Winter 2009
Assigned Sunday 02/01/09
Problems 1, 2, 3 due Wednesday 02/04/09, 5pm
Problems 4, 5, 6, 7 due Friday 02/06/09, 5pm

Structure of this homework: Problems 1-3 are fairly mechanical, designed for routine practice. They are due Wednesday. Problems 4-6 require a bit more thought, and are practice for material we discuss later in the week. You should work on them all through the week. You may have to read ahead to get an idea of how to solve these problems.

Reminder: The paradigms courses are 2-unit courses, but they meet for only three weeks in the term. Thus they are the time equivalent of two 3-unit courses running for a full term. *The time equivalent is thus 6 units or 2 courses.*

Animating functions of two variables: You will find it very useful to be able to view functions of 2 variables with as an animated sequence. Maple and Mathematica and other programs are very good at this. So I recommend you become familiar with this animated plotting capability.

CAVEAT: Maple keeps changing the syntax for the "animate" command! Figure out the best way to use it! If you use Maple, here is one way to do it that worked in Maple 7, 8, and 9.5. There are other ways to define functions, too.

In order for the `animate` function to work in Maple, you must first execute the command

```
> with(plots):
```

You need to define a function of two variables, say *g(x,t)*. I use this method:

```
>g:=(x,t)->A*sin(k*x-omega*t);
```

This reads "*g* is defined such that (*x*,*t*) maps onto *A**sin(*k**x-omega**t*)". If you execute the command `g(x,t);` Maple will return $A\sin(kx-\omega t)$. The `animate` command creates several frames, each containing a plot of *g* as a function of the space variable *x* at a fixed time - a different time for each frame. The frames are then played back like a movie, using the tape recorder buttons that appear when you click on the plot. The basic syntax is

```
>animate(g(x,t),x=0..10,t=1..2);
```

When you execute the command, the first plot will be of the function *g* between the positions (1st variable) $x = 0$ and 10 at time (2^{nd} variable) $t = 1$. When you hit the "play" button on the top ruler in Maple, subsequent frames show the functions at later times (2^{nd} variable) up to $t = 2$. There are many options - defining the number of time steps and formatting the plot, for example – look these up in the help files.

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REQUIRED

1. (a) Plot two spatial cycles of the following waveform at each of t = 0, 0.25, 0.5, 1, 0.75, 2 seconds:

$\psi(x,t) = A\cos(-kx + \omega t + \pi/4)$ for A = 1 unit; $k = 2\pi\ \mathrm{m}^{-1}$; $\omega = \pi\ \mathrm{rad\ s}^{-1}$

(b) What is the wavelength of the disturbance?

(c) What is the amplitude?

(d) Which direction does the wave travel and with what speed? Which direction does it travel if you change the sign of ω? Of k? Of both?

(e) Focus on the position x = 0 m. At what rate is the quantity represented by ψ changing at each of the times listed above? Discuss the units of A and give examples from your experience.

2. Write down a sinusoidal waveform $\psi(x,t)$ that has the following properties:

(a) Amplitude 2m, wavelength 10m, travels to the right at 1m/s, ψ = 2m at x = 5m and t = 0s.

(b) Standing wave, amplitude 5 m, period 1 s, wavelength 1 m that is momentarily flat at t = 0s.

3. Describe the following waveforms in words (waveform, period, phase angle, direction & speed of travel … *etc*.). Demonstrate whether they are, or are not, solutions to the non-dispersive wave equation $\frac{\partial^2}{\partial t^2}\psi(x,t) = v^2\frac{\partial^2}{\partial x^2}\psi(x,t)$.

(a) $\psi(x,t) = 4\cos(4\pi x + 3\pi t) - 4\sin(4\pi x + 3\pi t)$

(b) $\psi(x,t) = 3\cos(2\pi x)\sin(\pi t)$

(c) $\psi(x,t) = 3e^{-\alpha x}\cos\left(\frac{2\pi}{3}x - \pi t\right)$

4. Standing Waves in a rope:
Tabulate the results from the in-class lab on Tuesday and plot the dispersion relation for the rope. Discuss the dispersion relation and obtain the phase velocity of waves in the rope. Is this consistent with the tension in the rope and the mass density of the string?

5. Traveling and Standing Waves:
"A standing wave is the sum of two traveling waves propagating at the same speed in opposite directions".
By explicitly adding the waves described by $\psi_1 = A_1\sin(kx - \omega t)$ and $\psi_2 = A_2\sin(kx + \omega t)$, decide whether this is a true statement, and prove it analytically. (You could look at an animated function to try some possibilities, but you need an analytical proof.)

6. **Main 9.9**

7. **Propagation of electromagnetic waves in vacuum:**
(This is relevant for the lab exercise. This is really a 3-dimesnsional example because the wave has components in two spatial dimensions and propagates in the third. However, the components are described by the 1-d wave equation. You will meet this particular example again in the E&M and optics courses.)
Show that in a medium in which there is *no* free charge and *no* free current, electromagnetic waves propagate with a velocity of propagation $v = \frac{1}{\sqrt{\mu\varepsilon}}$, where ε is the permittivity and μ the permeability of the vacuum. (In vacuum, $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$, in which case the velocity has the special symbol *c*.)

   The following guides you through the problem. The basic approach is to show that the electric and magnetic field vectors separately obey the 1-d non-dispersive wave equation, and identify the velocity.

   To begin this problem, think back to previous courses -- *all* electric and magnetic fields obey the Maxwell equations, and you will find them in GEM. PH320 and PH422 dealt with electro*statics* and magneto*statics*, but you will recall from PH213 that there are *dynamic* (time varying) terms in two of the equations, describing motional EMF (Faraday's Law) and a "displacement current", respectively. These are crucial for discussing propagation of electromagnetic waves.

   Write down the 4 Maxwell equations with the source (current and charge) terms set to zero. This problem is most easily done with the equations in *differential* form, not *integral* form (you must relate derivatives to each other). The equations are coupled -- *E* and *B* appear together in some. You must manipulate the equations to obtain equations in *E* alone and *B* alone that relate the second time derivative to the second space derivative. Find the constant that relates the two and hence identify the velocity. [Hint: you will have to look up a vector identity for the "curl of the curl" of a vector: $\nabla \times \nabla \times \vec{A} = ?$]

EXAMPLES FOR EXTRA PRACTICE

Main 9.1 through 9.10; 9.13; 9.14