PH424 - 1-DIMENSIONAL WAVES Proposed EQN SHEET for FINAL EXAM

- Work with symbols first, and put in numbers (with units) as a last step.
- Always explain your reasoning to demonstrate insight. Yes/no answers, statements without explanation, and answers without working shown, are not distinguishable from guesses.
- Sketches should be large, clear, and neatly drawn, with labels and indicators highlighting the points to which you wish to draw attention. Sketches that are untidy, smudged or otherwise of poor quality will be disregarded.
- Calculators: Usually not allowed. If they are, it is for simple number crunching only – no graphing, integrals, text storage etc. Have a simple calculator on hand.
- Make suggestions for other quantities you think are needed on this sheet.
- Check for typos and report them.

$$\frac{\partial^2\psi(x,t)}{\partial t^2} + \Gamma\frac{\partial\psi(x,t)}{\partial t} = v^2\frac{\partial^2\psi(x,t)}{\partial x^2}$$

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \hat{H}\psi(x,t) \quad \text{where} \quad \psi(x,t) = \sum_n \varphi_n(x)e^{-iE_nt/\hbar} \quad \text{and} \quad \hat{H}\varphi(x) = E_n\varphi_n(x)$$

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}|\psi(t)\rangle \quad \text{where} \quad |\psi(t)\rangle = \sum_n |\varphi_n\rangle e^{-iE_nt/\hbar} \quad \text{and} \quad \hat{H}|\varphi_n\rangle = E_n|\varphi_n\rangle$$

$$\langle Q\rangle = \langle\Phi|\hat{Q}|\Phi\rangle = \int_{-\infty}^{\infty}\Phi^*(x)\hat{Q}\Phi(x)dx$$

$$j = \frac{\hbar}{2im}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right]$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$$Z = \frac{F_{appl}}{v_{trans}} \qquad w(x,t) = \frac{Z}{2v}\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + \frac{Zv}{2}\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2$$

Uniform string:

$$v = \sqrt{\frac{\tau}{\mu}} \qquad Z = \sqrt{\tau\mu} \qquad Z = \sqrt{\tau\mu}\sqrt{\frac{\Gamma}{2\omega}}\left[1 - i\right]\text{(with attenuation)}$$

Cable electromagnetic waves:

$$v = \sqrt{\frac{1}{LC}} \qquad Z = \sqrt{\frac{L}{C}} \qquad Z = \sqrt{\frac{L}{C}}\left[1 - i\frac{R}{2\omega L}\right]\text{(with attenuation)}$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}g(k)e^{ikx}dk \Leftrightarrow g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-ikx}dx$$

Constants:

$h = 6.6 \times 10^{-34} Js = 4.1 \times 10^{-15} eVs$

$\hbar = 1.1 \times 10^{-34} Js = 6.5 \times 10^{-16} eVs$

$m_{electron} = 9.1 \times 10^{-31} kg = 0.5 MeV / c^2$

$m_{proton} = 1.7 \times 10^{-27} kg = 1.0 GeV / c^2$

$c = 3.0 \times 10^8 m / s = 3.0 \times 10^{10} cm / s$

$hc = 1240\ eV\ nm$

Integrals

$$\int x^2 \sin^2(ax)dx = \frac{x^3}{6} - \left[\frac{x^2}{4a} - \frac{1}{8a^3}\right]\sin(2ax) - \frac{x\cos(2ax)}{4a^2}$$

$$\int x \sin^2(ax)dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

Trig identities

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos^2 A + \sin^2 A = 1$

$\cos^2 A - \sin^2 A = \cos 2A$

Rope/cable analogies (from Main 10.3)

Rope		Cable	
Displacement	ψ	Charge	ψ
Transverse velocity	$\frac{\partial \psi}{\partial t}$	Current	$\frac{\partial \psi}{\partial t}$
Slope of rope	$\frac{\partial \psi}{\partial x}$	Charge density	$\frac{\partial \psi}{\partial x}$
Mass/length	μ	Inductance/length	L_0
Tension	τ	(Capacitance/length)$^{-1}$	$1/C_0$
Resistance/length	R_0	Resistance/length	R_0
PE density	$\frac{1}{2}\tau\left(\frac{\partial \psi}{\partial x}\right)^2$	electric energy density	$\frac{1}{2C_0}\left(\frac{\partial \psi}{\partial x}\right)^2$
KE density	$\frac{1}{2}\mu\left(\frac{\partial \psi}{\partial t}\right)^2$	Magnetic energy density	$\frac{1}{2}L_0\left(\frac{\partial \psi}{\partial t}\right)^2$
Impedance (no damping)	$\sqrt{\tau\mu}$	Impedance (no damping)	$\sqrt{\frac{L_0}{C_0}}$
Velocity	$\sqrt{\frac{\tau}{\mu}}$	Propagation velocity	$\sqrt{\frac{1}{L_0 C_0}}$