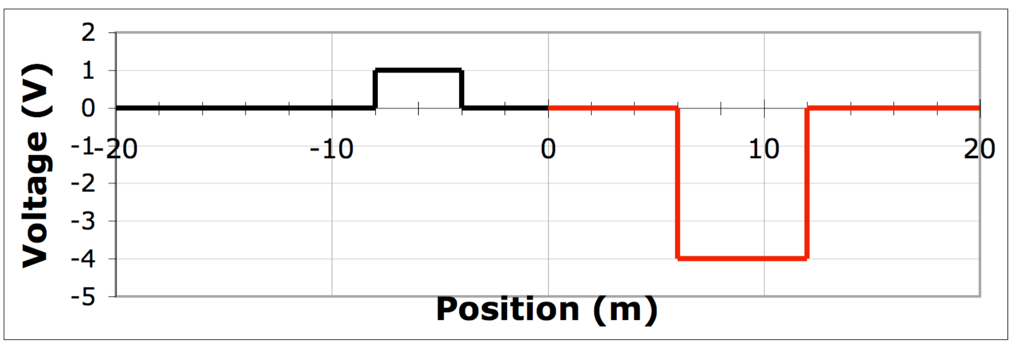
- Work with symbols first, and put in numbers (with units) as a last step.
- Always explain your reasoning to demonstrate insight. Yes/no answers, statements without explanation, and answers without working shown, are not distinguishable from guesses.
- Answer all questions. The best strategy is to attempt every problem.
- Sketches should be large, clear, and neatly drawn, with labels and indicators highlighting the points to which you wish to draw attention. Sketches that are untidy, smudged or otherwise of poor quality will be disregarded.

[1] [25 points] A very long coaxial cable of characteristic impedance 75 Ω (on the left) is joined to another very long coaxial cable of unknown characteristic impedance (on the right) at the position $x = 0$. The resistance of both cables is negligible, and they have the same capacitance per unit length. A voltage pulse of rectangular spatial profile is incident from the left (the 75-Ω cable) and encounters the discontinuity. The graph below displays the voltage difference between the inner conductor and the outer conducting shield of the cables <u>as a function of position</u> at some later time *after* the pulse has encountered the discontinuity. (Note that this is a spatial profile, NOT an oscilloscope trace, which would show the pulses at a function of time).

(a) What is the characteristic impedance of the second cable?
(b) Describe the original pulse (height, polarity, length), showing qualitative and quantitative reasoning.



[2] [20 points] A single-frequency harmonic wave in a rope stretched in the x direction is viewed at time $t = 0$. The waveform at $t = 0$ can be represented by: $\psi(x,0) = A_0\cos(kx+\delta)$ where A_0 and k and δ are known constants. The transverse (or material) velocity of each point in the rope at that time is $\frac{\partial\psi}{\partial t}(x,0) = \omega A_0\sin(kx)$ where ω is another known constant.
(a) Write an expression for the general waveform as a function of space and time in terms of the known quantities.
(b) Explain what sort of waveform it is for the cases δ = 0 and δ = π/2.

[3]. [30 points] A particle of mass m in an infinite square well (potential energy is zero for $0 < x < L$ and infinite elsewhere). For this Hamiltonian, the eigenvalues of the Hamiltonian operator are labeled E_n. The normalized eigenstates are labeled $\varphi_n(x)$.

(a) Write down the expressions for E_n and $\varphi_n(x)$ in terms of the given parameters for the problem. (For this part, you do not need to derive the expressions, but if you are uncertain, you should show your reasoning.)

By some means, many identical states of this system are prepared, and at time $t = 0$, energy measurements are made. The results are that 36% of measurements yield E_1 and 64% yield E_2.

(b) What is the expression for normalized original state at $t = 0$? Explain.

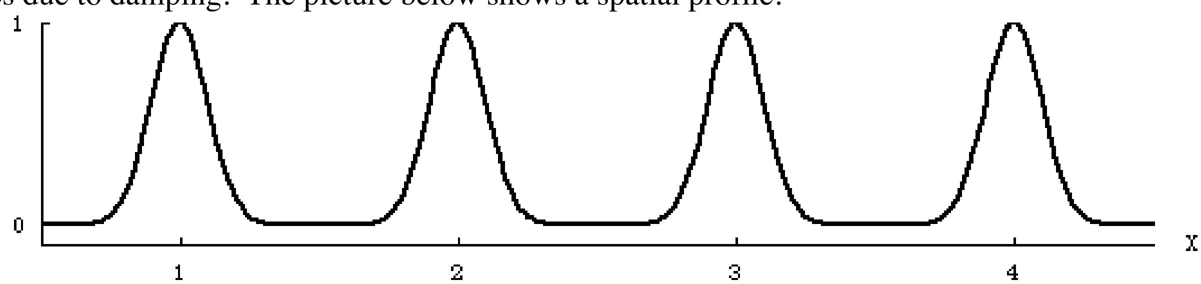
(c) What is the probability density of this particle at some later time t? Simplify this expression until there are no imaginary parts in it (remember the probability density is real).

(d) Is the state an eigenstate at $t = 0$? At later times t? Why or why not?

(e) Is the state normalized at this later time?

(f) What is the average value of the position of the particle $\langle x \rangle$ as a function of time? Do not evaluate non-obvious integrals, but do indicate whether they are zero or non-zero.

[4] [15 points] Gaussian pulses evenly spaced by $x_0 = 1$m propagate with constant speed with no dispersion in an infinite rope of mass density μ under tension τ. At time $t = 0$, each pulse has the form $\psi(x) = \sum_n A e^{-\frac{(x-nx_0)^2}{2\sigma^2}}$ where the nx_0 represent the positions of the peaks (n integer). Assume no energy losses due to damping. The picture below shows a spatial profile.



Reproduce the picture in your blue book, and explain:

(a) Where (at $t = 0$) the potential energy density is maximal and minimal and why.

(b) Where (at $t = 0$) the kinetic energy density is maximal and minimal and why.

(c) What happens to these maxima and minima as time goes on? Why?