Central Forces Homework 3  
Due 3/3/10

REQUIRED:

1. NASA has launched a satellite into a circular orbit around the earth. The scientists are going to change the orbit slightly by firing the engines briefly, applying a small impulse to the satellite. One scientist says that it doesn’t matter if the impulse is applied in a direction tangential to the motion or perpendicular to the motion, as both will have the same effect. Another scientist disagrees, thinking the situations will have two different effects. Which scientist would you side with, and why?

2. Consider the motion of a hockey puck of mass $m$ on a perfectly circular ice rink with radius $a$. You may neglect the effects of friction. The sides of rink are of height $h$. Think of the "rink" as a circular depression in a very smooth ice field. The transition from the base of the rink to the top must be gradual enough that a moving puck would be able to slide up the ice walls and maintain contact with the ice, but with a small enough and appropriately-shaped puck, this transition could be made as sharp as desired. For calculation purposes, it is easiest to idealize to a very sharp transition.

   (a) Draw a sketch of the potential for this system. Set the zero of potential energy at the top of the walls. Why is it important that the rink is circular and not some other shape?

   (b) Situation 1: the puck is initially moving radially outward from the exact center of the rink. What minimum velocity does the puck need to escape the rink? (This is not meant to be a trick question. Do not worry about the puck getting caught in the corner. You may consider the sides of the rink as angled very, very slightly outward and connected smoothly to the surface of the rink. You may also consider the puck to have no sharp corners.)

   (c) Situation 2: a stationary puck, at a distance $\frac{a^2}{2}$ from the center of the rink, is hit in such a way that its initial velocity $\vec{v}_0$ is perpendicular to its position vector as measured from the center of the rink. What is the total energy of the puck immediately after it is struck?

   (d) In situation 2, what is the angular momentum of the puck about the origin immediately after it is struck?

   (e) Draw a sketch of the effective potential for situation 2.

   (f) In situation 2, for what minimum value of $\vec{v}_0$ does the puck just escape the rink?

3. The figure below shows the position vector $\vec{r}$ and the orbit of a “fictitious” reduced mass, $\mu$, traveling around the center-of-mass at the origin.
(a) Assuming that $m_2 = m_1$, draw on the figure the position vectors for $m_1$ and $m_2$ corresponding to $\mathbf{r}$. Also draw the orbits for $m_1$ and $m_2$. Describe a common physics example of central force motion for which $m_1 = m_2$.

Note that in this view, ALL vectors are drawn from the origin. You will have to reconcile this view with the picture where $\mathbf{r}$ points from the tip of $\mathbf{r}_1$ to the tail of $\mathbf{r}_2$.

(b) Repeat the previous problem for $m_2 > m_1$. 