PH421/521 - Paradigms in Physics: Oscillations — Fall 2009
Homework #2 — Due Monday 11/02/09

1. A. PRACTICE

Read Main and Taylor on the topic of damped and driven oscillators. You may wish to check out this website (which I'll also show in class). Be warned that at first sight, it is very busy and confusing! But after you study it for a while, you'll realize it has a great deal of good information about the circuit we'll study.
http://www.ngsir.netfirms.com/englishhtm/RLC.htm

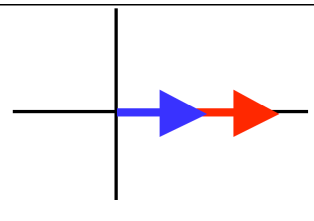
Taylor 5.29, Main 3.1 – 3.8

B. PROBLEMS

1. Taylor, problem 5.26

2. The current in a circuit is represented by the complex number $I(t) = I_0 e^{i(\omega t+\phi)}$. Current cannot be complex, but we represent it this way with the understanding that the real part will represent the actual current in the circuit.
(a) Represent the current on an Argand diagram at t=0 and your choice of ϕ.
(b) Represent $\dot{I}$ on the same diagram at t=0 and the same choice of ϕ. What is the phase of I relative to $\dot{I}(t)$ at any time?
(c) Represent the charge q flowing in that circuit on the same diagram at t = 0 and the same choice of ϕ. What is the phase of I relative to $q(t)$ at any time?

3. A series LRC circuit is driven by a sinusoidal voltage that by convention we write as $V_0\cos(\omega t)$. Draw phasor diagrams representing the driving voltage and each of the voltages across the capacitor V_C, resistor V_R, inductor V_L in a driven LRC circuit for three different cases: (1) $\omega \ll \omega_0$, (2) $\omega = \omega_0$ (resonance frequency), (3) $\omega \gg \omega_0$. And example is done for you. For each of the 3 frequency regimes, explain whether and why the circuit as a whole behaves predominantly as a resistor, capacitor, or inductor.

	V_R	V_L	V_C
$\omega \ll \omega_0$			
$\omega \approx \omega_0$			

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$\omega >> 0$			

4. A forced oscillator characterized by a damping β and frequency ω_0 has a generalized displacement response $x(t) = \dfrac{F_0 / m}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2\beta^2}} \cos\left(\omega t + \delta\right)$.

   This function is of the form $x(t) = A\left(\omega\right)\cos\left(\omega t + \delta\right)$, where $A(\omega)$ measures the amplitude of the response and it is a (strong) function of frequency.
   (a) Sketch the form of $A(\omega)$.
   (b) Find the exact frequency at which the amplitude $A(\omega)$ is maximal.
   (c) If the damping is light ($\omega_0\beta << 1$), show that the resonance frequency differs from ω_0 by a term of order β^2.
   (d) In the light damping approximation, what is the value of the maximum value of $A(\omega)$?
   (e) In the light damping approximation, find the "width" of the $A(\omega)$ function, by finding the two frequencies at which $A(\omega)$ drops to $\frac{1}{\sqrt{2}}$ of its maximum. Be careful not to make such a stringent approximation that the width is zero!
   (f) Define the difference between these two frequencies as $\Delta\omega$, and find an expression for $Q \equiv \dfrac{\omega_0}{\Delta\omega}$ in terms of β.
   [Note: Generally, it is the *power* that is measured (proportional to the square of the displacement), not the displacement itself. Thus $\Delta\omega$ as defined above corresponds to the full-width at half-maximum (FWHM) of the *power* response curve.]

5. Main, problem 3.5

6. FM radio stations have broadcast frequencies of approximately 100 MHz. Assume that your radio uses a series *LRC* circuit similar to the one you used in the lab as part of the receiver electronics. The quality factor *Q* of the receiver circuit determines the spacing of the broadcast frequencies of the stations your receiver pick up without interference from other stations. Estimate the spacing of the broadcast frequencies of FM stations if typical receivers have a *Q* of 500 or better. Explain your reasoning, and include a graph.