1. Plasma frequency and color.
   (a) Look up the crystal structure, atomic weight, the density, and the valence of Cu and Al.
       Assume a free electron model with the effective mass of electrons equal to the ordinary rest mass.
   (b) From (a), calculate values for the Fermi energy, $E_F$, the Fermi temperature, $T_F$, the Fermi wave vector, $k_F$, the density of states at the Fermi energy, $D(E_F)$, and the Fermi velocity, $v_F$. Put these values in context of other well-known physical quantities of the same type.
   (c) Calculate the plasma frequency based on the free electron theory. Based on your result, what color would you expect Al to be? Cu? What colors are they?
   (d) The band structures of Cu (below, left) and Al (below, right) hold a clue to the colors. Can you come up with an explanation for the colors of Cu and Al based on this information?

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2. (From Kittel Question 6.6, and similar to Sutton 27)
   (a) Use the equation $m^* \left( \frac{d\tilde{v}}{dt} + \frac{\tilde{v}}{\tau} \right) = q\xi$ for the electron drift velocity $\nu$ in the presence of an electric field $\tilde{E} = \tilde{E}_0 e^{i\omega t}$, to show that the *ac* conductivity of free electrons in a solid is

   $$\sigma(\omega) = \sigma_1 - i\sigma_2 = \sigma_0 \left( \frac{1 - i\omega\tau}{1 + \omega^2\tau^2} \right)$$

   where

   $$\sigma_0 = \frac{nq^2\tau}{m^*}.$$  

   (b) The figure on the right, taken from a 2006 research paper on a “heavy-fermion metal”, shows a direct measurement of the two components of the optical conductivity by microwave reflection. What parameters in the Drude model can you determine from this graph (give numerical estimates and explain).
3. (a) Start from the equations of motion for the Hall effect, and derive an expression for the Hall voltage

\[ V_H = \mu B I R_s \]

where \( \mu \) is the mobility, \( B \) the applied magnetic field, \( I \) the applied current, and \( R_s \) is the sheet resistance (what is this?).

(b) Your lab needs a magnetic field sensor and you're assigned the job of building it. You find a source of cheap Hall sensors, and the accompanying information states that these GaAs Hall effect devices "presented the following characteristics: \( R_s = 500 \Omega/\text{square}, \mu_n = 2640 \text{ cm}^2/\text{V s}, t = 0.25 \mu\text{m} \)."

You test the device and find that it will allow current passage of up to 10 mA.

(i) Generate a plot of Hall voltage vs. magnetic field for currents of 1 and 10 mA and see what voltmeter range you need for various fields. You'll have to decide what size fields this sensor will measure with generally available lab equipment. Be warned that, like in all real-world situations, there could be extraneous or unexpected information. For instance, you may have to become familiar with sheet resistance, which is measured in ohm/square.

(ii) What is the carrier density in this GaAs sensor?

4. This question concerns B-doped Ge; use a band diagram to illustrate your answers. Look up any needed material parameters (quote your sources).

(a) Describe the nature of the charge carriers in the system.

(b) Estimate the ionization energy of the defect state. What is meant by "ionization energy" in this context?

(b) Estimate the “Bohr radius” of the hydrogenic state of the impurity. How many atom sites feel the influence of this impurity?

(c) Read a little about impurity band conduction. What is it, and estimate the minimum impurity concentration that is required for this phenomenon.

5. ZnO has been growing in the research spotlight as evidenced by this graph:

(a) Why are researchers interested in ZnO and why \( p \)-ZnO in particular? (1 paragraph, not one page)

(b) What are possible native defects in ZnO, and are they donor or acceptor defects and why?

(c) What are possible substitutional dopants on the Zn and O sites respectively, that might produce \( p \)-conductivity and why?

(d) Provide some references to useful peer-reviewed articles that you used for this question.

6. A Si-Ge alloy is used to make thermoelectric (TE) devices that power instrumentation in spacecraft that are destined for deep space. The heat is generated by radioactive
elements, and the TE device converts the heat to electricity. The figure of merit for a TE device is $ZT = S^2\sigma T/\kappa$. ($T$= temperature of operation, $S$ = Seebeck coefficient, $\sigma$ = electrical conductivity). Please cite sources (preferably peer-reviewed) for your answers to the questions below if you did not come up with the answer by yourself.

(a) Is pure Si-Ge a semiconductor, a metal, an insulator, or something else?
(b) Why are $n$- and $p$-type Si-Ge alloys needed? What are the dopants used and explain why they are $n$- and $p$-type? What are typical carrier concentrations?
(c) Why is the thermal conductivity of Si-Ge relatively low compared to pure Si or pure Ge? What are the relevant thermal conductivities at the device operating temperature?
(d) Find typical values of $ZT$ at the operating temperature in these radioactive thermal devices.

7. I am going to add a phonon question or two to this set, but I will give that to you later. This will be the last homework set.