Why does math work?
A talk for the Society for Reason and Logic
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I am interested in a lot of things that most people don't care about, so usually I keep quiet about my passions – but to be invited to speak, that's a real treat.

I should explain my interests briefly. There seems to be a vague, murky borderland between science and philosophy. By "philosophy" I mean the sort of speculative philosophy that we usually call "metaphysics," and by "science," of course I mean physics. There is a similar borderland between science and theology and yet another between philosophy and theology. This is where I like to live. I suppose I'm a borderline personality or perhaps a swamp dweller. I like cloudy, boggy places between disciplines.

I notice that you call yourselves the Society for Reason and Logic. I'm afraid that I must disqualify myself so far as reason is concerned. I really don't know what reason is, and I suspect that no one else does either. I suppose we all have some vague ideas about it; for example, I know that I use reason all the time, whereas people who disagree with me do not. Unfortunately, I don't know how to make that more precise.

I do know a bit about logic, but that may only be because I regard it as a branch of mathematics, which I do know something about. (Just to give you a mathematician's eye view of logic, I am passing around a light introductory text on the subject.) Mathematics and logic both touch on our understanding of the physical world, but they do so in a way that is subtle and often misunderstood. Katie thought this would be a good topic, so here goes. I am going to start with Euclid, make a few inflammatory remarks about logic, knowledge, and mathematics, and finish with a profound question and two possible answers to it. The question, of course, is why does mathematics work?

First, what is mathematics? Math is different things to different people. For the engineers, it's a tool. When I am lecturing to physics students, it's a language for conveying ideas that cannot easily be expressed with spoken language. All of us use math to get numbers: to balance the checkbook, for example. All these are examples of math plus something else, and this talk is really about the borderland between the math and the "something else." We have to understand math without the something else before we get to the borderland.

What I am about to describe is technically called *formalism,* but we usually just call it "pure math." This is not intended as a value judgment. It's just the name of a viewpoint regarding math for its own sake. From this point of view, **math is a game played by moving symbols around on paper according to a definite set of rules**. Chess is a good analogy. The symbols are called king, queen, etc. The paper in this case is the chessboard, and the rules are excruciatingly precise. If you break the rules you are not necessarily a bad person, you're just not playing chess. The main difference is that chess is a competitive game between two people. Math at its best is a collaborative game played by a community of experts. Why play this game? Because it's fun. What is the object if not to win? To create a game of such elegance and originality, such *beauty,* that you and your colleagues want to write it down and play over it again and again.

There is a story that I half believe about Bobby Fisher, the legendary chess master of the 60's. They say he is still alive somewhere and that he plays chess anonymously over the Internet. Grandmasters who have played this shadowy figure say that the games are of such breathtaking beauty and power that there is no question in their minds regarding "who was that masked man?" on the other side of the board. The fact that they lost is of no consequence. This is the ideal of pure mathematics.

If you were to ask a practitioner of this kind of math what was "useful" about his work, he would reply, "nothing." What does it mean? Nothing. What does it prove about the real world? **Nothing**. If you were to ask the mathematician if his conclusions are true, he would reply that he did not break the rules.

I must emphasize that the key word here, both for chess and math, is *beauty,* but it a beauty that is only accessible to a few very talented people.

Now keep this in mind while I tell you about most the famous mathematician of all time, Euclid. We don't know much about the man himself. He lived in Greece about the time of Aristotle, i.e. around 300 b.c. He seemed to have collated the work of other geometers, but he produced such an impressive work, the *Elements of Geometry,* that the subject has survived in much the same form for the last 2000 years. It is taught in every high school and is the foundation of modern geometry and much of physics.

The *Elements* is impressive in part because of its logical structure and in part because of usefulness of its results. It starts with a few postulates that seem incontrovertible, like "it is possible to draw a straight line from any point to any point," and goes on to prove many remarkable theorems. The arguments seem to be "logical" in the mathematical sense. You can imagine formulating these arguments so that they had the form of a game played by moving symbols around on paper. Even in the original version, most of the proofs are in the form of "constructions," i.e. triangles and circles drawn on paper. People were so impressed with this that Euclidean geometry became throughout the Middle Ages the preeminent paradigm for the use of logic to arrive at truth and understanding. It strongly influenced science and mathematics and to some extent even theology. Saint Aquinas, as you know, devised five proofs of the existence of God (and one for his non-existence), and these proofs have the "feel" or logical structure of a Euclidean proof of some geometric proposition.

Here is a useful term from philosophy, "epistemology." It's that branch of philosophy that deals with knowledge. It tries to answer the question, what does it mean to know something? Euclidean geometry gives a partial answer: we know that a statement is true if we can prove it with an argument that uses the sort of logic that Euclid used to prove geometric propositions. Now here's my inflammatory statement: that answer is so subtly, powerfully, and seductively wrong that it became impossible to think clearly about epistemology for the next 2000 years. I will argue that we are just now coming out of the spell cast by this awful curse, and the person most responsible for our recovery is Einstein.

Euclidean geometry is misleading because it really is math-plus-something-else, but the something-else is completely invisible unless you know where to look for it. Let's start with that postulate; "it is possible to draw a straight line from any point to any point." First of all, what is a straight line? According to the definitions, "a straight line is a line which lies evenly with the points on itself." No one knows what Euclid meant by this, but Aristotle defines a straight line by appealing to our sight: if you look down the length of a straight line it looks like a point. Proclus, an early commentator (410-485 A.D.), defined a straight line as the shortest distance between two points. The vast majority of people who read Euclid through the centuries never gave this a second thought: of course everyone knows what a straight line is. There is another hidden assumption in the postulate: it implicitly assumes that the straight line between two points is unique. If you draw any other line between the points it will lie exactly over the first.

We can duck these questions by assuming that Euclidean geometry is pure math. The "straight line" is preeminently a symbol drawn on paper. Or course, the line drawn on paper is not perfectly straight, but it is a symbol to which we attach the words "straight line," and certain abstract attributes. This is fine so far as it goes, but remember, in pure math we are not allowed to ask if the conclusions are true, only if the game was played without breaking the rules. Believe me, Euclid did not break the rules. If we are to judge the truth of his propositions, we must first ask whether his postulates are true and what his definitions mean; but then we cross the borderland into math plus something else.

Let's start with Aristotle's definition of a straight line. In modern language we would say that the straight line is the path taken by light between two points. If that's the definition, then the postulate is wrong. Light passing close to massive objects can take several paths. Einstein predicted this, but it is *observed* with modern telescopes. It is a *fact.* If a straight line is the shortest distances between two points, then on the surface of the earth, the postulate is wrong again. There are an infinite number of paths between the North Pole and the South Pole. They are called lines of longitude.

To summarize: if Euclidean geometry is regarded as pure math (or pure logic) then it is correct in the sense that it is consistent with its own rules. If it is regarded as making statements about the real world, then it enters the borderland of math plus something else, and in this realm it is at least partly wrong. The reason it has been so seductive is that the postulates seem so natural that no one realized that they were in fact profound summaries of real-life experience and observation. Logic does not produce knowledge. It can only explore the consequences of what we have already observed. Einstein said it well: "As far as the laws of mathematics refer to reality they are not certain; and as far as they are certain, they do not refer to reality.

Now I have to confess that I don't entirely believe what I just said. At least I don't think it's the whole truth. Here's the puzzle: modern physics is based on some very advanced and esoteric math. Mechanics is based on partial differential equations, relativity on differential geometry, particle physics on Lie group theory, etc. When I lecture to graduate students, the lectures seem to be entirely about math, even though the course might be called something like quantum mechanics or electricity and magnetism. In most cases the math started out as pure math in the sense that I have used the term. Much later it was discovered that the math was custom made to apply to some emerging branch of physics. To give an extreme example – all modern theory regarding elementary particles is based on the work of an obscure 19-th century Norwegian mathematician named Sophus Lie. He thought he was moving symbols around on paper. We imagine that he saw the underlying structure of reality, as we currently understand it. What's going on here? Why does the universe pay any attention to our mathematics? Why does mathematics work?

I'll give you two common answers: both of them seem to me to be simplistic. The first is called *Platonism.* It holds that the objects of mathematics *exist* just as Plato claimed the eternal forms exist. We do not invent mathematics. We discover it. As Einstein put it, "God is a mathematician." If we were to make contact with alien beings from outer space, we could communicate with them using mathematics, because they would have discovered the same mathematics themselves. Mathematics is beautiful because eternal and perfect things are beautiful. Mathematics works because the universe is mathematical.

The most serious objection to this is that is impossible say in exactly what sense these things exist. Nowadays we think of Plato as a mystic (though he would not have used that term himself), and this assertion about the existence of mathematical objects has a decidedly mystical feel to it. Many of the mathematicians who espoused this point of view were outspoken mystics. We could probably put Einstein in that category as well as Kurt Godel about whom I will say more later.

The second answer reflects a kind of anti-mysticism, though it usually goes under the name of *conceptualism.* It holds that we invent mathematics. It is entirely a product of our own minds. It works because Darwinian evolution has endowed us with certain survival skills. We need to be able to judge distance and speed, recon cause and effect, understand the seasons, and so forth, or we would not survive. Our mathematics simply encodes and generalizes these skills. God is not a mathematician, but what He does can be described to some fair accuracy with mathematical models.

This point of view tends to be popular with sociologists. Physicists and mathematicians instinctively reject it. It is open to the obvious objection that there is no conceivable connection between modern mathematics, differential geometry for example, and our survival as early hunter-gatherers. You all know that quote from Hamlet: "The lady doth protest too much, methinks." Conceptualism seems to me to be so much a protest against mysticism in mathematics that it verges on silliness. At any rate, the assertion that all the order we perceive in the universe is simply projected onto a formless and chaotic void by our own minds is itself a sort of theological statement that can never be proved. Any order we perceive in the cosmos is automatically rejected as inadmissible evidence.

The truth probably lies somewhere in between these two extremes, but I tend to side with the mystics. In that connection, I would like to conclude with a fascinating observation. The formalist mathematicians suffered a heavy blow in the 30's when Kurt Godel proved that every system of logic and mathematics such as Euclidian geometry contains an infinite number of propositions, perhaps propositions we like and believe, which cannot be proved true or false. John Barrow pointed out that if you define religion as a system of thought that contains certain attractive propositions that cannot be proved but must be taken "on faith," then mathematics is the only major world religion that can actually prove that it is a religion!

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