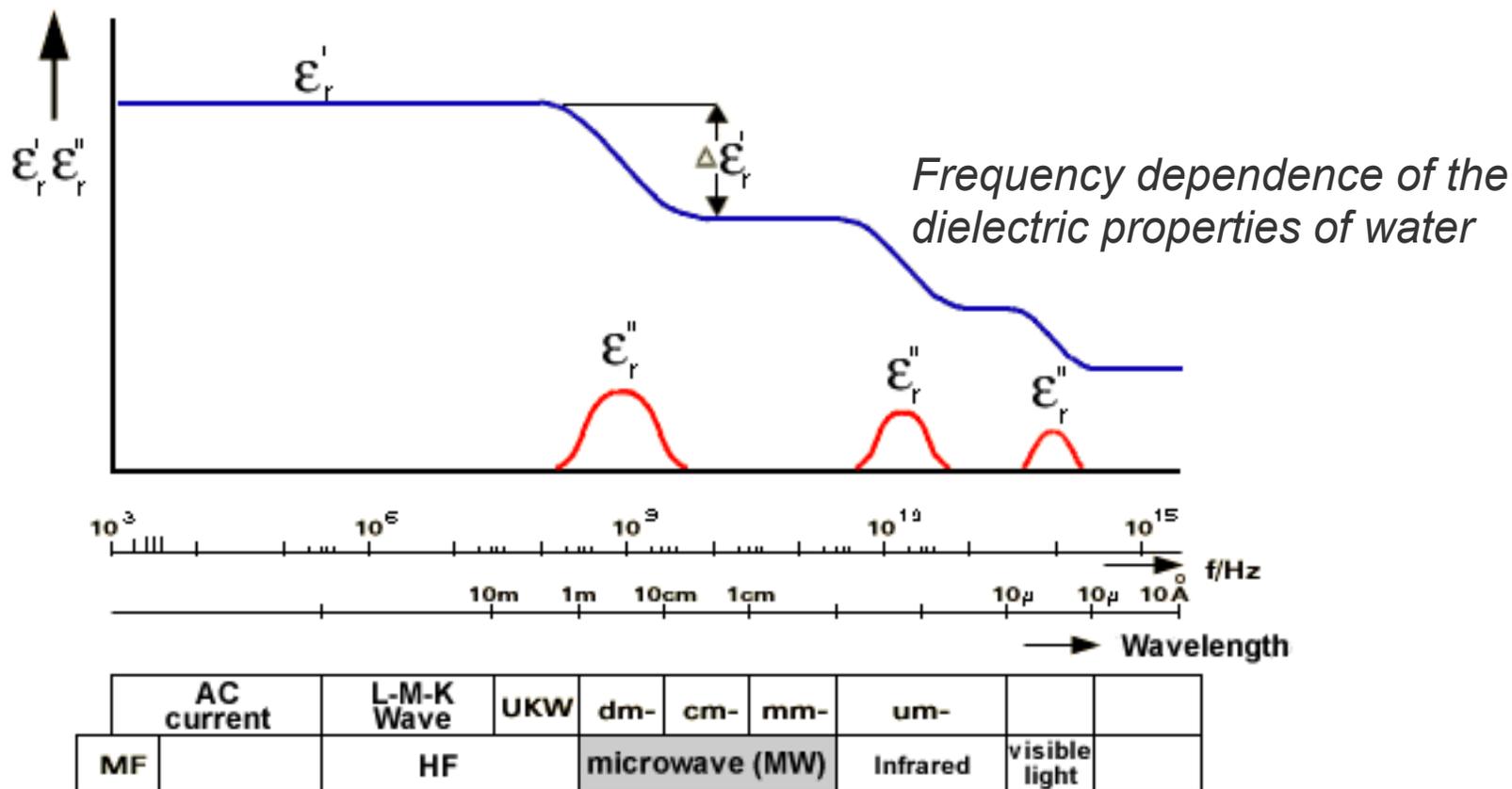


<https://physics.stackexchange.com/questions/72368/why-are-most-metals-gray-silver>

Dielectric function  $\epsilon$  of an electron gas:

In intro physics,  $\epsilon$  is the (real) factor measuring field enhancement in a dc capacitor. Related to polarizability of a material.

In general,  $\epsilon$  is **frequency-dependent**, because charges respond differently to different frequency electric fields. It is also **complex** because there is an in-phase and an out-of-phase response.



## Dielectric function of the free electron gas (i.e. electrons in a metal)

Material's response to electromagnetic radiation with frequency  $\omega$  and wave vector  $k = 2\pi/\lambda$ .

Long wavelength response (spatial electric field of light wave changes little over the mfp of an electron).

Visible light  $\lambda \approx 10\text{-}1000$  nm; mfp Cu RT  $\approx 80$  nm.

Free mobile electrons against fixed background of positive charge (total system is neutral).

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad \text{E: electric field; P: dipole moment /volume}$$

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E} \equiv \epsilon_{rel} \epsilon_0 \vec{E} \quad \begin{array}{l} \text{Permittivity of free space } 8.8 \times 10^{-12} \text{ F/m} \\ \chi: \text{ electric susceptibility} \end{array}$$

$$\epsilon_{rel} = (1 + \chi) \quad \text{Permittivity relative to free space}$$

Dielectric function of the (undamped) electron gas:

$$\text{Newton: } \vec{F} = m\vec{a} \Rightarrow q\vec{E} = m \frac{d^2 \vec{x}}{dt^2}$$

$$\text{Applied field: } \vec{E} = \vec{E}_0 e^{i\omega t} \qquad \text{Response: } \vec{x} = \vec{x}_0 e^{i\omega t}$$

$$-e\vec{E}_0 e^{i\omega t} = -m\omega^2 \vec{x}_0 e^{i\omega t}$$

$$\vec{x} = \frac{e\vec{E}}{m\omega^2}$$

$$\text{Polarization density: } \vec{P} = -ne\vec{x} = -\underbrace{\frac{ne^2}{m\omega^2}}_{\epsilon_0 \chi} \vec{E}$$

$$\epsilon_{rel} = (1 + \chi)$$

$$\epsilon_{rel}(\omega) = 1 - \frac{ne^2}{\epsilon_0 m\omega^2}$$

Dielectric function of the electron gas:

$$\epsilon_{rel}(\omega) = 1 - \frac{ne^2}{\epsilon_0 m^* \omega^2} \qquad \omega_p^2 = \frac{ne^2}{\epsilon_0 m^*}$$

$$\epsilon_{rel}(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \qquad \omega_p \text{ is called "plasma frequency"}$$

Magnitude of  $\omega_p$ ?

$$f_p = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\epsilon_0 m^*}} = ?$$

$$E_p = \hbar\omega_p = hf_p = ?$$

$$\lambda_p = \frac{c}{f_p} = ?$$

Maxwell, free space:

$$\frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E} \Rightarrow \vec{E} = \vec{E}_0 e^{i\omega \left( t - \frac{x}{c} \right)}$$

velocity:

$$c = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

Maxwell, medium:

$$\frac{1}{\mu_0} \frac{\partial^2 \vec{D}}{\partial t^2} = \nabla^2 \vec{E} \Rightarrow \vec{E} = \vec{E}_0 e^{i\omega \left( t - \frac{n_c x}{c} \right)}$$

velocity:

$$v = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n_c}$$

$\epsilon_r$  - relative permittivity

$n$  – refractive index (can be complex, real or imag)

# Optical constants

*E*, electric field

*k*: wave vector  
 $v = \omega/k = c/n_c$

$$E = E_0 e^{i(\omega t - kx)} = E_0 e^{i\omega \left( t - \frac{n_c x}{c} \right)} = E_0 e^{i\omega \left( t - \frac{nx}{c} \right)} e^{-\left( \frac{\omega \kappa}{c} \right) x}$$

$$I = I_0 e^{-\alpha x} \Rightarrow \alpha = \frac{2\omega \kappa}{c} = \frac{4\pi \kappa}{\lambda}$$

$$\epsilon = n_c^2$$

$$\epsilon = \epsilon_1 - i\epsilon_2; \quad n_c = n_r + in_i = n - i\kappa$$

$$\epsilon_1 = n^2 - \kappa^2; \quad \epsilon_2 = 2n\kappa$$

$$R = \left| \frac{n_c - 1}{n_c + 1} \right|^2 = \frac{(n - 1)^2 + \kappa^2}{(n + 1)^2 + \kappa^2}$$

$\epsilon$  = dielectric constant

$n$  = index of refraction

$\alpha$  = absorption coefficient

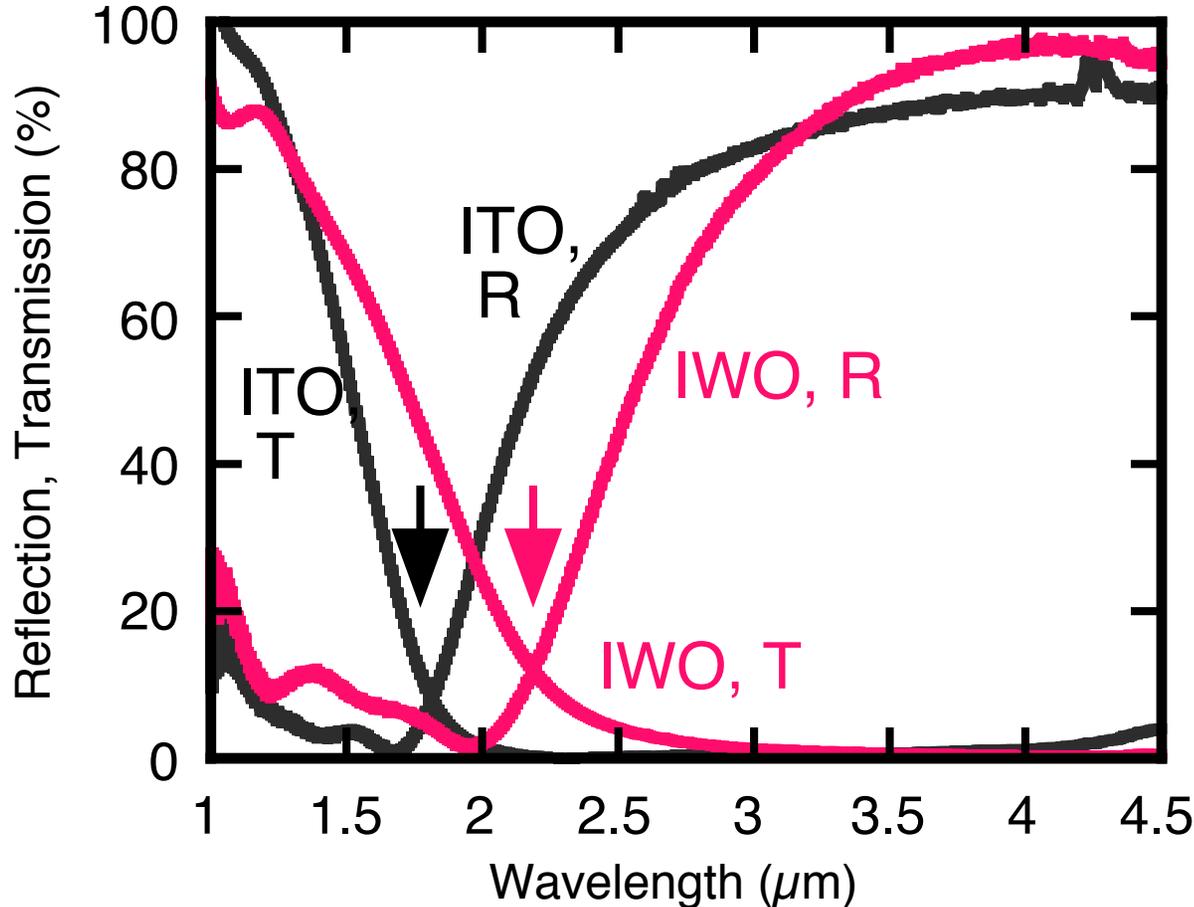
$\kappa$  = extinction coefficient

$R$  = reflection coefficient

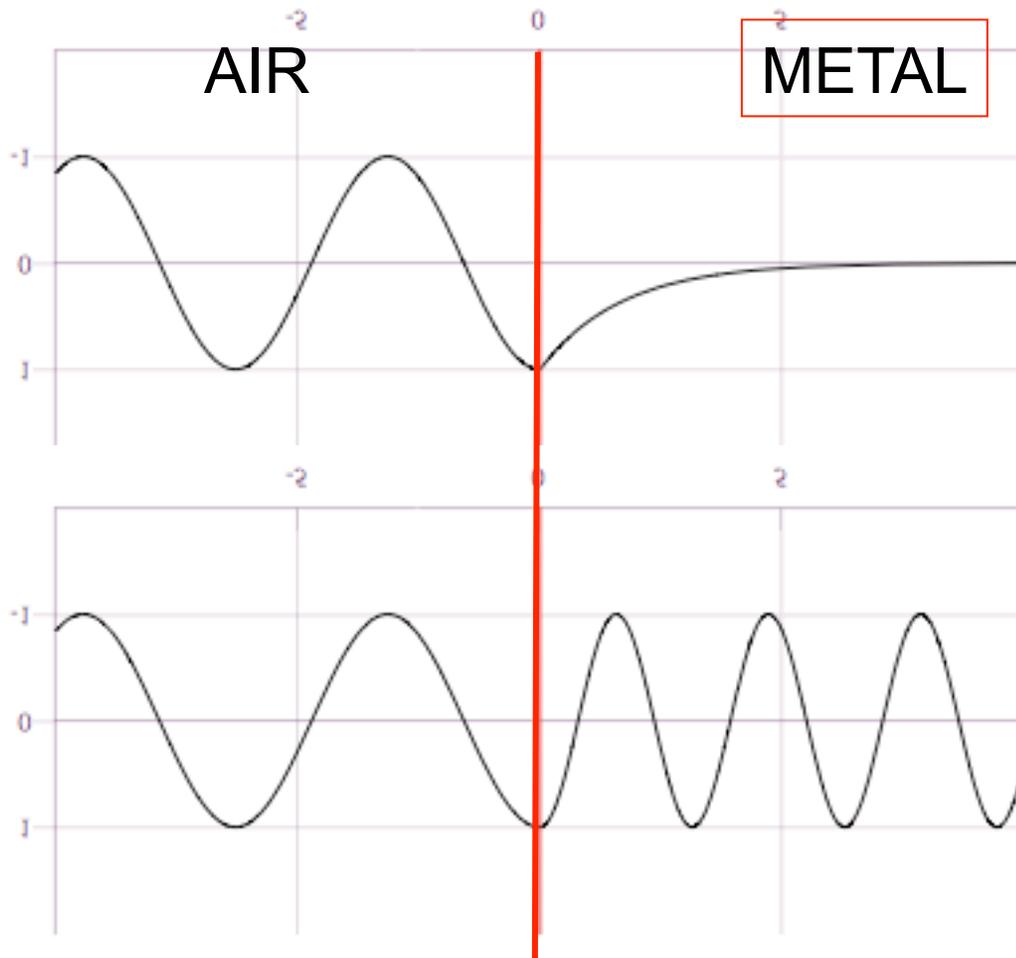
$$E = E_0 e^{i\omega\left(t - \frac{n_c x}{c}\right)} = E_0 e^{i\omega\left(t - \frac{nx}{c}\right)} e^{-\left(\frac{\omega\kappa}{c}\right)x}$$

$$\epsilon_{rel}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$\omega > \omega_p, \epsilon_{rel} > 0; n \text{ real} - \text{propagating wave}$   
 $\omega < \omega_p, \epsilon_{rel} < 0; n \text{ imag.} - \text{damped wave}$



"Transparent conductor"  $\text{In}_2\text{O}_3$ .  
 Arrows show plasma wavelengths for two different carrier concentrations.  
 (Which one is higher?)



$\omega < \omega_p$  No propagation;  
reflection

$\omega > \omega_p$  Transparent

Dielectric function of the electron gas: doped semiconductors

$$\vec{D} = \epsilon_0 \vec{\xi} + \vec{P}_{other} + \vec{P}_{free}$$

$$\vec{D} = \epsilon_0 \vec{\xi} + \vec{P}_{other} - \epsilon_0 \frac{ne^2}{\epsilon_0 m^* \omega^2} \vec{\xi}$$

$$\epsilon_{rel}(\omega) = 1 + \epsilon_{bound}(\omega) - \frac{ne^2}{\epsilon_0 m^* \omega^2}$$

Till now only the free electron part, but in semiconductors, dielectrics, molecular crystals, also other bound charge

The “background” dielectric constant is the one associated with the positive cores.

$$\epsilon_{rel}(\omega) = \epsilon_{r,opt} - \frac{\omega_p^2}{\omega^2} = \epsilon_{rel,opt} \left[ 1 - \frac{\tilde{\omega}_p^2}{\omega^2} \right] = \epsilon_{rel,opt} \left[ 1 - \frac{ne^2}{\epsilon_{rel,opt} \epsilon_0 m^* \omega^2} \right]$$