FINAL EXAM – JUNE 11, 2013

The following equations and constants may be helpful to you. You may fill the space below the periodic table on p2 with handwritten notes. A calculator with basic algebraic and trig capability is required. No text storage or internet access capability.
It is intended that answers to the descriptive questions be justified as deeply as possible given the time constraint of 10-15 minutes per question.

$$E(\vec{k}) = \sum_R e^{i\vec{k}\cdot\vec{R}} \langle \vec{0} | \hat{H} | \vec{R} \rangle \qquad E(\vec{k}) = \frac{\hbar^2 k^2}{2m^*} \qquad v(\vec{k}) = \frac{1}{\hbar}\nabla_k E(\vec{k})$$

$$D_\updownarrow(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \qquad dS = D(E)dE \qquad m^*(\vec{k}) = \frac{\hbar^2}{\nabla_k^2 E(\vec{k})}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\hbar = \frac{h}{2\pi} = 1.05\times10^{-34}\ \text{Js} = 6.58\times10^{-16}\ \text{eVs} \qquad hc = 1240\ \text{eVnm}$$

$$k_B = 1.38\times10^{-23}\ \text{J/K} = 8.6\times10^{-5}\ \text{eV/K} \qquad e = 1.6\times10^{-19}\ \text{C}$$

$$m_e = 9.1\times10^{-31}\ \text{kg} = 9.1\times10^{-28}\ \text{g} = 0.511\ \text{MeV/c}^2 \qquad N_A = 6.02\times10^{23}\ \text{atom/mol}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27\times10^{-24}\ \text{J/T} = 5.8\times10^{-5}\ \text{eV/T} \qquad \varepsilon_0 = 8.85\times10^{-12}\ \text{F/m}$$

$$\mu_0 = 4\pi\times10^{-7}\ \text{H/m}$$

$$f_{FD}(E,T) = \frac{1}{e^{(E-E_F)/k_BT}+1} \qquad f_{MB}(E,T) = Ae^{-E/k_BT} \qquad f_{BE}(E,T) = \frac{1}{e^{E/k_BT}+1}$$

$$\vec{g}_1 = 2\pi\frac{\vec{t}_2\times\vec{t}_3}{\vec{t}_1\cdot(\vec{t}_2\times\vec{t}_3)} \qquad \vec{g}_2 = 2\pi\frac{\vec{t}_3\times\vec{t}_1}{\vec{t}_1\cdot(\vec{t}_2\times\vec{t}_3)} \qquad \vec{g}_3 = 2\pi\frac{\vec{t}_1\times\vec{t}_2}{\vec{t}_1\cdot(\vec{t}_2\times\vec{t}_3)}$$

$$\sigma = \frac{ne^2\tau}{m^*} \qquad R_H = \frac{1}{nq} \qquad \omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m^*}}$$

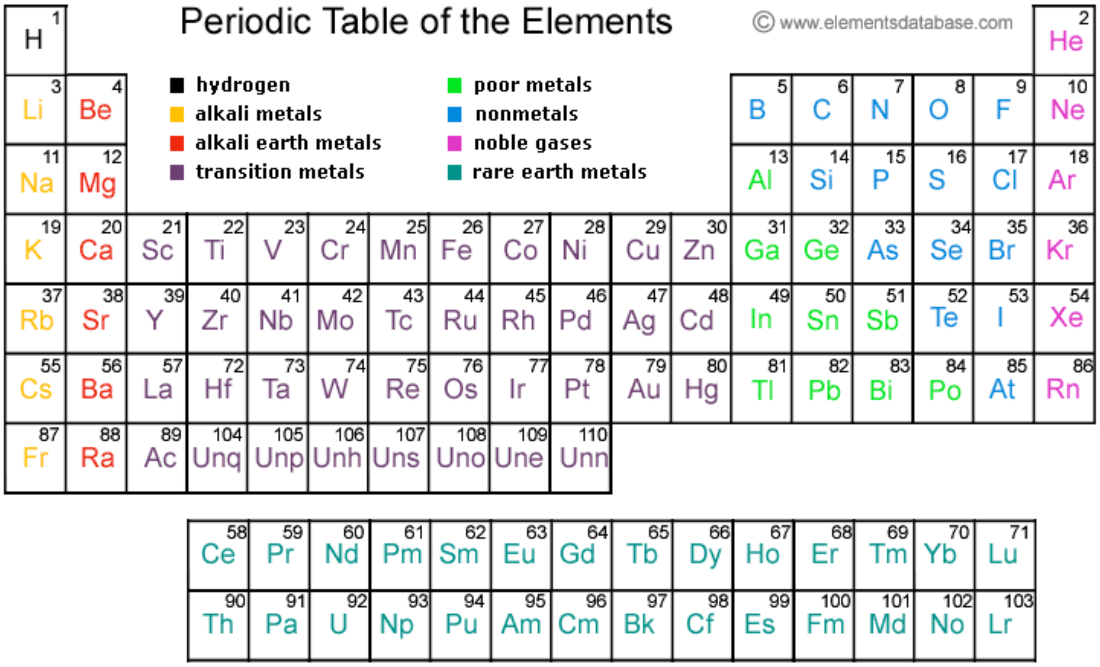
$$\Delta V = S\Delta T$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad \vec{D} = \varepsilon_0(\vec{E} + \vec{P}) \qquad \chi = \frac{M}{H} \approx \frac{\mu_0 M}{B}$$

$$C_V = \frac{\pi^2}{3} D_\updownarrow(E_F) k_B^2 T \qquad \frac{K_{el}}{\sigma T} = L = 2.45\times10^{-8} W\Omega/K^2$$

$$n_c = n - i\kappa \qquad \varepsilon = \varepsilon_1 - i\varepsilon_2 \qquad n^2 = \varepsilon$$

FINAL EXAM – JUNE 11, 2013



You may fill the space below with handwritten notes.

 This exam is for the use of students in PH575 in S2013 only. Do not distribute in any form, now or in the future.

FINAL EXAM – JUNE 11, 2013

1. **Short answers – general [15 points]**
   a) What is the size is the band gap in the semiconductors Si and GaAs? Is the gap direct or indirect?
   b) Crystalline La_2CuO_4 is an insulator at room temperature. If some La atoms are replaced by Sr atoms to make, for example, $La_{1.9}Sr_{0.1}CuO_4$, the material becomes slightly conductive. What sign of charge carriers do you expect, and why?
   c) Sketch the band structure of an insulator, a metal and a semi-metal, emphasizing the important features.
2. **Metal optics [15 points]**
   Discuss how the presence of free carriers in a metal leads to large reflectivity at certain photon energies and estimate the plasma frequency in a typical metal. Why is copper reddish in color?
3. **Semiconductor optics [15 points]**
   Explain why GaAs is a better choice for a light emitting diode than Si. Your explanation should include quantitative statements and order-of-magnitude calculations to justify them.
4. **Metal transport [15 points]**

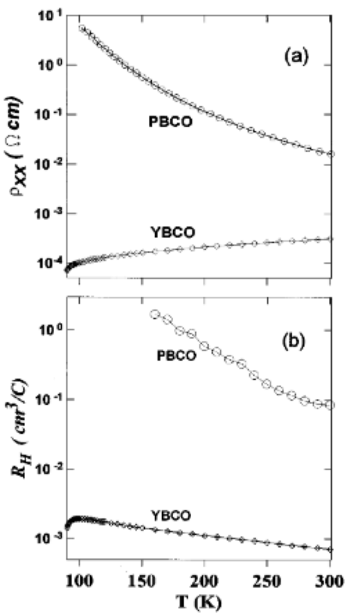


FIG. 2. (a) Resistivities as function of temperature for a PBCO film and a YBCO film. (b) The Hall coefficients as a function of temperature for a PBCO film and a YBCO film.

$YBa_2Cu_3O_7$, also abbreviated YBCO, is a famous material that is superconducting below the relatively high temperature of 92K. This problem concerns its normal (*non*-superconducting) state properties.

At the left is a figure from a paper* on the normal state properties of YBCO.
(a) Calculate the carrier density in YBCO at 100K from the lower graph.
(b) Using the information from both graphs, calculate the scattering time in ps for carriers at 100K and estimate the mean free path in nm.
(c) Are the carriers in YBCO electrons or holes? How do you know from the information in the figure?

*[Wang *et al*. PRB 56 (1997) 6231].

5. **Semiconductor transport [15 points]**
   (a) Draw energy diagrams that illustrate (i) an intrinsic semiconductor, (ii) an extrinsic, *n*-type, non-degenerate semiconductor and (iii) an extrinsic, *p*-type, non-degenerate semiconductor. Label important energies and give values of energies and/or energy differences that are typical for silicon.
   (b) Calculate the *np* product in non-degenerate Si and show that it is independent of the Fermi level/chemical potential. Explain any approximations.
   $$\int_0^\infty x^{1/2} e^{-x}\, dx = \frac{\sqrt{\pi}}{2}.$$

FINAL EXAM – JUNE 11, 2013

6. Phonons [15 points]

a) Fig. 6 depicts the phonon dispersion curve for MgO, whose structure is cubic as shown in the inset. Focus only on the Γ-X panel of Fig. 6. What is the nature of the phonons at the points indicated? Answer this question by using words and sketches to elucidate the difference between

i. (1 and 2) on the one hand and (3 and 4) on the other

ii. 1 and 2

iii. 3 and 4

b) How are phonon dispersion curves measured experimentally?

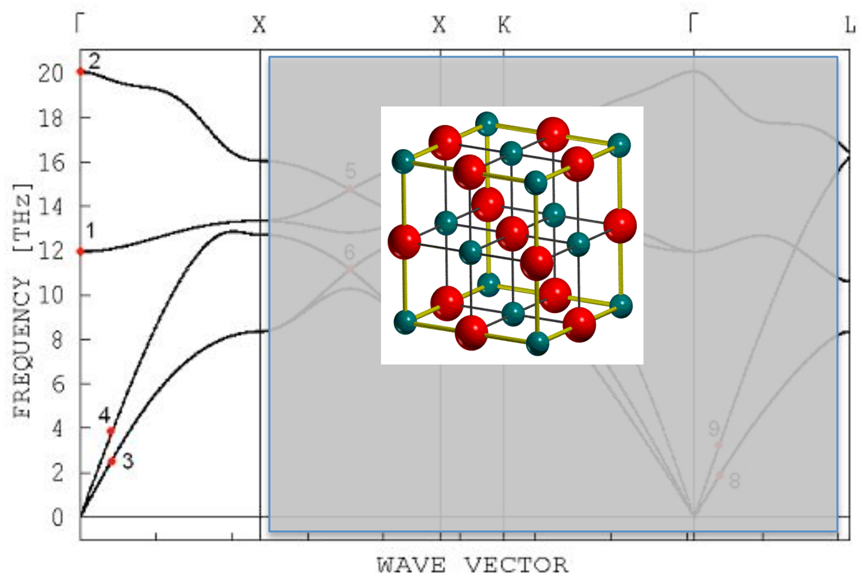
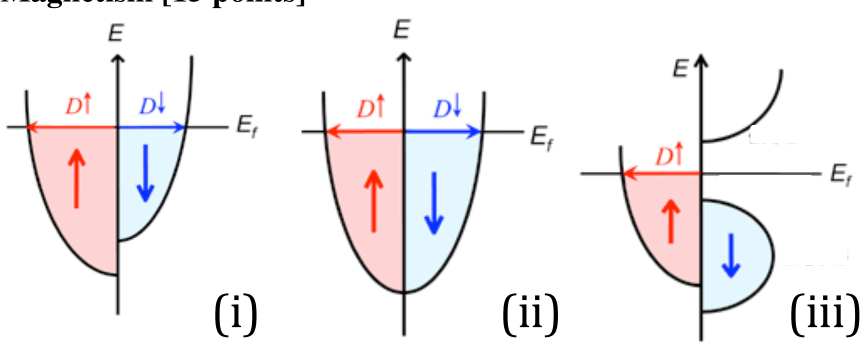


Fig. 6: Phonon dispersion curve of MgO with (inset) crystal structure of MgO. Red=O; Green = Mg

7. Magnetism [15 points]



The diagrams above depict the density of states in zero applied magnetic field (horizontal axes) for up-spin, $D\uparrow$, and down-spin, $D\downarrow$, as a function of energy (vertical axis) for three different materials. Discuss the magnetic behavior of each material and explain why you draw these conclusions.

