

PH424/524: 1-DIMENSIONAL WAVES

PH424/525: 1-Dimensional Waves
Homework #2

Winter 2009

Assigned Friday 02/06/09

Q1 (lab write up) – due Wednesday 02/11/09

2,3,4,5 - Due Friday 02/13/09

Do not leave Q2 and Q3 until Wednesday. It will take you longer than 2 days to solve them. Work on them all week.

1. Co-ax cable lab: reflection, transmission, propagation, attenuation of waves

This part of the homework is due on Wednesday.

In a nutshell, the experiment was this: You generated a voltage pulse at one end of a coaxial cable. You measured its height (and polarity) at this end, and again when it returned. (Some also measured it at the other end of the cable.) You did this for several values of a terminating resistance. Can you explain why the pulses you measured have the size and sign that they do? You also measured the speed of propagation.

At minimum, you must present your results and a sensible model. Describe the experiment clearly but succinctly, being careful to include a circuit diagram and a sketch of the oscilloscope traces that clearly indicate the quantities you measured. You should present your results in both graphical and tabular form, paying attention to significant figures and to presenting clear, informative and sensible graphs. Present the model that you propose to explain your results. Your description should clearly demonstrate that you understand the concepts of wave propagation, impedance mismatch, matching termination, and damping. Statements of the sort, "In class we showed that," or, "The theory says that ...," do not demonstrate your understanding; they demonstrate mine.

Some thoughts:

- Show that the damped 1-d wave equation is relevant to your system.
- Identify the parameters that appear in the equation. In principle, you need values for all the parameters, but you should be able to show that the solution to the equation can be written in

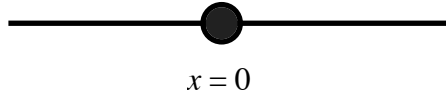
the form $\psi_V(x,t) = e^{i\left(-\omega t + kx + i\frac{R_0}{Z}x\right)}$ where R_0 is the resistance per unit length of the coax cable and Z is its impedance. Thus you should need to measure only the total resistance of the cable, its length and its impedance.

- Demonstrate that you understand what happens to the wave at the cable boundary by calculating the voltage reflection coefficient.
- Put it all together to calculate the pulse height ratio(s) that you measured.
- Include the report of the measurement of the propagation speed.
- Optional: if you did the extra depth part of the lab to look at standing waves, you may report on that.

This part of the homework is due on Wednesday.

2. Reflection and transmission of waves from a point mass on a rope:

Consider a rope has a uniform mass density μ and has a mass (a ball bearing or bead or something similar) m attached to it at position $x = 0$. A harmonic, single frequency, traveling wave is incident from the left. You are to discuss, qualitatively and quantitatively by following (a) – (f), the reflection and transmission of the wave. (We are to understand that this mass is a point mass – that is, its mass is concentrated at a true mathematical point.)



(a) First, think about the limits your answer should yield. What would happen if the mass were not there (*i.e.* $m = 0$)? What if the mass were very heavy?

(b) Second, set up the general problem ...

On the left, there is an incident wave (with unit amplitude) and a reflected one $\psi_{left}(x,t) = \psi_{inc}(x,t) + \psi_{ref}(x,t)$. On the right there is a transmitted wave $\psi_{right}(x,t)$. Write down an expression for $\psi_{right}(x,t)$ and $\psi_{ref}(x,t)$. It is understood that when we actually plot the displacement, the *real* part of these expressions will be plotted.

(c) What can you say about the frequencies and k vectors of the incident, transmitted and reflected waves, and why is this so?

(d) Consider what conditions must be placed on the displacement $\psi(0,t)$ at $x = 0$, and on the slope $\left. \frac{\partial \psi}{\partial x} \right|_{x=0}$. (It is this second part that is different from what we did in class ... you have to apply Newton's law to the *mass*).

(e) Solve your equations to show that $R = \frac{-i\varepsilon}{2 + i\varepsilon}$; $T = \frac{2}{2 + i\varepsilon}$, where $\varepsilon = \frac{m\omega^2}{k\tau}$ and τ is the tension in the rope. Show that the phase angle of the reflected and transmitted waves changes in general. Was your intuition in (a) justified?

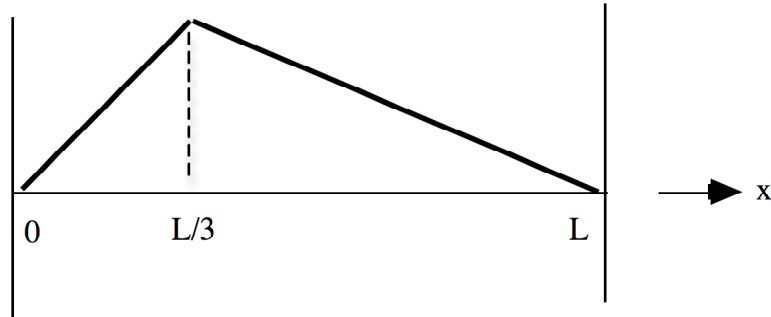
(f) Plot and animate the string profile for various values of ε (small intermediate, large) and describe what you see. In particular for large values of epsilon, can you see when the discontinuity in the slope at $x = 0$ is greatest?

Maple notes: I found it most useful to plot a little more than one wavelength on either side of zero and to animate for a time about 2 periods of the oscillation. I also increased the number of points and used *style=point* in the animate argument, because the point style seemed better at highlighting discontinuities in the slope, which is what you want to see. You may also wish to use *axes=boxed*; this displace the axes from the interesting region, and does not obscure the view.

(g) An extra challenge is to consider conservation of energy (not required, but can be considered for extra credit).

3. Analysis of normal modes of a string:

A string (mass per unit length μ , under tension T) is anchored at $x = 0$ and $x = L$. It is then displaced so that it has the following profile at $t = 0$, and its transverse velocity is zero. Give a complete mathematical description of the motion of the string, including a Maple or other animation (printouts of a few frames is fine).



Here is a guide to help you:

- (a) Write this wave form very generally as a superposition of the allowed modes of vibration of this string.
 - (b) Make sure that the *time* dependence satisfies what you know about the velocity at $t = 0$, and that the *space* dependence satisfies what you know about where the string is anchored.
 - (c) Turning to the space dependence only, find out what contribution each "normal mode" makes at $t = 0$. (Fourier series will be useful here even though this is not a periodic function – why?)
 - (d) Now return to the full wave function, including the time dependence. Plot the function (you need to include enough terms to form a reasonable approximation to the function, but not too many that Maple dumps you out of the system), and animate it. Describe what you see.
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 5. Water waves that propagate in the deep ocean obey the following dispersion relation: $\omega = \sqrt{gk}$ where g is the acceleration due to gravity and as usual ω and k are the angular frequency and wavevector of the wave.
 - (i) Sketch this dispersion relation and on the same plot, graph a dispersion relation for a non-dispersive medium ~~with the same wave velocity at low frequencies.~~
 - (ii) On a new plot, sketch the group velocity of a wave packet in both media.
 - (iii) describe qualitatively what happens to a wavepacket that propagates in each medium.